A NOVEL RESULT ON ANALYSIS FOR TIME-VARYING DELAY SYSTEMS WITH NON-LINEAR PERTURBATIONS

NATTHARIKARN SOMPENG
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SOMKAMON RUENMOON

An Independent Study Submitted in Partial Fulfillment
of the Requirements for the degree of Bachelor
of Science Program in Mathematics
April 2018
University of Phayao

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Advisor and Dean of School of Science have considered the independent study entitled "A novel result on analysis for time-varying delay systems with non-linear perturbations "submitted in partial fulfillment of the requirements for the degree of Bachelor of science Program in Mathematics is hereby approved.

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Nattharikarn Sompeng
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ชื่อเรื่อง	ผลลัพธ์ใหม่ของการวิเคราะห์เสถียรภาพของระบบที่มีตัวหน่วงแปรผันตามเวลาและมีการรบกวนแบบไม่เชิงเส้น
ผู้ศึกษาค้นคว้า	นางสาวณัฐริกานต์ สมเป็ง นางสาวพัทธ์ธีรา กิจตาวงค์ นางสาวสมกมล เรือนมูล
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บทคัดย่อ

ในการศึกษาอิสระนี้ผู้จัยได้ศึกษาเกี่ยวกับปัญหาเสถียรภาพของระบบที่มีตัวหน่วงแปรผันตามเวลาภายใต้การรบกวนแบบไม่เชิงเส้น โดยการใช้วิธีทางไลปูนอฟคราซอฟกี้และใช้สูตรของไลบ์นิซ-นิวตัน รวมถึงใช้อสมการปริพันธ์เมทริกซ์อิสระ เป็นผลให้เราได้ผลลัพธ์ที่ดีขึ้น เนื่องจากวิธีการดังกล่าวทำให้เราได้ค่าประมาณของขอบเขตบนของบางพจน์ใกล้เคียงกับค่าจริง นอกจากนี้ในตอนท้ายของการศึกษาอิสระได้แสดงตัวอย่างเชิงตัวเลข เพื่อแสดงให้เห็นถึงประสิทธิภาพของวิธีการที่เราได้กล่าวไว้ข้างต้น

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ABSTRACT

In this project, we study linear time-varying delay system under nonlinear perturbations. By using new integral inequality approach, the relationship of Leibniz-Newton formula terms has been expressed within the framework of free-matrix-based integral inequality. Merits of the proposed results lie in lesser conservatism, which are realized by introducing appropriated Lyapunov-Krasovskii functionals and estimating the upper bound of some cross term more exactly. Numerical examples are given to illustrate effectiveness of the proposed method.

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CHAPTER 1
INTRODUCTION AND PRELIMINARIES

1.1 Introduction

Time delay is a natural phenomenon in real word. It is well known that time delay often causes the oscillation deterioration of system performance, and even instability, so the stability analysis of time-delay systems strongly requires before experimental stage. As these reason, the stability analysis of time-delay system have become a study research field during the past years.

In many applications, chemical or physical engineering system are governed by perturbations. Due to inaccuracy in model parameter measurements, data input, disturbance and any kind of unpredictability, such systems always involve uncertainties and perturbations. Generally speaking, these perturbations may cause the oscillation deterioration and give rise to instability of the system, such as linear and time-delay would do, even if the perturbations are tiny [2-6],[8-9],[11-14],[23].

In stability problem of time-delay system, to derive less conservative criteria guaranteeing the stability of the system is key purpose. The maximal allowable upper bound (MAUB) of time-delay is one of the important indexes to check conservatism of stability criteria in the system. Therefore, many researchers have tried to develop such conditions which ensure the stability for MAUB of time-delay as large as possible. In line with this, several remarkable approaches have been reported such as free-weighting matrix approaches [20] , delay partitioning approach, reciprocally convex approach, augmented Lyapunov method. [17]

Motivated by the above discussions, we shall derive new criteria for time-varying delay systems. The main contributions of our studies are the followings: (i) The time-delay functions are only required to be continuous but necessarily differentiable. (ii) By employing an improved integral inequality in [22], we derive less conservative for time-varying delay systems with non-linear perturbation.

In independent study is organized as follow : Section 1 presents definitions and some well-known technical propositions needed for the proof of the main results in Section

2. Time-varying delay systems with non-linear perturbation, given illustrative numerical examples are show in Section 3. Section 4 give the conclusion of paper.

1.2 Problem formulation and preliminaries

Definition 2.1 [17] The trivial solution $(x(t) = 0)$ of system (1) is said to be asymptotically stable (A.S.) if it is stable and $\|x(t)\| \to 0$ as $t \to \infty$.

Proposition 2.1 [18] Let E, H and F be any constant matrices with appropriate dimensions and $F^T F \leq I$. For any $\varepsilon > 0$, we have

$$EFH + H^T F^T E^T \leq \varepsilon E E^T + \varepsilon^{-1} H^T H.$$

Proposition 2.2 [1] (Schur complement lemma.) Given constant matrices X, Y, Z with appropriate dimensions satisfying$X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -Y & Z \\ Z^T & X \end{bmatrix} < 0.$$

In this project, we consider time-varying delay systems with non-linear perturbations that can be described by linear differential difference equation:

$$\dot{x}(t) = Ax(t) + Bx(t - h(t)) + Ff(x(t), t) + Gg(x(t - h(t)), t), t > 0 \quad (1)$$

$$x(t + \eta) = \phi(\eta), \forall \eta \in [-h, 0] \quad (2)$$

with $x(t) \in R^n$ as state vector of the system $A, B, F, G \in R^{n \times n}$ constant matrices $\phi(\bullet)$ continuous vector-valued initial function the time-varying delay $h(t)$ is satisfying

$$0 \leq h_a \leq h(t) \leq h_b \quad (3)$$

h_a, h_b being constants $f(x(t), t)$ and $g(x(t - h(t)), t)$ are unknown nonlinear perturbations with respect to $x(t)$ and $x(t - h(t))$, respectively assumed as

$$f^T(x(t), t) f(x(t), t) \leq \alpha^2 x^T(t) x(t) \quad (4)$$

$$g^T(x(t - h(t)), t) g(x(t - h(t)), t) \leq \beta^2 x^T(t - h(t)) x(t - h(t)) \quad (5)$$

where $\alpha \geq 0, \beta \geq 0$ are known scalars, F and G are known constant matrices, $\forall x \in R^n$.

The following lemma is useful for our main result :

Lemma 1. [20] Let x be a differentiable function : $[\alpha, \beta] \to R^n$. For symmetric matrices $R \in R^{n\times n}$ and $Z_1, Z_3 \in R^{3n\times 3n}$, and any matrices $Z_2 \in R^{3n\times 3n}$ and $N_1, N_2 \in R^{3n\times n}$ satisfying

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0, \tag{6}$$

the following inequality holds :

$$-\int_\alpha^\beta \dot{x}^T(s) R \dot{x}(s) ds \leq \varpi_1^T(\alpha, \beta) \Psi_1 \varpi_1(\alpha, \beta),$$

where

$$\varpi_1(\alpha, \beta) = \begin{bmatrix} x^T(\beta), & x^T(\alpha), & \frac{1}{\beta - \alpha} \int_\alpha^\beta x^T(s) ds \end{bmatrix}^T,$$
$$\Psi_1 = (\beta - \alpha)(Z_1 + \tfrac{1}{3} Z_3) + Sym\{N_1 \begin{bmatrix} I, & -I, & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I, & -I, & 2I \end{bmatrix}\}.$$

CHAPTER 2
MAIN RESULTS

2.1 Main Results

Theorem 1

The system (1) is asymptotically stable if there exist positive definite matrices, $P, Q_i, R_i,\ (i = 1,2,3)$, positive semi-definite matrices Z_1, Z_3 and any matrices N_1, N_2 such that the following LMIs hold :

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0, \tag{7}$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & 0 & \Xi_{110} & \Xi_{111} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 & 0 & 0 & 0 & \Xi_{210} & 0 \\ * & * & \Xi_{33} & \Xi_{34} & 0 & 0 & 0 & 0 & 0 & \Xi_{310} & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 & \Xi_{410} & 0 \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & 0 & \Xi_{59} & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & \Xi_{68} & \Xi_{69} & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Xi_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Xi_{1111} \end{bmatrix} < 0, \tag{8}$$

where

$$Z_1 = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}, \qquad Z_3 = \begin{bmatrix} \frac{1}{3}K_{11} & \frac{1}{3}K_{12} & \frac{1}{3}K_{13} \\ \frac{1}{3}K_{21} & \frac{1}{3}K_{22} & \frac{1}{3}K_{23} \\ \frac{1}{3}K_{31} & \frac{1}{3}K_{32} & \frac{1}{3}K_{33} \end{bmatrix},$$

$$N_1 = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \end{bmatrix} \begin{bmatrix} I & -I & 0 \end{bmatrix} = \begin{bmatrix} L_{11} & -L_{11} & 0 \\ L_{21} & -L_{21} & 0 \\ L_{31} & -L_{31} & 0 \end{bmatrix},$$

$$N_1^T = \begin{bmatrix} L_{11}^T & -L_{11}^T & L_{31}^T \\ -L_{21}^T & -L_{21}^T & -L_{31}^T \\ 0 & 0 & 0 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} M_{11} \\ M_{21} \\ M_{31} \end{bmatrix} \begin{bmatrix} -I & -I & 2I \end{bmatrix} = \begin{bmatrix} -M_{11} & -M_{11} & 2M_{11} \\ -M_{21} & -M_{21} & 2M_{21} \\ -M_{31} & -M_{31} & 2M_{31} \end{bmatrix},$$

$$N_2^T = \begin{bmatrix} -M_{11}^T & -M_{21}^T & -M_{31}^T \\ -M_{11}^T & -M_{21}^T & -M_{31}^T \\ 2M_{11}^T & 2M_{21}^T & 2M_{31}^T \end{bmatrix},$$

$$\begin{aligned}
\Xi_{11} &= (A^TP + PA) + (Q_1 + Q_2) + (2L_{11} + 2L_{11}^T - 2M_{11} - 2M_{11}^T + h_a(J_{11} + \tfrac{K_{11}}{3})) \\
&\quad + h_b(J_{11} + \tfrac{K_{11}}{3})) + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)A + \varepsilon_1\alpha^2 I - \tfrac{AW_1}{2} - \tfrac{W_1^TA^T}{2}, \\
\Xi_{12} &= PB + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \tfrac{AW_1}{2} + \tfrac{BW_1}{2}, \\
\Xi_{13} &= PF + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \tfrac{FW_1}{2}, \\
\Xi_{14} &= PG + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \tfrac{GW_1}{2}, \\
\Xi_{15} &= L_{21} - L_{11} - M_{11} - M_{21} + h_a(J_{12} + \tfrac{K_{12}}{3}), \\
\Xi_{16} &= L_{21} - L_{11} - M_{11} - M_{21} + h_b(J_{12} + \tfrac{K_{12}}{3}), \\
\Xi_{17} &= L_{31} + 2M_{11} - M_{31} + h_a(J_{13} + \tfrac{K_{13}}{3}),
\end{aligned}$$

$\Xi_{18} = L_{31} + 2M_{11} - M_{31} + h_b(J_{13} + \frac{K_{13}}{3}),$

$\Xi_{110} = \frac{AW_1}{2} - \frac{W_1}{2},$

$\Xi_{111} = T,$

$\Xi_{22} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \frac{\varepsilon_2}{2}\beta^2 I + I\frac{\varepsilon_2^T}{2}\beta^2 + I\varepsilon_3 + \frac{BW_1}{2} + \frac{BW_1^T}{2} - T,$

$\Xi_{23} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \frac{FW_1}{2},$

$\Xi_{24} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \frac{GW_1}{2},$

$\Xi_{210} = \frac{BW_1}{2} - \frac{W_1}{2},$

$\Xi_{33} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F - \varepsilon_1 I,$

$\Xi_{34} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G,$

$\Xi_{310} = \frac{FW_1}{2},$

$\Xi_{44} = G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G - \varepsilon_2 I,$

$\Xi_{410} = \frac{GW_1}{2},$

$\Xi_{55} = (Q_3 - Q_1) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_a(J_{22} + \frac{K_{22}}{3}) + L_{11} - M_{11}L_{11}^T - M_{11}^T$
$\quad + (h_b - h_a)(J_{11} + \frac{K_{11}}{3}),$

$\Xi_{56} = L_{21} - L_{11} - M_{11} - M_{21} + (h_b - h_a)(J_{12} + \frac{K_{12}}{3}),$

$\Xi_{57} = -L_{31} + 2M_{21} - M_{31} + h_a(J_{23} + \frac{K_{23}}{3}),$

$\Xi_{59} = L_{31} + 2M_{11} - M_{31} + (h_b - h_a)(J_{13} + \frac{K_{13}}{3}),$

$\Xi_{66} = (-Q_2 - Q_3) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_b(J_{22} + \frac{K_{22}}{3}) - L_{21} - M_{21} - L_{21}^T - M_{21}^T$
$\quad + (h_b - h_a)(J_{22} + \frac{K_{22}}{3}),$

$\Xi_{68} = -L_{31} + 2M_{21} - M_{31} + h_b(J_{23} + \frac{K_{23}}{3}),$

$\Xi_{69} = -L_{31} + 2M_{21} - M_{31} + (h_b - h_a)(J_{23} + \frac{K_{23}}{3}),$

$\Xi_{77} = 2M_{31} + 2M_{31}^T + h_a(J_{33} + \frac{K_{33}}{3}),$

$\Xi_{88} = 2M_{31} + 2M_{31}^T + h_b(J_{33} + \frac{K_{33}}{3}),$

$\Xi_{99} = 2M_{31} + 2M_{31}^T + (h_b - h_a)(J_{33} + \frac{K_{33}}{3}),$

$\Xi_{1010} = -\frac{W_1}{2} - \frac{W_1^T}{2},$

$\Xi_{1111} = -\frac{I}{\varepsilon_3}.$

Proof. We introduce a following Lyapunov-Krasovskii functional :

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{9}$$

where

$$\begin{aligned}
V_1(t) &= x^T(t)Px(t),\\
V_2(t) &= \int_{t-h_a}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_b}^{t} x^T(s)Q_2x(s)ds + \int_{t-h_b}^{t-h_a} x^T(s)Q_3x(s)ds,\\
V_3(t) &= \int_{-h_a}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{-h_b}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta\\
&\quad + \int_{-h_b}^{-h_a}\int_{t+\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\theta.
\end{aligned}$$

From
$$\begin{aligned}
V(t,x_t) &= x^T(t)Px(t) + \int_{t-h_a}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_b}^{t} x^T(s)Q_2x(s)\\
&\quad + \int_{t-h_b}^{t-h_a} x^T(s)Q_3x(s) + \int_{-h_a}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta\\
&\quad + \int_{-h_b}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta + \int_{-h_b}^{-h_a}\int_{t+\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\theta
\end{aligned}$$
consider from deffination of $V_{11} = \lambda_{min}(P)\|x_t\|^2 \leq x^T(t)Px(t) \leq V(t,x_t)$,

$$\begin{aligned}
V_{22} &\leq \int_{t-h_a}^{t} x^T(s)(Q_1)\|x(t)\|^2ds\\
&= \lambda_{max}(Q_1)\|x(t)\|^2\int_{t-h_a}^{t} 1ds\\
&= \lambda_{max}(Q_1)\|x(t)\|^2(h_a)\\
&\leq \lambda_{max}(Q_1)(h_a)\|x(t)\|^2,\\
V_{33} &\leq \int_{t-h_b}^{t} x^T(s)(Q_2)\|x(t)\|^2ds\\
&= \lambda_{max}(Q_2)\|x(t)\|^2\int_{t-h_b}^{t} 1ds\\
&= \lambda_{max}(Q_2)\|x(t)\|^2(h_b)\\
&\leq \lambda_{max}(Q_2)(h_b)\|x(t)\|^2,\\
V_{44} &\leq \int_{t-h_b}^{t-h_a} x^T(s)(Q_3)\|x(t)\|^2ds\\
&= \lambda_{max}(Q_3)\|x(t)\|^2\int_{t-h_b}^{t-h_a} 1ds\\
&= \lambda_{max}(Q_3)\|x(t)\|^2(h_b-h_a)\\
&\leq \lambda_{max}(Q_3)(h_b-h_a)\|x(t)\|^2,
\end{aligned}$$

$$
\begin{aligned}
V_{55} \quad &\leq \int_{-h_a}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
&\leq \int_{-h_a}^{0}\int_{t-h_a}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta \\
&= \int_{t-h_a}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \int_{-h_a}^{0} 1ds \\
&= \int_{t-h_a}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds(h_a) \\
&\leq (h_a)\int_{t-h_a}^{t} \lambda_{max}R_1\|\dot{x}(t)\|^2dt \\
&= (h_a)\lambda_{max}R_1\|\dot{x}(t)\|^2\int_{t-h_a}^{t} 1dt \\
&= (h_a)^2\lambda_{max}R_1\|\dot{x}(t)\|^2,
\end{aligned}
$$

consider

$$
\begin{aligned}
\|\dot{x}(t)\|^2 &= \|Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t)\|^2 \\
&\leq [\|A\|\|x(t)\| + \|B\|\|x(t-h(t))\| + \|F\|\|f(x(t),t)\| \\
&+\|G\|\|g(x(t-h(t)),t)\|] \cdot [\|A\|\|x(t)\| + \|B\|\|x(t-h(t))\| \\
&+\|F\|\|f(x(t),t)\| + \|G\|\|g(x(t-h(t)),t)\|] \\
&= \|A\|^2\|x(t)\|^2 + \|A\|\|x(t)\| \cdot \|B\|\|x(t-h(t))\| \\
&\ +\|A\|\|x(t)\| \cdot \|F\|\|f(x(t),t)\| \\
&\ +\|A\|\|x(t)\| \cdot \|G\|\|g(x(t-h(t)),t)\| \\
&\ +\|B\|\|x(t-h(t))\| \cdot \|A\|\|x(t)\| + \|B\|^2\|x(t-h(t))\|^2 \\
&\ +\|B\|\|x(t-h(t))\| \cdot \|F\|\|f(x(t),t)\| \\
&\ +\|B\|\|x(t-h(t))\| \cdot \|G\|\|g(x(t-h(t)),t)\| \\
&\ +\|F\|\|f(x(t),t)\| \cdot \|A\|\|x(t)\| \\
&\ +\|F\|\|f(x(t),t)\| \cdot \|B\|\|x(t-h(t))\| + \|F\|^2\|f(x(t),t)\|^2 \\
&\ +\|F\|\|f(x(t),t)\| \cdot \|G\|\|g(x(t-h(t)),t)\| \\
&\ +\|G\|\|g(x(t-h(t)),t)\| \cdot \|A\|\|x(t)\| \\
&\ +\|G\|\|g(x(t-h(t)),t)\| \cdot \|B\|\|x(t-h(t))\| \\
&\ +\|G\|\|g(x(t-h(t)),t)\| \cdot \|F\|\|f(x(t),t)\| \\
&\ +\|G\|^2\|g(x(t-h(t)),t)\|^2 \\
&\leq \|A\|^2\|x_t\|^2 + \|A\|\|x_t\| \cdot \|B\|\|x_t\| + \|A\|\|x_t\| \cdot \|F\|\|f_t\| \\
&+\|A\|\|x_t\| \cdot \|G\|\|g_t\| + \|B\|\|x_t\| \cdot \|A\|\|x_t\| + \|B\|^2\|x_t\|^2 \\
&+\|B\|\|x_t\| \cdot \|F\|\|f_t\| + \|B\|\|x_t\| \cdot \|G\|\|g_t\|
\end{aligned}
$$

$$
\begin{aligned}
&+\|F\|\|f_t\|\cdot\|A\|\|x_t\ | + \|F\|\|f_t\|\cdot\|B\|\|x_t\| + \|F\|^2\|f_t\|^2 \\
&+\|F\|\|f_t\|\cdot\|G\|\|g_t\| + \|G\|\|g_t\|\cdot\|A\|\|x_t\| \\
&+\|G\|\|g_t\|\cdot\|B\|\|x_t\| + \|G\|\|g_t\|\cdot\|F\|\|f_t\| + \|G\|^2\|g_t\|^2 \\
&= \|A\|^2\|x_t\|^2 + 2(\|A\|\|x_t\|\cdot\|B\|\|x_t\|) + 2(\|A\|\|x_t\|\cdot\|F\|\|f_t\|) \\
&+2(\|A\|\|x_t\|\cdot\|G\|\|g_t\|) + \|B\|^2\|x_t\|^2 + 2(\|B\|\|x_t\|\cdot\|F\|\|f_t\|) \\
&+2(\|B\|\|x_t\|\cdot\|G\|\|g_t\|) + \|F\|^2\|f_t\|^2 + 2(\|F\|\|f_t\|\cdot\|G\|\|g_t\|) \\
&+\|G\|^2\|g_t\|^2 \\
&\leq \|A\|^2\|x_t\|^2 + 2(\|A\|\|x_t\|\cdot\|B\|\|x_t\|) + 2(\|A\|\|x_t\|\cdot\|F\|\alpha\|x_t\|) \\
&+2(\|A\|\|x_t\|\cdot\|G\|\beta\|x_t\|) + \|B\|^2\|x_t\|^2 + 2(\|B\|\|x_t\|\cdot\|F\|\alpha\|x_t\|) \\
&+2(\|B\|\|x_t\|\cdot\|G\|\beta\|x_t\|) + \|F\|^2\alpha^2\|x_t\|^2 \\
&+2(\|F\|\alpha\|x_t\|\cdot\|G\|\beta\|x_t\|) + \|G\|^2\alpha^2\|x_t\|^2 \\
&= \|A\|^2\|x_t\|^2 + 2\|x_t\|^2(\|A\|\|B\|) + 2\|x_t\|^2(\alpha\|A\|\|F\|)+ \\
&2\|x_t\|^2(\beta\|A\|\|G\|) + \|B\|^2\|x_t\|^2 + 2\|x_t\|^2(\alpha\|B\|\|F\|) \\
&+2\|x_t\|^2(\beta\|B\|\|G\|) + \|F\|^2\alpha^2\|x_t\|^2 + 2\|X_t\|^2(\alpha\beta\|F\|\|G\|) \\
&+\|G\|^2\beta^2\|x_t\|^2 \\
&\leq \|x_t\|^2[\|A\|^2 + 2(\|A\|\|B\|) + 2\alpha(\|A\|\|F\|) + 2\beta(\|A\|\|G\|) \\
&+\|B\|^2 2\alpha(\|B\|\|F\|) + 2\beta(\|B\|\|G\|) + \alpha^2\|F\|^2 \\
&+2\alpha\beta(\|F\|\|G\|)+\beta^2\|G\|^2. \qquad (9.1)
\end{aligned}
$$

From (9.1) we obtain

$$
\begin{aligned}
V_{55} &= (h_a)^2\lambda_{max}(R_1)[\|A\|^2+2(\|A\|\|B\|)+2\alpha(\|A\|\|F\|)+2\beta(\|A\|\|G\|) \\
&+\|B\|^2+2\alpha(\|B\|\|F\|)+2\beta(\|B\|\|G\|)+\alpha^2\|F\|^2+2\alpha\beta(\|F\|\|G\|) \\
&+\beta^2\|G\|^2]\|x_t\|^2.
\end{aligned}
$$

Similarly, we obtain

$$
\begin{aligned}
V_{66} &\leq \int_{-h_b}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta \\
&\leq \int_{-h_b}^{0}\int_{t-h_b}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta \\
&= \int_{t-h_b}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds \int_{-h_b}^{0} 1d\theta \\
&= \int_{t-h_b}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds(h_b)
\end{aligned}
$$

$$
\begin{aligned}
& \leq (h_b) \int_{t-h_b}^{t} \lambda_{max}(R_2) \|\dot{x}(t)\|^2 dt \\
& = (h_b) \lambda_{max}(R_2) \|\dot{x}(t)\|^2 \int_{t-h_b}^{t} 1 dt \\
& = (h_b)^2 \lambda_{max}(R_2) \|\dot{x}(t)\|^2, \\
V_{66} & = (h_b)^2 \lambda_{max}(R_2)[\|A\|^2 + 2(\|A\|\|B\|) + 2\alpha(\|A\|\|F\|) + 2\beta(\|A\|\|G\|) \\
& + \|B\|^2 + 2\alpha(\|B\|\|F\|) + 2\beta(\|B\|\|G\|) + \alpha^2\|F\|^2 + 2\alpha\beta(\|F\|\|G\|) \\
& + \beta^2\|G\|^2]\|X_t\|^2, \\
V_{77} & \leq \int_{-h_b}^{-h_a} \int_{t+\theta}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds d\theta \\
& \leq \int_{-h_b}^{-h_a} \int_{t-h_b}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds d\theta \\
& = \int_{t-h_b}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds \int_{-h_b}^{-h_[a]} 1 d\theta \\
& = \int_{t-h_b}^{t} \dot{x}^T(s) R_3 \dot{x}(s) ds (h_b - h_a) \\
& \leq (h_b - h_a) \int_{t-h_b}^{t} \lambda_{max}(R_3) \|\dot{x}(t)\|^2 dt \\
& = (h_b - h_a) \lambda_{max}(R_3) \|\dot{X}(t)\|^2 \int_{t-h_b}^{t} 1 dt \\
& = (h_b - h_a)(h_b) \lambda_{max}(R_3) \|\dot{x}(t)\|^2, \\
V_{77} & = (h_b - h_a)(h_b)^2 \lambda_{max}(R_3)[\|A\|^2 + 2(\|A\|\|B\|) \\
& + 2\alpha(\|A\|\|F\|) + 2\beta(\|A\|\|G\|) + \|B\|^2 + 2\alpha(\|B\|\|F\|) + 2\beta(\|B\|\|G\|) \\
& + \alpha^2\|F\|^2 + 2\alpha\beta(\|F\|\|G\|) + \beta^2\|G\|^2]\|x_t\|^2.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
V_{11} & \leq \lambda_{min}(P) \\
V_{22} & \leq (h_a) \lambda_{max}(Q_1) \\
V_{33} & \leq (h_b) \lambda_{max}(Q_2) \\
V_{44} & \leq (h_b - h_a) \lambda_{max}(Q_3) \\
V_{55} & \leq (h_a)^2 \lambda_{max}(R_1)[\|A\|^2 + 2(\|A\|\|B\|) + 2\alpha(\|A\|\|F\|) + 2\beta(\|A\|\|G\|) \\
& + \|B\|^2 + 2\alpha(\|B\|\|F\|) + 2\beta(\|B\|\|G\|) + \alpha^2\|F\|^2 + 2\alpha\beta(\|F\|\|G\|) \\
& + \beta^2\|G\|^2]\|x_t\|^2 \\
V_{66} & \leq (h_b)^2 \lambda_{max}(R_2)[\|A\|^2 + 2(\|A\|\|B\|) + 2\alpha(\|A\|\|F\|) + 2\beta(\|A\|\|G\|) \\
& + \|B\|^2 + 2\alpha(\|B\|\|F\|) + 2\beta(\|B\|\|G\|) + \alpha^2\|F\|^2 + 2\alpha\beta(\|F\|\|G\|) \\
& + \beta^2\|G\|^2]\|x_t\|^2. \\
V_{77} & \leq (h_b - h_a)(h_b)^2 \lambda_{max}(R_3)[\|A\|^2 + 2(\|A\|\|B\|) \\
& + 2\alpha(\|A\|\|F\|) + 2\beta(\|A\|\|G\|)
\end{aligned}
$$

$+\|B\|^2 + 2\alpha(\|B\|\|F\|) + 2\beta(\|B\|\|G\|) + \alpha^2\|F\|^2 + 2\alpha\beta(\|F\|\|G\|)$
$+\beta^2\|G\|^2]\|x_t\|^2.$

We conclude that $\quad V(t) \quad \leq \lambda_1\|x_t\|^2.$

By taking derivative of $V(t)$ for $t \in [0,\infty]$ along the trajectory solution of (1) yields

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t). \tag{10}$$

From (1) and (9), we obtain

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\
&= x^T(t)P\dot{x}(t) + x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P[Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t)] \\
&= [x^T(t)2PAx(t)] + [x^T(t)2PBx(t-h(t))] + [x^T(t)2PFf(x(t),t)] \\
&\quad +[x^T(t)2PGg(x(t-h(t)),t)] \\
&= [x^T(t)(PA+PA)x(t)] + [x^T(t)(PB+PB)x(t-h(t))] \\
&\quad +[x^T(t)(PF+PF)f(x(t),t)] + [x^T(t)(PG+PG)g(x(t-h(t)),t)] \\
&= [(x^T(t)(PA)x(t))^T + (x^T(t)(PA)x(t))] \\
&\quad +[(x^T(t)(PB)x(t-h(t)))^T + (x^T(t)(PB)x(t-h(t)))] \\
&\quad +[(x^T(t)(PF)f(x(t),t))^T + (x^T(t)(PF)f(x(t),t))] \\
&\quad +[(x^T(t)(PG)g(x(t-h(t)),t))^T + (x^T(t)(PG)g(x(t-h(t)),t))] \\
&= [(x^T(t)(A^TP)x(t)) + (x^T(t)(PA)x(t))] \\
&\quad +[(x^T(t)(B^TP)x(t-h(t))) + (x^T(t)(PB)x(t-h(t)))] \\
&\quad +[(x^T(t)(F^TP)f(x(t),t)) + (x^T(t)(PF)f(x(t),t))] \\
&\quad +[(x^T(t)(G^TP)g(x(t-h(t)),t)) + (x^T(t)(PG)g(x(t-h(t)),t))] \\
&= [(x^T(t)(A^TP+PA)x(t))] + x^T(t)PBx(t-h(t)) + x^T(t)PFf(x(t),t) \\
&\quad +x^T(t)PGg(x(t-h(t)),t) + x^T(t-h(t))B^TPx(t) + f^T(x(t),t)F^TPx(t) \\
&\quad +g^T(x(t-h(t)),t)G^TPx(t).
\end{aligned} \tag{11}$$

From (9), we have

$$\begin{aligned}\dot{V}_2(t) &= [x^T(t)Q_1x(t) - x^T(t-h_a)Q_1x(t-h_a)] \\ &\quad +[x^T(t)Q_2x(t) - x^T(t-h_b)Q_2x(t-h_b)] \\ &\quad +[x^T(t-h_a)Q_3x(t-h_a) - x^T(t-h_b)Q_3x(t-h_b)] \\ &= x^T(t)(Q_1+Q_2)x(t) + x^T(t-h_a)(Q_3-Q_1)x(t-h_a) \\ &\quad +x^T(t-h_b)(-Q_2-Q_3)x(t-h_b). \end{aligned} \tag{12}$$

From (9),we obtain

$$\begin{aligned}\dot{V}_3(t) &= [h_a(\dot{x}^T(t)R_1\dot{x}(t)) - \dot{x}^T(t)R_1\dot{x}(t) + \dot{x}^T(t-h_a)R_1\dot{x}(t-h_a)] \\ &\quad +[h_b(\dot{x}^T(t)R_2\dot{x}(t)) - \dot{x}^T(t)R_2\dot{x}(t) + \dot{x}^T(t-h_b)R_2\dot{x}(t-h_b)] \\ &\quad +[(h_b-h_a)(\dot{x}^T(t)R_3\dot{x}(t)) - \dot{x}^T(t-h_a)R_3\dot{x}(t-h_a) \\ &\quad +\dot{x}^T(t-h_b)R_3\dot{x}(t-h_b)] \\ &= \dot{x}^T(t)(h_aR_1 + h_bR_2 + (h_b-h_a)R_3)\dot{x}(t) - \int_{t-h_a}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \\ &\quad - \int_{t-h_b}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-h_b}^{t-h_a} \dot{x}^T(s)R_3\dot{x}(s)ds. \end{aligned} \tag{13}$$

From $\Psi_1 = (\beta-\alpha)(Z_1 + \frac{1}{3}Z_3) + Sym\{N_1 \begin{bmatrix} I & -I & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I & -I & 2I \end{bmatrix}\}$, we have

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix},$$

where

$$\Omega_{11} = L_{11} + L_{11}^T + M_{11} + M_{11}^T + (\beta - \alpha)(J_{11} + \tfrac{1}{3}K_{11}),$$
$$\Omega_{12} = L_{21}^T - L_{11} - M_{11} - M_{21}^T + (\beta - \alpha)(J_{12} + \tfrac{1}{3}K_{12}),$$
$$\Omega_{13} = L_{31}^T + 2M_{11} - M_{31}^T + (\beta - \alpha)(J_{13} + \tfrac{1}{3}K_{13}),$$
$$\Omega_{21} = L_{21} - L_{11}^T - M_{21} - M_{11}^T + (\beta - \alpha)(J_{21} + \tfrac{1}{3}K_{21}),$$
$$\Omega_{22} = -L_{21} - L_{21}^T - M_{21} - M_{21}^T + (\beta - \alpha)(J_{22} + \tfrac{1}{3}K_{22}),$$
$$\Omega_{23} = -L_{31}^T + 2M_{21} - M_{31}^T + (\beta - \alpha)(J_{23} + \tfrac{1}{3}K_{23}),$$
$$\Omega_{31} = L_{31} + 2M_{11}^T - M_{31} + (\beta - \alpha)(J_{31} + \tfrac{1}{3}K_{31}),$$
$$\Omega_{32} = -L_{31} - M_{31} + 2M_{21}^T + (\beta - \alpha)(J_{32} + \tfrac{1}{3}K_{32}),$$
$$\Omega_{33} = 2M_{31} + 2M_{31}^T + (\beta - \alpha)(J_{33} + \tfrac{1}{3}K_{33}).$$

From **Lemma 1.** and we let

$$\Psi_{11} = (h_a)(Z_1 + \tfrac{1}{3}Z_3) + Sym\{N_1 \begin{bmatrix} I & -I & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I & -I & 2I \end{bmatrix}\},$$

we have $-\int_{t-h_a}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds$

$$\leq \begin{bmatrix} x^T(t) & x^T(t-h_a) & \frac{1}{h_a}\int_{t-h_a}^{t} x^T(s)ds \end{bmatrix} \Psi_{11} \begin{bmatrix} x(t) \\ x(t-h_a) \\ \frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds \end{bmatrix}$$

$$\begin{aligned}
&= x^T(t)(L_{11} + L_{11}^T + M_{11} + M_{11}^T + (h_a)(J_{11} + \tfrac{1}{3}K_{11}))x(t) \\
&+ x^T(t-h_a)(L_{21} - L_{11}^T - M_{21} - M_{11}^T + (h_a)(J_{21} + \tfrac{1}{3}K_{21}))x(t) \\
&+ \frac{1}{h_a}\int_{t-h_a}^{t} x^T(s)ds(L_{31} + 2M_{11}^T - M_{31} + (h_a)(J_{31} + \frac{1}{3}K_{31}))x(t) \\
&+ x^T(t)(L_{21}^T - L_{11} - M_{11} - M_{21}^T + (h_a)(J_{12} + \tfrac{1}{3}K_{12}))x(t-h_a) \\
&+ x^T(t-h_a)(-L_{21} - L_{21}^T - M_{21} - M_{21}^T + (h_a)(J_{22} + \tfrac{1}{3}K_{22}))x(t-h_a) \\
&+ \frac{1}{h_a}\int_{(}t-h_a)^t x^T(s)ds(-L_{31} - M_{31} + 2M_{21}^T + (h_a)(J_{32} + \frac{1}{3}K_{32}))x(t-h_a) \\
&+ x^T(t)(L_{31}^T + 2M_{11} - M_{31}^T + (h_a)(J_{13} + \frac{1}{3}K_{13}))\frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds \\
&+ x^T(t-h_a)(-L_{31}^T + 2M_{21} - M_{31}^T + (h_a)(J_{23} + \frac{1}{3}K_{23}))\frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds \\
&+ \frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds(2M_{31} + 2M_{31}^T + (h_a)(J_{33} + \frac{1}{3}K_{33}))\frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds.
\end{aligned} \tag{14}$$

By using the same approach as in (14), we obtain $-\int_{t-h_b}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds$

$$\le \begin{bmatrix} x^T(t) & x^T(t-h_b) & \frac{1}{h_b}\int_{t-h_b}^{t} x^T(s)ds \end{bmatrix} \Psi_{12} \begin{bmatrix} x(t) \\ x(t-h_b) \\ \frac{1}{h_b}\int_{t-h_b}^{t} x(s)ds \end{bmatrix}$$

$$\begin{aligned}
&= x^T(t)(L_{11} + L_{11}^T + M_{11} + M_{11}^T + (h_b)(J_{11} + \tfrac{1}{3}K_{11}))x(t) \\
&+x^T(t-h_b)(L_{21} - L_{11}^T - M_{21} - M_{11}^T + (h_b)(J_{21} + \tfrac{1}{3}K_{21}))x(t) \\
&+\frac{1}{h_b}\int_{t-h_b}^{t} x^T(s)ds(L_{31} + 2M_{11}^T - M_{31} + (h_b)(J_{31} + \frac{1}{3}K_{31}))x(t) \\
&+x^T(t)(L_{21}^T - L_{11} - M_{11} - M_{21}^T + (h_b)(J_{12} + \tfrac{1}{3}K_{12}))x(t-h_b) \\
&+x^T(t-h_b)(-L_{21} - L_{21}^T - M_{21} - M_{21}^T + (h_b)(J_{22} + \tfrac{1}{3}K_{22}))x(t-h_b) \\
&+\frac{1}{h_b}\int_{(} t-h_b)^t x^T(s)ds(-L_{31} - M_{31} + 2M_{21}^T + (h_b)(J_{32} + \frac{1}{3}K_{32}))x(t-h_b) \\
&+x^T(t)(L_{31}^T + 2M_{11} - M_{31}^T + (h_b)(J_{13} + \frac{1}{3}K_{13}))\frac{1}{h_b}\int_{t-h_b}^{t} x(s)ds \\
&+x^T(t-h_b)(-L_{31}^T + 2M_{21} - M_{31}^T + (h_b)(J_{23} + \frac{1}{3}K_{23}))\frac{1}{h_b}\int_{t-h_b}^{t} x(s)ds \\
&+\frac{1}{h_b}\int_{t-h_b}^{t} x(s)ds(2M_{31}+2M_{31}^T+(h_b)(J_{33}+\frac{1}{3}K_{33}))\frac{1}{h_b}\int_{t-h_b}^{t} x(s)ds, \qquad (15)
\end{aligned}$$

where $\Psi_{12} = (h_b)(Z_1 + \frac{1}{3}Z_3) + Sym\{N_1 \begin{bmatrix} I & -I & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I & -I & 2I \end{bmatrix}\}$.

Similarly, we have $-\int_{t-h_b}^{t-h_a} \dot{x}^T(s)R_3\dot{x}(s)ds$

$$\le \begin{bmatrix} x^T(t-h_a) & x^T(t-h_b) & \frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x^T(s)ds \end{bmatrix} \Psi_{13} \begin{bmatrix} x(t-h_a) \\ x(t-h_b) \\ \frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x(s)ds \end{bmatrix}$$

$$\begin{aligned}
&= x^T(t-h_a)(L_{11} + L_{11}^T + M_{11} + M_{11}^T + (h_b-h_a)(J_{11} + \tfrac{1}{3}K_{11}))x(t-h_a) \\
&+x^T(t-h_b)(L_{21} - L_{11}^T - M_{21} - M_{11}^T + (h_b-h_a)(J_{21} + \tfrac{1}{3}K_{21}))x(t-h_a) \\
&+\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x^T(s)ds(L_{31} + 2M_{11}^T - M_{31} + (h_b-h_a)(J_{31} + \frac{1}{3}K_{31}))x(t-h_a) \\
&+x^T(t-h_a)(L_{21}^T - L_{11} - M_{11} - M_{21}^T + (h_b-h_a)(J_{12} + \tfrac{1}{3}K_{12}))x(t-h_b) \\
&+x^T(t-h_b)(-L_{21}-L_{21}^T-M_{21}-M_{21}^T+(h_b-h_a)(J_{22}+\tfrac{1}{3}K_{22}))x(t-h_b) \qquad (16)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x^T(s)ds(-L_{31}-M_{31}+2M_{21}^T+(h_b-h_a)(J_{32}+\frac{1}{3}K_{32}))x(t-h_b)\\
&+x^T(t-h_a)(L_{31}^T+2M_{11}-M_{31}^T+(h_b-h_a)(J_{13}+\frac{1}{3}K_{13}))\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x(s)ds\\
&+x^T(t-h_b)(-L_{31}^T+2M_{21}-M_{31}^T+(h_b-h_a)(J_{23}+\frac{1}{3}K_{23}))\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x(s)ds\\
&+\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x(s)ds(2M_{31}+2M_{31}^T+(h_b-h_a)(J_{33}+\frac{1}{3}K_{33}))\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x(s)ds,
\end{aligned}
$$

where $\Psi_{13} = (h_b)(Z_1 + \frac{1}{3}Z_3) + Sym\{N_1 \begin{bmatrix} I & -I & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I & -I & 2I \end{bmatrix}\}$.

We may express the term $\dot{x}^T(t)(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)\dot{x}(t)$ as follows :

$$
\begin{aligned}
&\dot{x}^T(t)(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)\dot{x}(t)\\
&\quad= [Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t)]^T\\
&\qquad (h_aR_1 + h_bR_2 + (h_b - h_a)R_3)\dot{x}(t)[Ax(t) + Bx(t-h(t)) + Ff(x(t),t)\\
&\qquad +Gg(x(t-h(t)),t)]\\
&\quad= x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t)\\
&\qquad +x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t))\\
&\qquad +x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t)\\
&\qquad +x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t)\\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t)\\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t))\\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t)\\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t)\\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t)\\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t))\\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t) \qquad (17)\\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t)\\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t)\\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t))\\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t)\\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t)
\end{aligned}
$$

Note that for any $\varepsilon_1 \geq 0, \varepsilon_2 \geq 0$, it follows from (4) and (5) that

$$\varepsilon_1[\alpha^2 x^T(t)x(t) - f^T(x(t),t)f(x(t),t)] \geq 0 \tag{18}$$

and

$$\varepsilon_2[\beta^2 x^T(t-h(t))x(t-h(t)) - g^T(x(t-h(t)),t)g(x(t-h(t)),t)] \geq 0. \tag{19}$$

By using the following identity relation

$$\dot{x}(t) - (Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t)) = 0,$$

we have

$[x^T(t) + x^T(t-h(t)) + \dot{x}^T(t)](-W_1)[\dot{x}(t) - (Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t))] = 0$, where W_1 is a positive definite matrix.

It follows that

$$\begin{aligned}
&[x^T(t) + x^T(t-h(t)) + \dot{x}^T(t)](-W_1)[\dot{x}(t) - Ax(t) - Bx(t-h(t)) \\
&-Ff(x(t),t) - Gg(x(t-h(t)),t)] \\
&= x^T(t)(-W_1)(\dot{x}(t)) + x^T(t)(-W_1)(-Ax(t)) \\
&+x^T(t)(-W_1)(-Bx(t-h(t))) + x^T(t)(-W_1) - (Ff(x(t),t)) \\
&+x^T(t)(-W_1) - (Gg(x(t-h(t)),t)) \\
&+x^T(t-h(t))(-W_1)(\dot{x}(t)) + x^T(t-h(t))(-W_1)(-Ax(t)) \\
&+x^T(t-h(t))(-W_1)(-Bx(t-h(t))) + x^T(t-h(t))(-W_1) - (Ff(x(t),t)) \\
&+x^T(t-h(t))(-W_1) - (Gg(x(t-h(t)),t)) \\
&+\dot{x}^T(t)(-W_1)(\dot{x}(t)) + x^T(t-h(t))(-W_1)(-Ax(t)) \\
&+\dot{x}^T(t)(-W_1)(-Bx(t-h(t))) + x^T(t-h(t))(-W_1) - (Ff(x(t),t)) \\
&+\dot{x}^T(t)(-W_1) - (Gg(x(t-h(t)),t)) = 0.
\end{aligned} \tag{20}$$

By using the following identity relation

$$x^T(t-h(t))Tx(t-h(t)) - x^T(t-h(t))Tx(t-h(t)) = 0, T > 0. \tag{21}$$

From **Proposition 2.1**, we obtain

$$x^T(t-h(t))x(t-h(t)) \leq (\tfrac{1}{\varepsilon})T^T x^T(t-h(t))Tx(t-h(t)) \\ +\varepsilon x(t-h(t))x^T(t-h(t)), \qquad (22)$$

Combine (10) and (14)-(22),

$$\dot{V}(t) \leq \xi^T(t)\overline{\Xi}\xi(t)\ , \qquad (23)$$

where

$$\xi^T(t) = [\ x^T(t), x^T(t-h(t))\ ,\ f^T(x(t),t)\ ,\ g^T(x(t-h(t)),t)\ ,\ x^T(t-h_a)\ ,\\ x^T(t-h_b)\ ,\ \frac{1}{h_a}\int_{t-h_a}^{t} x^T(s)ds\ ,\ \frac{1}{h_b}\int_{t-h_b}^{t} x^T(s)ds\ ,\ \frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a} x^T(s)ds\ ,\\ \dot{x}^T(t)\ ],$$

$$\overline{\Xi} = \begin{bmatrix} \overline{\Xi}_{11} & \overline{\Xi}_{12} & \overline{\Xi}_{13} & \overline{\Xi}_{14} & \overline{\Xi}_{15} & \overline{\Xi}_{16} & \overline{\Xi}_{17} & \overline{\Xi}_{18} & 0 & \overline{\Xi}_{110} \\ * & \overline{\Xi}_{22} & \overline{\Xi}_{23} & \overline{\Xi}_{24} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}_{210} \\ * & * & \overline{\Xi}_{33} & \overline{\Xi}_{34} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}_{310} \\ * & * & * & \overline{\Xi}_{44} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}_{410} \\ * & * & * & * & \overline{\Xi}_{55} & \overline{\Xi}_{56} & \overline{\Xi}_{57} & 0 & \overline{\Xi}_{59} & 0 \\ * & * & * & * & * & \overline{\Xi}_{66} & 0 & \overline{\Xi}_{68} & \overline{\Xi}_{69} & 0 \\ * & * & * & * & * & * & \overline{\Xi}_{77} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \overline{\Xi}_{88} & 0 & 0 \\ * & * & * & * & * & * & * & * & \overline{\Xi}_{99} & 0 \\ * & * & * & * & * & * & * & * & * & \overline{\Xi}_{1010} \end{bmatrix} < 0,$$

$$\overline{\Xi}_{11} = (A^TP+PA)+(Q_1+Q_2)+(2L_{11}+2L_{11}^T-2M_{11}-2M_{11}^T+h_a(J_{11}+\tfrac{K_{11}}{3})) \\ +h_b(J_{11}+\tfrac{K_{11}}{3}))+A^T(h_aR_1+h_bR_2+(h_b-h_a)R_3)A+\varepsilon_1\alpha^2I-\tfrac{AW_1}{2}-\tfrac{W_1^TA^T}{2},$$

$$\overline{\Xi}_{12} = PB + A^T(h_aR_1 + h_bR_2 + (h_b-h_a)R_3)B + \tfrac{AW_1}{2} + \tfrac{BW_1}{2},$$

$$\overline{\Xi}_{13} = PF + A^T(h_aR_1 + h_bR_2 + (h_b-h_a)R_3)F + \tfrac{FW_1}{2},$$

$$\overline{\Xi}_{14} = PG + A^T(h_aR_1 + h_bR_2 + (h_b-h_a)R_3)G + \tfrac{GW_1}{2},$$

$$\overline{\Xi}_{15} = L_{21} - L_{11} - M_{11} - M_{21} + h_a(J_{12} + \tfrac{K_{12}}{3}),$$

$$\overline{\Xi}_{16} = L_{21} - L_{11} - M_{11} - M_{21} + h_b(J_{12} + \tfrac{K_{12}}{3}),$$

$$\overline{\Xi}_{17} = L_{31} + 2M_{11} - M_{31} + h_a(J_{13} + \tfrac{K_{13}}{3}),$$

$$\overline{\Xi}_{18} = L_{31} + 2M_{11} - M_{31} + h_b(J_{13} + \tfrac{K_{13}}{3}),$$

$\overline{\Xi}_{110} = \frac{AW_1}{2} - \frac{W_1}{2}$,

$\overline{\Xi}_{22} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \frac{\varepsilon_2}{2}\beta^2 I + I\frac{\varepsilon_2^T}{2}\beta^2 + I\varepsilon_3 + \frac{BW_1}{2} + \frac{BW_1^T}{2}$,

$\overline{\Xi}_{23} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \frac{FW_1}{2}$,

$\overline{\Xi}_{24} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \frac{GW_1}{2}$,

$\overline{\Xi}_{210} = \frac{BW_1}{2} - \frac{W_1}{2}$,

$\overline{\Xi}_{33} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F - \varepsilon_1 I$,

$\overline{\Xi}_{34} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G$,

$\overline{\Xi}_{310} = \frac{FW_1}{2}$,

$\overline{\Xi}_{44} = G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G - \varepsilon_2 I$,

$\overline{\Xi}_{410} = \frac{GW_1}{2}$,

$\overline{\Xi}_{55} = (Q_3 - Q_1) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_a(J_{22} + \frac{K_{22}}{3}) + L_{11} - M_{11}L_{11}^T - M_{11}^T$
$+(h_b - h_a)(J_{11} + \frac{K_{11}}{3})$,

$\overline{\Xi}_{56} = L_{21} - L_{11} - M_{11} - M_{21} + (h_b - h_a)(J_{12} + \frac{K_{12}}{3})$,

$\overline{\Xi}_{57} = -L_{31} + 2M_{21} - M_{31} + h_a(J_{23} + \frac{K_{23}}{3})$,

$\overline{\Xi}_{59} = L_{31} + 2M_{11} - M_{31} + (h_b - h_a)(J_{13} + \frac{K_{13}}{3})$,

$\overline{\Xi}_{66} = (-Q_2 - Q_3) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_b(J_{22} + \frac{K_{22}}{3}) - L_{21} - M_{21} - L_{21}^T - M_{21}^T$
$+(h_b - h_a)(J_{22} + \frac{K_{22}}{3})$,

$\overline{\Xi}_{68} = -L_{31} + 2M_{21} - M_{31} + h_b(J_{23} + \frac{K_{23}}{3})$,

$\overline{\Xi}_{69} = -L_{31} + 2M_{21} - M_{31} + (h_b - h_a)(J_{23} + \frac{K_{23}}{3})$,

$\overline{\Xi}_{77} = 2M_{31} + 2M_{31}^T + h_a(J_{33} + \frac{K_{33}}{3})$,

$\overline{\Xi}_{88} = 2M_{31} + 2M_{31}^T + h_b(J_{33} + \frac{K_{33}}{3})$,

$\overline{\Xi}_{99} = 2M_{31} + 2M_{31}^T + (h_b - h_a)(J_{33} + \frac{K_{33}}{3})$,

$\overline{\Xi}_{1010} = -\frac{W_1}{2} - \frac{W_1^T}{2}$.

From (23) and **Proposition 2.2**, it is easy to see that $\dot{V}(t) < 0$.

Hence, from **Definition 2.1**, we conclude that system (1) is asymptotically stable.

CHAPTER 3
NUMERICAL EXAMPLES

In this section, four numerical examples are given to illustrate the validity and superiority of the proposed scheme.

Example 1. In order to demonstrate effectiveness of the method, we set the following parameters :

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

It is assumed that non-linear perturbations satisfy

$$f^T(x(t),t)f(x(t),t) \leq \alpha^2 x^T(t)x(t),$$
$$g^T(x(t-h(t)),t)g(x(t-h(t)),t) \leq \beta^2 x^T(t-h(t))x(t-h(t))$$

and $0 \leq h_a \leq h(t) \leq h_b$.

We assume that with satisfy

$$f = \begin{bmatrix} \sqrt{x_1^2(t)+x_2^2(t)} \\ \sqrt{x_1^2(t)+x_2^2(t)} \end{bmatrix}, g = \begin{bmatrix} \sqrt{x_1^2(t-h(t))+x_2^2(t-h(t))} \\ \sqrt{x_1^2(t-h(t))+x_2^2(t-h(t))} \end{bmatrix},$$

$$h(t) = 1 + sin^2(t).$$

By taking parameters $\alpha = 0$ and $\beta = 0.1$, we get Example 1. remains feasible for any delay time $h_b \leq 4.3159$. In case of $h_b = 4.3159$, Theorem 1 yields the following set of feasible solutions :

$$P = \begin{bmatrix} 2.3830 & -0.0418 \\ -0.0418 & 1.6576 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 4.0744 & 0.0003 \\ 0.0003 & 4.0750 \end{bmatrix},$$
$$Q_2 = \begin{bmatrix} 2.4251 & 0.0001 \\ 0.0001 & 2.4252 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 5.9909 & -0.0002 \\ -0.0002 & 5.9904 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 0.0046 & -0.0001 \\ -0.0001 & 0.0024 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.0011 & -0.0000 \\ -0.0000 & 0.0005 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 0.0014 & -0.0000 \\ -0.0000 & 0.0007 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 0.0268 & -0.0041 \\ -0.0041 & 0.0585 \end{bmatrix},$$

$$J_{11} = \begin{bmatrix} 1.1447 & 0.0000 \\ 0.0000 & 1.1447 \end{bmatrix}, \qquad J_{12} = \begin{bmatrix} -1.3109 & -0.0000 \\ -0.0000 & -1.3109 \end{bmatrix},$$

$$J_{13} = \begin{bmatrix} 3.7044 & 0.1637 \\ 0.1642 & 3.9969 \end{bmatrix}, \qquad J_{22} = \begin{bmatrix} 1.5015 & 0.0000 \\ 0.0000 & 1.5015 \end{bmatrix},$$

$$J_{23} = \begin{bmatrix} -2.2487 & -0.0170 \\ -0.0168 & -2.2791 \end{bmatrix}, \qquad J_{33} = \begin{bmatrix} 1.6108 & -0.001 \\ -0.0001 & 1.6108 \end{bmatrix},$$

$$K_{11} = \begin{bmatrix} 5.8900 & 0.0000 \\ 0.0000 & 5.8920 \end{bmatrix}, \qquad K_{12} = \begin{bmatrix} 3.3082 & 0.0007 \\ 0.0023 & 3.3130 \end{bmatrix},$$

$$K_{13} = \begin{bmatrix} -9.7268 & -0.0032 \\ -0.0041 & -9.7374 \end{bmatrix}, \qquad K_{22} = \begin{bmatrix} 1.0215 & -0.0012 \\ -0.0012 & 1.0196 \end{bmatrix},$$

$$K_{23} = \begin{bmatrix} -1.2643 & 0.0006 \\ 0.0008 & -1.2634 \end{bmatrix}, \qquad K_{33} = \begin{bmatrix} 2.2426 & 0.0002 \\ 0.0002 & 2.2438 \end{bmatrix},$$

$$\varepsilon_1 = 72.1078, \quad \varepsilon_2 = 40.4170, \quad \varepsilon_3 = 450.0000.$$

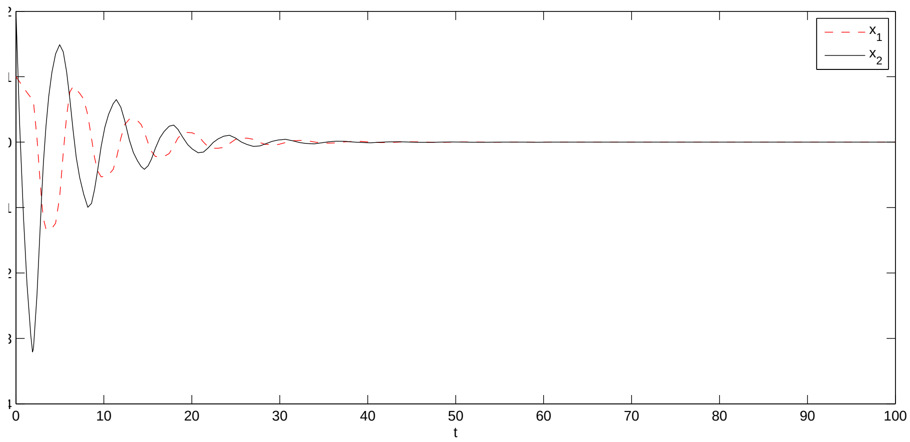

Figure 1 The trajectory of the solution of system (1) in Example 1 with $h_b = 4.3159$.

Table 1 : MADBs h_b for different α and β for Example 1.

Method	h_b $\alpha = 0$, $\beta = 0.1$	h_b $\alpha = 0.1$, $\beta = 0.1$
Cao and Lam[2]	0.6811	0.6129
Han[4]	1.3279	1.2503
Zuo and Wang[23]	2.7422	1.8753
Qiu et al.[12]	2.7423	1.8753
Chen et al.[3]	2.7423	1.8753
Qiu et al.[13]	2.7757	1.8959
Kwon et al.[5]	2.7758	1.8959
Kwon and park[6]	2.7753	1.8959
Liu[9]	2.7429	1.8895
Rakkiyappan[14] et al.	2.9816	1.9805
Lakshmanan et al.[8]	3.0853	2.0974
P.-L.Liu[11]	3.4863	2.6144
Theorem 1	4.3159	4.3158

Example 2. Consider the system (1) with the following parameters :

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Table 2 : MADBs h_b for different methods for Example 2.

Method	h_b
Park ans Kwon[10]	1.0
Kwon et al.[5]	3.4039
P.-L.Liu[11]	3.6654
Theorem 1	4.3143

Example 3. Consider the system (1) with the following parameters :

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Table 3 : MAUB h_b for various Method for Example 3.

Method	h_b
Seuret and Gouaisbaut[15]	4.703
Kwon et al.[7]	4.8117
Zeng et al.[20]	4.788
T.H.Lee et al.(Remark3)[16]	4.8076
T.H.Lee et al.(Corollary1)[16]	4.8257
T.H.Lee et al.(Theorem1)[16]	4.8313
Theorem 1	4.9252

Example 4. Consider the system (1) with the following parameters :

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Table 4 : MAUB h_b for various Method for Example 4.

Method	h_b
Seuret and Gouaisbaut[15]	6.5906
Kwon et al.[7]	7.1250
Zeng et al.[20]	7.1480
T.H.Lee et al.(Remark3)[16]	7.1550
T.H.Lee et al.(Corollary1)[16]	7.1582
T.H.Lee et al.(Theorem1)[16]	7.1672
Theorem 1	7.1799

CHAPTER 4
CONCLUSION

We obtained a new criteria for asymptotical stability for system (1) as follow :

Theorem 1

The system (1) is asymptotically stable if there exist positive definite matrices, $P, Q_i, R_i,\ (i = 1, 2, 3)$, positive semi-definite matrices Z_1, Z_3 and any matrices N_1, N_2 such that the following LMIs hold :

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0, \tag{7}$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & 0 & \Xi_{110} & \Xi_{111} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 & 0 & 0 & 0 & \Xi_{210} & 0 \\ * & * & \Xi_{33} & \Xi_{34} & 0 & 0 & 0 & 0 & 0 & \Xi_{310} & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 & \Xi_{410} & 0 \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & 0 & \Xi_{59} & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & \Xi_{68} & \Xi_{69} & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Xi_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Xi_{1111} \end{bmatrix} < 0, \tag{8}$$

where

$$Z_1 = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}, \qquad Z_3 = \begin{bmatrix} \frac{1}{3}K_{11} & \frac{1}{3}K_{12} & \frac{1}{3}K_{13} \\ \frac{1}{3}K_{21} & \frac{1}{3}K_{22} & \frac{1}{3}K_{23} \\ \frac{1}{3}K_{31} & \frac{1}{3}K_{32} & \frac{1}{3}K_{33} \end{bmatrix},$$

$$N_1 = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \end{bmatrix} \begin{bmatrix} I & -I & 0 \end{bmatrix} = \begin{bmatrix} L_{11} & -L_{11} & 0 \\ L_{21} & -L_{21} & 0 \\ L_{31} & -L_{31} & 0 \end{bmatrix},$$

$$N_1^T = \begin{bmatrix} L_{11}^T & -L_{11}^T & L_{31}^T \\ -L_{21}^T & -L_{21}^T & -L_{31}^T \\ 0 & 0 & 0 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} M_{11} \\ M_{21} \\ M_{31} \end{bmatrix} \begin{bmatrix} -I & -I & 2I \end{bmatrix} = \begin{bmatrix} -M_{11} & -M_{11} & 2M_{11} \\ -M_{21} & -M_{21} & 2M_{21} \\ -M_{31} & -M_{31} & 2M_{31} \end{bmatrix},$$

$$N_2^T = \begin{bmatrix} -M_{11}^T & -M_{21}^T & -M_{31}^T \\ -M_{11}^T & -M_{21}^T & -M_{31}^T \\ 2M_{11}^T & 2M_{21}^T & 2M_{31}^T \end{bmatrix},$$

$$\begin{aligned}
\Xi_{11} &= (A^TP+PA)+(Q_1+Q_2)+(2L_{11}+2L_{11}^T-2M_{11}-2M_{11}^T+h_a(J_{11}+\tfrac{K_{11}}{3})) \\
&\quad +h_b(J_{11}+\tfrac{K_{11}}{3}))+A^T(h_aR_1+h_bR_2+(h_b-h_a)R_3)A+\varepsilon_1\alpha^2 I-\tfrac{AW_1}{2}-\tfrac{W_1^TA^T}{2}, \\
\Xi_{12} &= PB+A^T(h_aR_1+h_bR_2+(h_b-h_a)R_3)B+\tfrac{AW_1}{2}+\tfrac{BW_1}{2}, \\
\Xi_{13} &= PF+A^T(h_aR_1+h_bR_2+(h_b-h_a)R_3)F+\tfrac{FW_1}{2}, \\
\Xi_{14} &= PG+A^T(h_aR_1+h_bR_2+(h_b-h_a)R_3)G+\tfrac{GW_1}{2}, \\
\Xi_{15} &= L_{21}-L_{11}-M_{11}-M_{21}+h_a(J_{12}+\tfrac{K_{12}}{3}), \\
\Xi_{16} &= L_{21}-L_{11}-M_{11}-M_{21}+h_b(J_{12}+\tfrac{K_{12}}{3}), \\
\Xi_{17} &= L_{31}+2M_{11}-M_{31}+h_a(J_{13}+\tfrac{K_{13}}{3}),
\end{aligned}$$

$\Xi_{18} = L_{31} + 2M_{11} - M_{31} + h_b(J_{13} + \frac{K_{13}}{3}),$

$\Xi_{110} = \frac{AW_1}{2} - \frac{W_1}{2},$

$\Xi_{111} = T,$

$\Xi_{22} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \frac{\varepsilon_2}{2}\beta^2 I + I\frac{\varepsilon_2^T}{2}\beta^2 + I\varepsilon_3 + \frac{BW_1}{2} + \frac{BW_1^T}{2} - T,$

$\Xi_{23} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \frac{FW_1}{2},$

$\Xi_{24} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \frac{GW_1}{2},$

$\Xi_{210} = \frac{BW_1}{2} - \frac{W_1}{2},$

$\Xi_{33} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F - \varepsilon_1 I,$

$\Xi_{34} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G,$

$\Xi_{310} = \frac{FW_1}{2},$

$\Xi_{44} = G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G - \varepsilon_2 I,$

$\Xi_{410} = \frac{GW_1}{2},$

$\Xi_{55} = (Q_3 - Q_1) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_a(J_{22} + \frac{K_{22}}{3}) + L_{11} - M_{11}L_{11}^T - M_{11}^T$
$+ (h_b - h_a)(J_{11} + \frac{K_{11}}{3}),$

$\Xi_{56} = L_{21} - L_{11} - M_{11} - M_{21} + (h_b - h_a)(J_{12} + \frac{K_{12}}{3}),$

$\Xi_{57} = -L_{31} + 2M_{21} - M_{31} + h_a(J_{23} + \frac{K_{23}}{3}),$

$\Xi_{59} = L_{31} + 2M_{11} - M_{31} + (h_b - h_a)(J_{13} + \frac{K_{13}}{3}),$

$\Xi_{66} = (-Q_2 - Q_3) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_b(J_{22} + \frac{K_{22}}{3}) - L_{21} - M_{21} - L_{21}^T - M_{21}^T$
$+ (h_b - h_a)(J_{22} + \frac{K_{22}}{3}),$

$\Xi_{68} = -L_{31} + 2M_{21} - M_{31} + h_b(J_{23} + \frac{K_{23}}{3}),$

$\Xi_{69} = -L_{31} + 2M_{21} - M_{31} + (h_b - h_a)(J_{23} + \frac{K_{23}}{3}),$

$\Xi_{77} = 2M_{31} + 2M_{31}^T + h_a(J_{33} + \frac{K_{33}}{3}),$

$\Xi_{88} = 2M_{31} + 2M_{31}^T + h_b(J_{33} + \frac{K_{33}}{3}),$

$\Xi_{99} = 2M_{31} + 2M_{31}^T + (h_b - h_a)(J_{33} + \frac{K_{33}}{3}),$

$\Xi_{1010} = -\frac{W_1}{2} - \frac{W_1^T}{2},$

$\Xi_{1111} = -\frac{I}{\varepsilon_3}.$

By choosing an appropriate Lyapunov-Krasovskii functional and using an improved Free-matrix-based integral inequality for stability analysis of systems with time-varying delay,it has been show by four examples that the obtain stability criteria are effective and less conservative the some existing results.

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APPENDIX

A novel result on analysis for time-varying delay systems with non-linear perturbations.

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Abstract

In this project, we study linear time-varying delay system under nonlinear perturbations. By using new integral inequality approach, the relationship of Leibniz-Newton formula terms has been expressed within the framework of free-matrix-based integral inequality. Merits of the proposed results lie in lesser conservatism, which are realized by introducing appropriated Lyapunov-Krasovskii functionals and estimating the upper bound of some cross term more exactly. Numerical examples are given to illustrate effectiveness of the proposed method.

1 Introduction

Time delay is a natural phenomenon in real word. It is well known that time delay often causes the oscillation deterioration of system performance, and even instability, so the stability analysis of time-delay systems strongly requires before experimental stage. As these reason, the stability analysis of time-delay system have become a study research field during the past years.

In many applications, chemical or physical engineering system are governed by perturbations. Due to inaccuracy in model parameter measurements, data input, disturbance and any kind of unpredictability, such systems always involve uncertainties and perturbations. Generally speaking, these perturbations may cause the oscillation deterioration and give rise to instability of the system, such as linear and time-delay would do, even if the perturbations are tiny [2-6],[8-9],[11-14],[23].

In stability problem of time-delay system, to derive less conservative criteria guaranteeing the stability of the system is key purpose. The maximal allowable upper bound (MAUB) of time-delay is one of the important indexes to check conservatism of stability criteria in the system. Therefore, many researchers have tried to develop such conditions which ensure the stability for MAUB of time-delay as large as possible. In line with this, several remarkable approaches have been reported such as free-weighting matrix approaches [20] , delay partitioning approach, reciprocally convex approach, augmented Lyapunov method. [17]

Motivated by the above discussions, we shall derive new criteria for time-varying delay systems. The main contributions of our studies are the followings: (i) The time-delay functions are only required to be continuous but necessarily differentiable. (ii) By employing an improved integral inequality in [22], we derive less conservative for time-varying delay systems with non-linear perturbation.

In independent study is organized as follow : Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results in Section 3. Time-varying delay systems with non-linear perturbation, given illustrative numerical examples are show in Section 4. Section 5 give the conclusion of paper.

2 Problem formulation and preliminaries

Definition 2.1 [17] The trivial solution ($x(t) = 0$) of system (1) is said to be asymptotically stable (A.S.) if it is stable and $\|x(t)\| \to 0$ as $t \to \infty$.

Proposition 2.1 [18] Let E, H and F be any constant matrices with appropriate dimensions and $F^T F \leq I$. For any $\varepsilon > 0$, we have

$$EFH + H^T F^T E^T \leq \varepsilon E E^T + \varepsilon^{-1} H^T H.$$

Proposition 2.2 [1] (Schur complement lemma.) Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0 \text{ or } \begin{bmatrix} -Y & Z \\ Z^T & X \end{bmatrix} < 0.$$

In this project, we consider time-varying delay systems with non-linear perturbations that can be described by linear differential difference equation:

$$\dot{x}(t) = Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t), t > 0 \tag{1}$$

$$x(t+\eta) = \phi(\eta), \forall \eta \in [-h, 0] \tag{2}$$

with $x(t) \in R^n$ as state vector of the system $A, B, F, G \in R^{n \times n}$ constant matrices $\phi(\bullet)$ continuous vector-valued initial function the time-varying delay $h(t)$ is satisfying

$$0 \leq h_a \leq h(t) \leq h_b \tag{3}$$

h_a, h_b being constants $f(x(t), t)$ and $g(x(t-h(t)), t)$ are unknown nonlinear perturbations with respect to $x(t)$ and $x(t-h(t))$, respectively assumed as

$$f^T(x(t), t)f(x(t), t) \leq \alpha^2 x^T(t)x(t) \tag{4}$$

$$g^T(x(t-h(t)), t)g(x(t-h(t)), t) \leq \beta^2 x^T(t-h(t))x(t-h(t)) \tag{5}$$

where $\alpha \geq 0, \beta \geq 0$ are known scalars, F and G are known constant matrices, $\forall x \in R^n$. The following lemma is useful for our main result :

Lemma 1. [20] Let x be a differentiable function : $[\alpha, \beta] \to R^n$. For symmetric matrices $R \in R^{n\times n}$ and $Z_1, Z_3 \in R^{3n\times 3n}$, and any matrices $Z_2 \in R^{3n\times 3n}$ and $N_1, N_2 \in R^{3n\times n}$ satisfying

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0, \tag{6}$$

the following inequality holds :

$$-\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds \leq \varpi_1^T(\alpha, \beta)\Psi_1\varpi_1(\alpha, \beta),$$

where

$$\varpi_1(\alpha, \beta) = \left[x^T(\beta), \quad x^T(\alpha), \quad \tfrac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds\right]^T,$$

$$\Psi_1 = (\beta - \alpha)(Z_1 + \tfrac{1}{3}Z_3) + Sym\{N_1 \begin{bmatrix} I, & -I, & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I, & -I, & 2I \end{bmatrix}\}.$$

3 Main result

Theorem 1

The system (1) is asymptotically stable if there exist positive definite matrices, P, Q_i, R_i, $(i = 1, 2, 3)$, positive semi-definite matrices Z_1, Z_3 and any matrices N_1, N_2 such that the following LMIs hold :

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & R \end{bmatrix} \geq 0, \tag{7}$$

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & \Xi_{16} & \Xi_{17} & \Xi_{18} & 0 & \Xi_{110} & \Xi_{111} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & 0 & 0 & 0 & 0 & 0 & \Xi_{210} & 0 \\ * & * & \Xi_{33} & \Xi_{34} & 0 & 0 & 0 & 0 & 0 & \Xi_{310} & 0 \\ * & * & * & \Xi_{44} & 0 & 0 & 0 & 0 & 0 & \Xi_{410} & 0 \\ * & * & * & * & \Xi_{55} & \Xi_{56} & \Xi_{57} & 0 & \Xi_{59} & 0 & 0 \\ * & * & * & * & * & \Xi_{66} & 0 & \Xi_{68} & \Xi_{69} & 0 & 0 \\ * & * & * & * & * & * & \Xi_{77} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Xi_{88} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Xi_{99} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Xi_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Xi_{1111} \end{bmatrix} < 0, \tag{8}$$

where

$$Z_1 = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}, \quad Z_3 = \begin{bmatrix} \frac{1}{3}K_{11} & \frac{1}{3}K_{12} & \frac{1}{3}K_{13} \\ \frac{1}{3}K_{21} & \frac{1}{3}K_{22} & \frac{1}{3}K_{23} \\ \frac{1}{3}K_{31} & \frac{1}{3}K_{32} & \frac{1}{3}K_{33} \end{bmatrix},$$

$$N_1 = \begin{bmatrix} L_{11} \\ L_{21} \\ L_{31} \end{bmatrix} \begin{bmatrix} I & -I & 0 \end{bmatrix} = \begin{bmatrix} L_{11} & -L_{11} & 0 \\ L_{21} & -L_{21} & 0 \\ L_{31} & -L_{31} & 0 \end{bmatrix},$$

$$N_1^T = \begin{bmatrix} L_{11}^T & -L_{11}^T & L_{31}^T \\ -L_{21}^T & -L_{21}^T & -L_{31}^T \\ 0 & 0 & 0 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} M_{11} \\ M_{21} \\ M_{31} \end{bmatrix} \begin{bmatrix} -I & -I & 2I \end{bmatrix} = \begin{bmatrix} -M_{11} & -M_{11} & 2M_{11} \\ -M_{21} & -M_{21} & 2M_{21} \\ -M_{31} & -M_{31} & 2M_{31} \end{bmatrix},$$

$$N_2^T = \begin{bmatrix} -M_{11}^T & -M_{21}^T & -M_{31}^T \\ -M_{11}^T & -M_{21}^T & -M_{31}^T \\ 2M_{11}^T & 2M_{21}^T & 2M_{31}^T \end{bmatrix},$$

$$\begin{aligned}
&\Xi_{11} = (A^TP + PA) + (Q_1 + Q_2) + (2L_{11} + 2L_{11}^T - 2M_{11} - 2M_{11}^T + h_a(J_{11} + \tfrac{K_{11}}{3})) \\
&\quad +h_b(J_{11} + \tfrac{K_{11}}{3})) + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)A + \varepsilon_1\alpha^2 I - \tfrac{AW_1}{2} - \tfrac{W_1^TA^T}{2}, \\
&\Xi_{12} = PB + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \tfrac{AW_1}{2} + \tfrac{BW_1}{2}, \\
&\Xi_{13} = PF + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \tfrac{FW_1}{2}, \\
&\Xi_{14} = PG + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \tfrac{GW_1}{2}, \Xi_{15} = L_{21} - L_{11} - M_{11} - M_{21} + h_a(J_{12} + \tfrac{K_{12}}{3}), \\
&\Xi_{16} = L_{21} - L_{11} - M_{11} - M_{21} + h_b(J_{12} + \tfrac{K_{12}}{3}), \Xi_{17} = L_{31} + 2M_{11} - M_{31} + h_a(J_{13} + \tfrac{K_{13}}{3}), \\
&\Xi_{18} = L_{31} + 2M_{11} - M_{31} + h_b(J_{13} + \tfrac{K_{13}}{3}), \Xi_{110} = \tfrac{AW_1}{2} - \tfrac{W_1}{2}, \Xi_{111} = T, \\
&\Xi_{22} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \tfrac{\varepsilon_2}{2}\beta^2 I + I\tfrac{\varepsilon_2^T}{2}\beta^2 + I\varepsilon_3 + \tfrac{BW_1}{2} + \tfrac{BW_1^T}{2} - T, \\
&\Xi_{23} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \tfrac{FW_1}{2}, \Xi_{24} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \tfrac{GW_1}{2}, \\
&\Xi_{210} = \tfrac{BW_1}{2} - \tfrac{W_1}{2}, \Xi_{33} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F - \varepsilon_1 I, \\
&\Xi_{34} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G,\ \Xi_{310} = \tfrac{FW_1}{2}, \\
&\Xi_{44} = G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G - \varepsilon_2 I, \Xi_{410} = \tfrac{GW_1}{2}, \\
&\Xi_{55} = (Q_3 - Q_1) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_a(J_{22} + \tfrac{K_{22}}{3}) + L_{11} - M_{11}L_{11}^T - M_{11}^T \\
&\quad +(h_b - h_a)(J_{11} + \tfrac{K_{11}}{3}), \Xi_{56} = L_{21} - L_{11} - M_{11} - M_{21} + (h_b - h_a)(J_{12} + \tfrac{K_{12}}{3}), \\
&\Xi_{57} = -L_{31} + 2M_{21} - M_{31} + h_a(J_{23} + \tfrac{K_{23}}{3}), \Xi_{59} = L_{31} + 2M_{11} - M_{31} + (h_b - h_a)(J_{13} + \tfrac{K_{13}}{3}), \\
&\Xi_{66} = (-Q_2 - Q_3) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_b(J_{22} + \tfrac{K_{22}}{3}) - L_{21} - M_{21} - L_{21}^T - M_{21}^T \\
&\quad +(h_b - h_a)(J_{22} + \tfrac{K_{22}}{3}), \Xi_{68} = -L_{31} + 2M_{21} - M_{31} + h_b(J_{23} + \tfrac{K_{23}}{3}), \\
&\Xi_{69} = -L_{31} + 2M_{21} - M_{31} + (h_b - h_a)(J_{23} + \tfrac{K_{23}}{3}), \Xi_{77} = 2M_{31} + 2M_{31}^T + h_a(J_{33} + \tfrac{K_{33}}{3}), \\
&\Xi_{88} = 2M_{31} + 2M_{31}^T + h_b(J_{33} + \tfrac{K_{33}}{3}), \Xi_{99} = 2M_{31} + 2M_{31}^T + (h_b - h_a)(J_{33} + \tfrac{K_{33}}{3}), \\
&\Xi_{1010} = -\tfrac{W_1}{2} - \tfrac{W_1^T}{2}, \Xi_{1111} = -\tfrac{I}{\varepsilon_3}.
\end{aligned}$$

Proof. We introduce a following Lyapunov-Krasovskii functional :

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{9}$$

where

$$V_1(t) = x^T(t)Px(t),$$

$$V_2(t) = \int_{t-h_a}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_b}^{t} x^T(s)Q_2x(s)ds + \int_{t-h_b}^{t-h_a} x^T(s)Q_3x(s)ds,$$

$$V_3(t) = \int_{-h_a}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_1\dot{x}(s)dsd\theta + \int_{-h_b}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R_2\dot{x}(s)dsd\theta + \int_{-h_b}^{-h_a}\int_{t+\theta}^{t} \dot{x}^T(s)R_3\dot{x}(s)dsd\theta.$$

By taking derivative of $V(t)$ for $t \in [0, \infty]$ along the trajectory solution of (1) yields

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t). \tag{10}$$

From (1) and (9), we obtain

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) \\
&= x^T(t)P\dot{x}(t) + x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P[Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t)] \\
&= [x^T(t)2PAx(t)] + [x^T(t)2PBx(t-h(t))] + [x^T(t)2PFf(x(t),t)] \\
&\quad + [x^T(t)2PGg(x(t-h(t)),t)] \\
&= [x^T(t)(PA+PA)x(t)] + [x^T(t)(PB+PB)x(t-h(t))] \\
&\quad + [x^T(t)(PF+PF)f(x(t),t)] + [x^T(t)(PG+PG)g(x(t-h(t)),t)] \\
&= [(x^T(t)(PA)x(t))^T + (x^T(t)(PA)x(t))] \\
&\quad + [(x^T(t)(PB)x(t-h(t)))^T + (x^T(t)(PB)x(t-h(t)))] \\
&\quad + [(x^T(t)(PF)f(x(t),t))^T + (x^T(t)(PF)f(x(t),t))] \\
&\quad + [(x^T(t)(PG)g(x(t-h(t)),t))^T + (x^T(t)(PG)g(x(t-h(t)),t))] \\
&= [(x^T(t)(A^TP)x(t)) + (x^T(t)(PA)x(t))] \\
&\quad + [(x^T(t)(B^TP)x(t-h(t))) + (x^T(t)(PB)x(t-h(t)))] \\
&\quad + [(x^T(t)(F^TP)f(x(t),t)) + (x^T(t)(PF)f(x(t),t))] \\
&\quad + [(x^T(t)(G^TP)g(x(t-h(t)),t)) + (x^T(t)(PG)g(x(t-h(t)),t))] \\
&= [(x^T(t)(A^TP+PA)x(t))] + x^T(t)PBx(t-h(t)) + x^T(t)PFf(x(t),t) \\
&\quad + x^T(t)PGg(x(t-h(t)),t) + x^T(t-h(t))B^TPx(t) + f^T(x(t),t)F^TPx(t) \\
&\quad + g^T(x(t-h(t)),t)G^TPx(t).
\end{aligned} \tag{11}$$

From (9), we have

$$\begin{aligned}
\dot{V}_2(t) &= [x^T(t)Q_1x(t) - x^T(t-h_a)Q_1x(t-h_a)] \\
&\quad + [x^T(t)Q_2x(t) - x^T(t-h_b)Q_2x(t-h_b)] \\
&\quad + [x^T(t-h_a)Q_3x(t-h_a) - x^T(t-h_b)Q_3x(t-h_b)] \\
&= x^T(t)(Q_1+Q_2)x(t) + x^T(t-h_a)(Q_3-Q_1)x(t-h_a)
\end{aligned}$$

$$+x^T(t-h_b)(-Q_2-Q_3)x(t-h_b). \tag{12}$$

From (9),we obtain

$$\begin{aligned}
\dot{V}_3(t) &= [h_a(\dot{x}^T(t)R_1\dot{x}(t)) - \dot{x}^T(t)R_1\dot{x}(t) + \dot{x}^T(t-h_a)R_1\dot{x}(t-h_a)] \\
&\quad +[h_b(\dot{x}^T(t)R_2\dot{x}(t)) - \dot{x}^T(t)R_2\dot{x}(t) + \dot{x}^T(t-h_b)R_2\dot{x}(t-h_b)] \\
&\quad +[(h_b-h_a)(\dot{x}^T(t)R_3\dot{x}(t)) - \dot{x}^T(t-h_a)R_3\dot{x}(t-h_a) \\
&\quad +\dot{x}^T(t-h_b)R_3\dot{x}(t-h_b)] \\
&= \dot{x}^T(t)(h_aR_1 + h_bR_2 + (h_b-h_a)R_3)\dot{x}(t) - \int_{t-h_a}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds \\
&\quad - \int_{t-h_b}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds - \int_{t-h_b}^{t-h_a} \dot{x}^T(s)R_3\dot{x}(s)ds.
\end{aligned} \tag{13}$$

From $\Psi_1 = (\beta-\alpha)(Z_1 + \frac{1}{3}Z_3) + Sym\{N_1 \begin{bmatrix} I & -I & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I & -I & 2I \end{bmatrix}\}$, we have

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix},$$

where

$$\begin{aligned}
\Omega_{11} &= L_{11} + L_{11}^T + M_{11} + M_{11}^T + (\beta-\alpha)(J_{11} + \tfrac{1}{3}K_{11}), \\
\Omega_{12} &= L_{21}^T - L_{11} - M_{11} - M_{21}^T + (\beta-\alpha)(J_{12} + \tfrac{1}{3}K_{12}), \\
\Omega_{13} &= L_{31}^T + 2M_{11} - M_{31}^T + (\beta-\alpha)(J_{13} + \tfrac{1}{3}K_{13}), \\
\Omega_{21} &= L_{21} - L_{11}^T - M_{21} - M_{11}^T + (\beta-\alpha)(J_{21} + \tfrac{1}{3}K_{21}), \\
\Omega_{22} &= -L_{21} - L_{21}^T - M_{21} - M_{21}^T + (\beta-\alpha)(J_{22} + \tfrac{1}{3}K_{22}), \\
\Omega_{23} &= -L_{31}^T + 2M_{21} - M_{31}^T + (\beta-\alpha)(J_{23} + \tfrac{1}{3}K_{23}), \\
\Omega_{31} &= L_{31} + 2M_{11}^T - M_{31} + (\beta-\alpha)(J_{31} + \tfrac{1}{3}K_{31}), \\
\Omega_{32} &= -L_{31} - M_{31} + 2M_{21}^T + (\beta-\alpha)(J_{32} + \tfrac{1}{3}K_{32}), \\
\Omega_{33} &= 2M_{31} + 2M_{31}^T + (\beta-\alpha)(J_{33} + \tfrac{1}{3}K_{33}).
\end{aligned}$$

From **Lemma 1.** and we let

$$\Psi_{11} = (h_a)(Z_1 + \tfrac{1}{3}Z_3) + Sym\{N_1 \begin{bmatrix} I & -I & 0 \end{bmatrix} + N_2 \begin{bmatrix} -I & -I & 2I \end{bmatrix}\},$$

we have $-\int_{t-h_a}^{t} \dot{x}^T(s)R_1\dot{x}(s)ds$

$$\leq \begin{bmatrix} x^T(t) & x^T(t-h_a) & \frac{1}{h_a}\int_{t-h_a}^{t} x^T(s)ds \end{bmatrix} \Psi_{11} \begin{bmatrix} x(t) \\ x(t-h_a) \\ \frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds \end{bmatrix}$$

$$= x^T(t)(L_{11} + L_{11}^T + M_{11} + M_{11}^T + (h_a)(J_{11} + \tfrac{1}{3}K_{11}))x(t)$$
$$+x^T(t-h_a)(L_{21} - L_{11}^T - M_{21} - M_{11}^T + (h_a)(J_{21} + \tfrac{1}{3}K_{21}))x(t)$$
$$+\frac{1}{h_a}\int_{t-h_a}^{t} x^T(s)ds(L_{31} + 2M_{11}^T - M_{31} + (h_a)(J_{31} + \frac{1}{3}K_{31}))x(t)$$
$$+x^T(t)(L_{21}^T - L_{11} - M_{11} - M_{21}^T + (h_a)(J_{12} + \tfrac{1}{3}K_{12}))x(t-h_a)$$
$$+x^T(t-h_a)(-L_{21} - L_{21}^T - M_{21} - M_{21}^T + (h_a)(J_{22} + \tfrac{1}{3}K_{22}))x(t-h_a)$$
$$+\frac{1}{h_a}\int_{(} t-h_a)^t x^T(s)ds(-L_{31} - M_{31} + 2M_{21}^T + (h_a)(J_{32} + \frac{1}{3}K_{32}))x(t-h_a)$$
$$+x^T(t)(L_{31}^T + 2M_{11} - M_{31}^T + (h_a)(J_{13} + \frac{1}{3}K_{13}))\frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds$$
$$+x^T(t-h_a)(-L_{31}^T + 2M_{21} - M_{31}^T + (h_a)(J_{23} + \frac{1}{3}K_{23}))\frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds$$
$$+\frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds(2M_{31}+2M_{31}^T+(h_a)(J_{33}+\frac{1}{3}K_{33}))\frac{1}{h_a}\int_{t-h_a}^{t} x(s)ds. \quad (14)$$

By using the same approach as in (14), we obtain $-\int_{t-h_b}^{t} \dot{x}^T(s)R_2\dot{x}(s)ds$

$$\leq \begin{bmatrix} x^T(t) & x^T(t-h_b) & \frac{1}{h_b}\int_{t-h_b}^{t} x^T(s)ds \end{bmatrix} \Psi_{12} \begin{bmatrix} x(t) \\ x(t-h_b) \\ \frac{1}{h_b}\int_{t-h_b}^{t} x(s)ds \end{bmatrix}$$

$$= x^T(t)(L_{11} + L_{11}^T + M_{11} + M_{11}^T + (h_b)(J_{11} + \tfrac{1}{3}K_{11}))x(t)$$
$$+x^T(t-h_b)(L_{21} - L_{11}^T - M_{21} - M_{11}^T + (h_b)(J_{21} + \tfrac{1}{3}K_{21}))x(t)$$
$$+\frac{1}{h_b}\int_{t-h_b}^{t} x^T(s)ds(L_{31} + 2M_{11}^T - M_{31} + (h_b)(J_{31} + \frac{1}{3}K_{31}))x(t)$$
$$+x^T(t)(L_{21}^T - L_{11} - M_{11} - M_{21}^T + (h_b)(J_{12} + \tfrac{1}{3}K_{12}))x(t-h_b)$$
$$+x^T(t-h_b)(-L_{21} - L_{21}^T - M_{21} - M_{21}^T + (h_b)(J_{22} + \tfrac{1}{3}K_{22}))x(t-h_b)$$
$$+\frac{1}{h_b}\int_{t-h_b}^{t} x^T(s)ds(-L_{31} - M_{31} + 2M_{21}^T + (h_b)(J_{32} + \frac{1}{3}K_{32}))x(t-h_b)$$

$$+x^T(t)(L_{31}^T+2M_{11}-M_{31}^T+(h_b)(J_{13}+\frac{1}{3}K_{13}))\frac{1}{h_b}\int_{t-h_b}^{t}x(s)ds$$
$$+x^T(t-h_b)(-L_{31}^T+2M_{21}-M_{31}^T+(h_b)(J_{23}+\frac{1}{3}K_{23}))\frac{1}{h_b}\int_{t-h_b}^{t}x(s)ds$$
$$+\frac{1}{h_b}\int_{t-h_b}^{t}x(s)ds(2M_{31}+2M_{31}^T+(h_b)(J_{33}+\frac{1}{3}K_{33}))\frac{1}{h_b}\int_{t-h_b}^{t}x(s)ds, \qquad (15)$$

where $\Psi_{12}=(h_b)(Z_1+\frac{1}{3}Z_3)+Sym\{N_1\begin{bmatrix}I & -I & 0\end{bmatrix}+N_2\begin{bmatrix}-I & -I & 2I\end{bmatrix}\}$.

Similarly, we have $-\int_{t-h_b}^{t-h_a}\dot{x}^T(s)R_3\dot{x}(s)ds$

$$\leq\begin{bmatrix}x^T(t-h_a) & x^T(t-h_b) & \frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x^T(s)ds\end{bmatrix}\Psi_{13}\begin{bmatrix}x(t-h_a)\\ x(t-h_b)\\ \frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x(s)ds\end{bmatrix}$$
$$=x^T(t-h_a)(L_{11}+L_{11}^T+M_{11}+M_{11}^T+(h_b-h_a)(J_{11}+\tfrac{1}{3}K_{11}))x(t-h_a)$$
$$+x^T(t-h_b)(L_{21}-L_{11}^T-M_{21}-M_{11}^T+(h_b-h_a)(J_{21}+\tfrac{1}{3}K_{21}))x(t-h_a)$$
$$+\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x^T(s)ds(L_{31}+2M_{11}^T-M_{31}+(h_b-h_a)(J_{31}+\frac{1}{3}K_{31}))x(t-h_a)$$
$$+x^T(t-h_a)(L_{21}^T-L_{11}-M_{11}-M_{21}^T+(h_b-h_a)(J_{12}+\tfrac{1}{3}K_{12}))x(t-h_b)$$
$$+x^T(t-h_b)(-L_{21}-L_{21}^T-M_{21}-M_{21}^T+(h_b-h_a)(J_{22}+\tfrac{1}{3}K_{22}))x(t-h_b)$$
$$+\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x^T(s)ds(-L_{31}-M_{31}+2M_{21}^T+(h_b-h_a)(J_{32}+\frac{1}{3}K_{32}))x(t-h_b)$$
$$+x^T(t-h_a)(L_{31}^T+2M_{11}-M_{31}^T+(h_b-h_a)(J_{13}+\frac{1}{3}K_{13}))\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x(s)ds$$
$$+x^T(t-h_b)(-L_{31}^T+2M_{21}-M_{31}^T+(h_b-h_a)(J_{23}+\frac{1}{3}K_{23}))\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x(s)ds$$
$$+\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x(s)ds(2M_{31}+2M_{31}^T+(h_b-h_a)(J_{33}+\frac{1}{3}K_{33}))\frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x(s)ds, \qquad (16)$$

where $\Psi_{13}=(h_b)(Z_1+\frac{1}{3}Z_3)+Sym\{N_1\begin{bmatrix}I & -I & 0\end{bmatrix}+N_2\begin{bmatrix}-I & -I & 2I\end{bmatrix}\}$.

We may express the term $\dot{x}^T(t)(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)\dot{x}(t)$ as follows :

$$
\begin{aligned}
&\dot{x}^T(t)(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)\dot{x}(t) \\
&\quad = [Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t)]^T \\
&\qquad (h_aR_1 + h_bR_2 + (h_b - h_a)R_3)\dot{x}(t)[Ax(t) + Bx(t-h(t)) + Ff(x(t),t) \\
&\qquad +Gg(x(t-h(t)),t)] \\
&\quad = x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t) \\
&\qquad +x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t)) \\
&\qquad +x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t) \\
&\qquad +x^T(t)A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t) \\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t) \\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t)) \\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t) \\
&\qquad +x^T(t-h(t))B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t) \\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t) \\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t)) \\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t) \qquad (17) \\
&\qquad +f^T(x(t),t)F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t) \\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ax(t) \\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Bx(t-h(t)) \\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Ff(x(t),t) \\
&\qquad +g^T(x(t-h(t)),t)G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)Gg(x(t-h(t)),t)
\end{aligned}
$$

Note that for any $\varepsilon_1 \geq 0, \varepsilon_2 \geq 0$, it follows from (4) and (5) that

$$\varepsilon_1[\alpha^2 x^T(t)x(t) - f^T(x(t),t)f(x(t),t)] \geq 0 \tag{18}$$

and

$$\varepsilon_2[\beta^2 x^T(t-h(t))x(t-h(t)) - g^T(x(t-h(t)),t)g(x(t-h(t)),t)] \geq 0. \tag{19}$$

By using the following identity relation

$$\dot{x}(t) - (Ax(t) + Bx(t-h(t)) + Ff(x(t),t) + Gg(x(t-h(t)),t)) = 0,$$

we have

$[x^T(t)+x^T(t-h(t))+\dot{x}^T(t)](-W_1)[\dot{x}(t)-(Ax(t)+Bx(t-h(t))+Ff(x(t),t)$
$+Gg(x(t-h(t)),t))]=0$, where W_1 is a positive definite matrix.
It follows that $[x^T(t)+x^T(t-h(t))+\dot{x}^T(t)](-W_1)[\dot{x}(t)-Ax(t)-Bx(t-h(t))$

$$
\begin{aligned}
&-Ff(x(t),t)-Gg(x(t-h(t)),t)]\\
&=x^T(t)(-W_1)(\dot{x}(t))+x^T(t)(-W_1)(-Ax(t))\\
&+x^T(t)(-W_1)(-Bx(t-h(t)))+x^T(t)(-W_1)-(Ff(x(t),t))\\
&+x^T(t)(-W_1)-(Gg(x(t-h(t)),t))\\
&+x^T(t-h(t))(-W_1)(\dot{x}(t))+x^T(t-h(t))(-W_1)(-Ax(t))\\
&+x^T(t-h(t))(-W_1)(-Bx(t-h(t)))+x^T(t-h(t))(-W_1)-(Ff(x(t),t))\\
&+x^T(t-h(t))(-W_1)-(Gg(x(t-h(t)),t))\\
&+\dot{x}^T(t)(-W_1)(\dot{x}(t))+x^T(t-h(t))(-W_1)(-Ax(t))\\
&+\dot{x}^T(t)(-W_1)(-Bx(t-h(t)))+x^T(t-h(t))(-W_1)-(Ff(x(t),t))\\
&+\dot{x}^T(t)(-W_1)-(Gg(x(t-h(t)),t))=0.
\end{aligned}
\tag{20}
$$

By using the following identity relation

$$x^T(t-h(t))Tx(t-h(t))-x^T(t-h(t))Tx(t-h(t))=0, T>0. \tag{21}$$

From **Proposition 2.1**, we obtain

$$x^T(t-h(t))x(t-h(t))\le(\tfrac{1}{\varepsilon})T^Tx^T(t-h(t))Tx(t-h(t))+\varepsilon x(t-h(t))x^T(t-h(t)), \tag{22}$$

Combine (10) and (14)-(22),

$$\dot{V}(t)\le\xi^T(t)\overline{\Xi}\xi(t)\ , \tag{23}$$

where

$$\xi^T(t)=[\ x^T(t),x^T(t-h(t))\ ,\ f^T(x(t),t)\ ,\ g^T(x(t-h(t)),t)\ ,\ x^T(t-h_a)\ ,\ x^T(t-h_b)\ ,$$

$$\frac{1}{h_a}\int_{t-h_a}^{t}x^T(s)ds\ ,\ \frac{1}{h_b}\int_{t-h_b}^{t}x^T(s)ds\ ,\ \frac{1}{h_b-h_a}\int_{t-h_b}^{t-h_a}x^T(s)ds\ ,\ \dot{x}^T(t)\ ],$$

$$
\overline{\Xi}=\begin{bmatrix}
\overline{\Xi}_{11} & \overline{\Xi}_{12} & \overline{\Xi}_{13} & \overline{\Xi}_{14} & \overline{\Xi}_{15} & \overline{\Xi}_{16} & \overline{\Xi}_{17} & \overline{\Xi}_{18} & 0 & \overline{\Xi}_{110}\\
* & \overline{\Xi}_{22} & \overline{\Xi}_{23} & \overline{\Xi}_{24} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}_{210}\\
* & * & \overline{\Xi}_{33} & \overline{\Xi}_{34} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}_{310}\\
* & * & * & \overline{\Xi}_{44} & 0 & 0 & 0 & 0 & 0 & \overline{\Xi}_{410}\\
* & * & * & * & \overline{\Xi}_{55} & \overline{\Xi}_{56} & \overline{\Xi}_{57} & 0 & \overline{\Xi}_{59} & 0\\
* & * & * & * & * & \overline{\Xi}_{66} & 0 & \overline{\Xi}_{68} & \overline{\Xi}_{69} & 0\\
* & * & * & * & * & * & \overline{\Xi}_{77} & 0 & 0 & 0\\
* & * & * & * & * & * & * & \overline{\Xi}_{88} & 0 & 0\\
* & * & * & * & * & * & * & * & \overline{\Xi}_{99} & 0\\
* & * & * & * & * & * & * & * & * & \overline{\Xi}_{1010}
\end{bmatrix}<0,
$$

$$\overline{\Xi}_{11} = (A^TP + PA) + (Q_1 + Q_2) + (2L_{11} + 2L_{11}^T - 2M_{11} - 2M_{11}^T + h_a(J_{11} + \tfrac{K_{11}}{3}))$$
$$+h_b(J_{11} + \tfrac{K_{11}}{3})) + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)A + \varepsilon_1\alpha^2 I - \tfrac{AW_1}{2} - \tfrac{W_1^TA^T}{2},$$
$$\overline{\Xi}_{12} = PB + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \tfrac{AW_1}{2} + \tfrac{BW_1}{2},$$
$$\overline{\Xi}_{13} = PF + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \tfrac{FW_1}{2},$$
$$\overline{\Xi}_{14} = PG + A^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \tfrac{GW_1}{2},$$
$$\overline{\Xi}_{15} = L_{21} - L_{11} - M_{11} - M_{21} + h_a(J_{12} + \tfrac{K_{12}}{3}), \overline{\Xi}_{16} = L_{21} - L_{11} - M_{11} - M_{21} + h_b(J_{12} + \tfrac{K_{12}}{3}),$$
$$\overline{\Xi}_{17} = L_{31} + 2M_{11} - M_{31} + h_a(J_{13} + \tfrac{K_{13}}{3}), \overline{\Xi}_{18} = L_{31} + 2M_{11} - M_{31} + h_b(J_{13} + \tfrac{K_{13}}{3}), \overline{\Xi}_{110} = \tfrac{AW_1}{2} - \tfrac{W_1}{2},$$
$$\overline{\Xi}_{22} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)B + \tfrac{\varepsilon_2}{2}\beta^2 I + I\tfrac{\varepsilon_2^T}{2}\beta^2 + I\varepsilon_3 + \tfrac{BW_1}{2} + \tfrac{BW_1^T}{2},$$
$$\overline{\Xi}_{23} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F + \tfrac{FW_1}{2},$$
$$\overline{\Xi}_{24} = B^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G + \tfrac{GW_1}{2}, \overline{\Xi}_{210} = \tfrac{BW_1}{2} - \tfrac{W_1}{2},$$
$$\overline{\Xi}_{33} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)F - \varepsilon_1 I, \overline{\Xi}_{34} = F^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G,$$
$$\overline{\Xi}_{310} = \tfrac{FW_1}{2}, \overline{\Xi}_{44} = G^T(h_aR_1 + h_bR_2 + (h_b - h_a)R_3)G - \varepsilon_2 I, \overline{\Xi}_{410} = \tfrac{GW_1}{2},$$
$$\overline{\Xi}_{55} = (Q_3 - Q_1) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_a(J_{22} + \tfrac{K_{22}}{3}) + L_{11} - M_{11}L_{11}^T - M_{11}^T$$
$$+(h_b - h_a)(J_{11} + \tfrac{K_{11}}{3}), \overline{\Xi}_{56} = L_{21} - L_{11} - M_{11} - M_{21} + (h_b - h_a)(J_{12} + \tfrac{K_{12}}{3}),$$
$$\overline{\Xi}_{57} = -L_{31} + 2M_{21} - M_{31} + h_a(J_{23} + \tfrac{K_{23}}{3}), \overline{\Xi}_{59} = L_{31} + 2M_{11} - M_{31} + (h_b - h_a)(J_{13} + \tfrac{K_{13}}{3}),$$
$$\overline{\Xi}_{66} = (-Q_2 - Q_3) - L_{21} - M_{21} - L_{21}^T - M_{21}^T + h_b(J_{22} + \tfrac{K_{22}}{3}) - L_{21} - M_{21} - L_{21}^T - M_{21}^T$$
$$+(h_b - h_a)(J_{22} + \tfrac{K_{22}}{3}), \overline{\Xi}_{68} = -L_{31} + 2M_{21} - M_{31} + h_b(J_{23} + \tfrac{K_{23}}{3}),$$
$$\overline{\Xi}_{69} = -L_{31} + 2M_{21} - M_{31} + (h_b - h_a)(J_{23} + \tfrac{K_{23}}{3}), \overline{\Xi}_{77} = 2M_{31} + 2M_{31}^T + h_a(J_{33} + \tfrac{K_{33}}{3}),$$
$$\overline{\Xi}_{88} = 2M_{31} + 2M_{31}^T + h_b(J_{33} + \tfrac{K_{33}}{3}), \overline{\Xi}_{99} = 2M_{31} + 2M_{31}^T + (h_b - h_a)(J_{33} + \tfrac{K_{33}}{3}),$$
$$\overline{\Xi}_{1010} = -\tfrac{W_1}{2} - \tfrac{W_1^T}{2}.$$

From (23) and **Proposition 2.2**, it is easy to see that $\dot{V}(t) < 0$.

Hence, from **Definition 2.1**, we conclude that system (1) is asymptotically stable.

4 Numerical Examples

In this section, four numerical examples are given to illustrate the validity and superiority of the proposed scheme.

Example 1. In order to demonstrate effectiveness of the method, we set the following parameters :

$$A = \begin{bmatrix} -1.2 & 0.1 \\ -0.1 & -1 \end{bmatrix}, B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

It is assumed that non-linear perturbations satisfy

$$f^T(x(t),t)f(x(t),t) \leq \alpha^2 x^T(t)x(t),$$
$$g^T(x(t-h(t)),t)g(x(t-h(t)),t) \leq \beta^2 x^T(t-h(t))x(t-h(t))$$

and $0 \leq h_a \leq h(t) \leq h_b$.

We assume that with satisfy

$$f = \begin{bmatrix} \sqrt{x_1^2(t) + x_2^2(t)} \\ \sqrt{x_1^2(t) + x_2^2(t)} \end{bmatrix}, g = \begin{bmatrix} \sqrt{x_1^2(t-h(t)) + x_2^2(t-h(t))} \\ \sqrt{x_1^2(t-h(t)) + x_2^2(t-h(t))} \end{bmatrix},$$

$$h(t) = 1 + sin^2(t).$$

By taking parameters $\alpha = 0$ and $\beta = 0.1$, we get Example 1. remains feasible for any delay time $h_b \leq 4.3159$. In case of $h_b = 4.3159$, Theorem 1 yields the following set of feasible solutions :

$$P = \begin{bmatrix} 2.3830 & -0.0418 \\ -0.0418 & 1.6576 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 4.0744 & 0.0003 \\ 0.0003 & 4.0750 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 2.4251 & 0.0001 \\ 0.0001 & 2.4252 \end{bmatrix}, \quad Q_3 = \begin{bmatrix} 5.9909 & -0.0002 \\ -0.0002 & 5.9904 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.0046 & -0.0001 \\ -0.0001 & 0.0024 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.0011 & -0.0000 \\ -0.0000 & 0.0005 \end{bmatrix},$$

$$R_3 = \begin{bmatrix} 0.0014 & -0.0000 \\ -0.0000 & 0.0007 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.0268 & -0.0041 \\ -0.0041 & 0.0585 \end{bmatrix},$$

$$J_{11} = \begin{bmatrix} 1.1447 & 0.0000 \\ 0.0000 & 1.1447 \end{bmatrix}, \quad J_{12} = \begin{bmatrix} -1.3109 & -0.0000 \\ -0.0000 & -1.3109 \end{bmatrix},$$

$$J_{13} = \begin{bmatrix} 3.7044 & 0.1637 \\ 0.1642 & 3.9969 \end{bmatrix}, \quad J_{22} = \begin{bmatrix} 1.5015 & 0.0000 \\ 0.0000 & 1.5015 \end{bmatrix},$$

$$J_{23} = \begin{bmatrix} -2.2487 & -0.0170 \\ -0.0168 & -2.2791 \end{bmatrix}, \quad J_{33} = \begin{bmatrix} 1.6108 & -0.001 \\ -0.0001 & 1.6108 \end{bmatrix},$$

$$K_{11} = \begin{bmatrix} 5.8900 & 0.0000 \\ 0.0000 & 5.8920 \end{bmatrix}, \qquad K_{12} = \begin{bmatrix} 3.3082 & 0.0007 \\ 0.0023 & 3.3130 \end{bmatrix},$$

$$K_{13} = \begin{bmatrix} -9.7268 & -0.0032 \\ -0.0041 & -9.7374 \end{bmatrix}, \qquad K_{22} = \begin{bmatrix} 1.0215 & -0.0012 \\ -0.0012 & 1.0196 \end{bmatrix},$$

$$K_{23} = \begin{bmatrix} -1.2643 & 0.0006 \\ 0.0008 & -1.2634 \end{bmatrix}, \qquad K_{33} = \begin{bmatrix} 2.2426 & 0.0002 \\ 0.0002 & 2.2438 \end{bmatrix},$$

$\varepsilon_1 = 72.1078, \quad \varepsilon_2 = 40.4170, \quad \varepsilon_3 = 450.0000.$

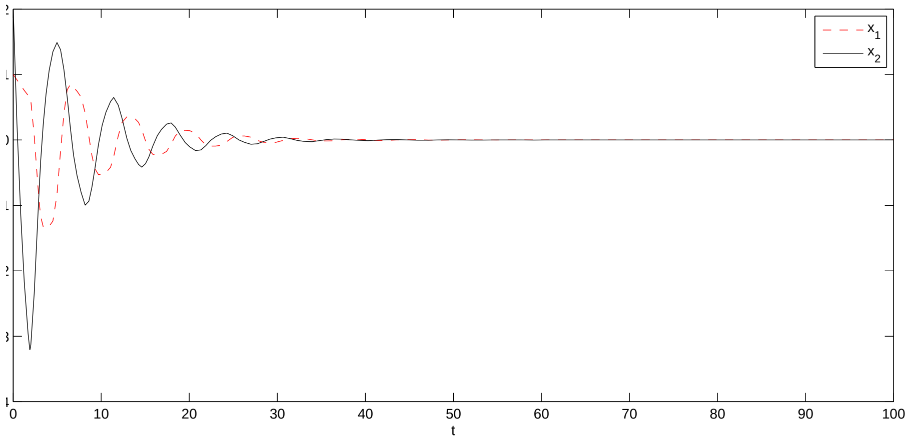

Figure 1 The trajectory of the solution of system (1) in Example 1 with $h_b = 4.3159$.

Table 1 : MADBs h_b for different α and β for Example 1.

Method	h_b $\alpha = 0$, $\beta = 0.1$	h_b $\alpha = 0.1$, $\beta = 0.1$
Cao and Lam[2]	0.6811	0.6129
Han[4]	1.3279	1.2503
Zuo and Wang[23]	2.7422	1.8753
Qiu et al.[12]	2.7423	1.8753
Chen et al.[3]	2.7423	1.8753
Qiu et al.[13]	2.7757	1.8959
Kwon et al.[5]	2.7758	1.8959
Kwon and park[6]	2.7753	1.8959
Liu[9]	2.7429	1.8895
Rakkiyappan[14] et al.	2.9816	1.9805
Lakshmanan et al.[8]	3.0853	2.0974
P.-L.Liu[11]	3.4863	2.6144
Theorem 1	4.3159	4.3158

Example 2. Consider the system (1) with the following parameters :

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Table 2 : MADBs h_b for different methods for Example 2.

Method	h_b
Park ans Kwon[10]	1.0
Kwon et al.[5]	3.4039
P.-L.Liu[11]	3.6654
Theorem 1	4.3143

Example 3. Consider the system (1) with the following parameters :

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Table 3 : MAUB h_b for various Method for Example 3.

Method	h_b
Seuret and Gouaisbaut[15]	4.703
Kwon et al.[7]	4.8117
Zeng et al.[20]	4.788
T.H.Lee et al.(Remark3)[16]	4.8076
T.H.Lee et al.(Corollary1)[16]	4.8257
T.H.Lee et al.(Theorem 1)[16]	4.8313
Theorem 1	4.9252

Example 4. Consider the system (1) with the following parameters :

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}, F = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Table 4 : MAUB h_b for various Method for Example 4.

Method	h_b
Seuret and Gouaisbaut[15]	6.5906
Kwon et al.[7]	7.1250
Zeng et al.[20]	7.1480
T.H.Lee et al.(Remark3)[16]	7.1550
T.H.Lee et al.(Corollary1)[16]	7.1582
T.H.Lee et al.(Theorem 1)[16]	7.1672
Theorem 1	7.1799

5 Conclusions

By choosing an appropriate Lyapunov-Krasovskii functional and using an improved Free-matrix-based integral inequality for stability analysis of systems with time-varying delay, it has been shown by four examples that our obtain stability criteria are effective and less conservative the some existing results.

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MATLAB CODE

1. MATLAB CODE for finding solution of examples 1

```
A=[-1.2,0.1;-0.1,-1]
B=[-0.6,0.7;-1,-0.8]
F=[1,0;0,1]
G=[1,0;0,1]
I=[1,0;0,1]
O=[0,0;0,0]
E1=72.1078
E2=40.4170
E3=450
V=0
S=0.1
ha=1
hb=4.3159
      setlmis([]);
P=lmivar(1,[2,1]);
Q1=lmivar(1,[2,1]);
Q2=lmivar(1,[2,1]);
Q3=lmivar(1,[2,1]);
R1=lmivar(1,[2,1]);
R2=lmivar(1,[2,1]);
R3=lmivar(1,[2,1]);
T=lmivar(1,[2,1]);
J11=lmivar(1,[2,1]);
J22=lmivar(1,[2,1]);
J33=lmivar(1,[2,1]);
H11=lmivar(1,[2,1]);
H22=lmivar(1,[2,1]);
H33=lmivar(1,[2,1]);
K11=lmivar(1,[2,1]);
K22=lmivar(1,[2,1]);
K33=lmivar(1,[2,1]);
R=lmivar(1,[2,1]);
W1=lmivar(1,[2,1]);
J12=lmivar(2,[2,2]);
J13=lmivar(2,[2,2]);
J23=lmivar(2,[2,2]);
H12=lmivar(2,[2,2]);
H13=lmivar(2,[2,2]);
H23=lmivar(2,[2,2]);
K12=lmivar(2,[2,2]);
K13=lmivar(2,[2,2]);
K23=lmivar(2,[2,2]);
L11=lmivar(2,[2,2]);
L21=lmivar(2,[2,2]);
L31=lmivar(2,[2,2]);
M11=lmivar(2,[2,2]);
```

```
M21=lmivar(2,[2,2]);
M31=lmivar(2,[2,2]);
lmiterm([-1 1 1 J11],1,1);                              % LMI #1: J11
lmiterm([-1 2 1 -J12],1,1);                             % LMI #1: J12'
lmiterm([-1 2 2 J22],1,1);                              % LMI #1: J22
lmiterm([-1 3 1 -J13],1,1);                             % LMI #1: J13'
lmiterm([-1 3 2 -J23],1,1);                             % LMI #1: J23'
lmiterm([-1 3 3 J33],1,1);                              % LMI #1: J33
lmiterm([-1 4 1 -H11],1,1);                             % LMI #1: H11'
lmiterm([-1 4 2 -H12],1,1);                             % LMI #1: H12'
lmiterm([-1 4 3 -H13],1,1);                             % LMI #1: H13'
lmiterm([-1 4 4 K11],.5*1,(1/3),'s');                   % LMI #1: K11*(1/3) (NON
SYMMETRIC?)
lmiterm([-1 5 1 -H12],1,1);                             % LMI #1: H12'
lmiterm([-1 5 2 -H22],1,1);                             % LMI #1: H22'
lmiterm([-1 5 3 -H23],1,1);                             % LMI #1: H23'
lmiterm([-1 5 4 -K12],1,(1/3));                         % LMI #1: K12'*(1/3)
lmiterm([-1 5 5 K22],.5*1,(1/3),'s');                   % LMI #1: K22*(1/3) (NON
SYMMETRIC?)
lmiterm([-1 6 1 -H13],1,1);                             % LMI #1: H13'
lmiterm([-1 6 2 -H23],1,1);                             % LMI #1: H23'
lmiterm([-1 6 3 -H33],1,1);                             % LMI #1: H33'
lmiterm([-1 6 4 -K13],1,(1/3));                         % LMI #1: K13'*(1/3)
lmiterm([-1 6 5 -K23],1,(1/3));                         % LMI #1: K23'*(1/3)
lmiterm([-1 6 6 K33],.5*1,(1/3),'s');                   % LMI #1: K33*(1/3) (NON
SYMMETRIC?)
lmiterm([-1 7 1 -L11],1,1);                             % LMI #1: L11'
lmiterm([-1 7 2 -L21],1,1);                             % LMI #1: L21'
lmiterm([-1 7 3 -L31],1,1);                             % LMI #1: L31'
lmiterm([-1 7 4 -M11],1,1);                             % LMI #1: M11'
lmiterm([-1 7 5 -M21],1,1);                             % LMI #1: M21'
lmiterm([-1 7 6 -M31],1,1);                             % LMI #1: M31'
lmiterm([-1 7 7 R],1,1);                                % LMI #1: R
lmiterm([2 1 1 P],A',1,'s');                            % LMI #2: A'*P+P*A
lmiterm([2 1 1 Q1],1,1);                                % LMI #2: Q1
lmiterm([2 1 1 Q2],1,1);                                % LMI #2: Q2
lmiterm([2 1 1 L11],.5*2,1,'s');                        % LMI #2: 2*L11 (NON
SYMMETRIC?)
lmiterm([2 1 1 -L11],.5*2,1,'s');                       % LMI #2: 2*L11' (NON
SYMMETRIC?)
lmiterm([2 1 1 M11],.5*2,-1,'s');                       % LMI #2: -2*M11 (NON
SYMMETRIC?)
lmiterm([2 1 1 -M11],.5*2,-1,'s');                      % LMI #2: -2*M11' (NON
SYMMETRIC?)
lmiterm([2 1 1 J11],.5*ha,1,'s');                       % LMI #2: ha*J11 (NON
SYMMETRIC?)
lmiterm([2 1 1 K11],.5*ha,(1/3),'s');                   % LMI #2: ha*K11*(1/3)
(NON SYMMETRIC?)
lmiterm([2 1 1 J11],.5*hb,1,'s');                       % LMI #2: hb*J11 (NON
SYMMETRIC?)
lmiterm([2 1 1 K11],.5*hb,(1/3),'s');                   % LMI #2: hb*K11*(1/3)
(NON SYMMETRIC?)
lmiterm([2 1 1 R1],.5*A'*ha,A,'s');                     % LMI #2: A'*ha*R1*A
(NON SYMMETRIC?)
```

```
lmiterm([2 1 1 R2],.5*A'*hb,A,'s');              % LMI #2: A'*hb*R2*A
(NON SYMMETRIC?)
lmiterm([2 1 1 R3],.5*A'*hb,A,'s');              % LMI #2: A'*hb*R3*A
(NON SYMMETRIC?)
lmiterm([2 1 1 R3],.5*A'*ha,-A,'s');             % LMI #2: -A'*ha*R3*A
(NON SYMMETRIC?)
lmiterm([2 1 1 W1],.5*A,(1/2),'s');              % LMI #2: A*W1*(1/2)
(NON SYMMETRIC?)
lmiterm([2 1 1 -W1],.5*1,A'*(1/2),'s');          % LMI #2: W1'*A'*(1/2)
(NON SYMMETRIC?)
lmiterm([2 1 1 0],E1*V^2*I);                     % LMI #2: E1*V^2*I
lmiterm([2 2 1 P],B',1);                         % LMI #2: B'*P
lmiterm([2 2 1 R1],B'*ha,A);                     % LMI #2: B'*ha*R1*A
lmiterm([2 2 1 R2],B'*hb,A);                     % LMI #2: B'*hb*R2*A
lmiterm([2 2 1 R3],B'*hb,A);                     % LMI #2: B'*hb*R3*A
lmiterm([2 2 1 R3],B'*ha,-A);                    % LMI #2: -B'*ha*R3*A
lmiterm([2 2 1 -W1],1,A'*(1/2));                 % LMI #2: W1'*A'*(1/2)
lmiterm([2 2 1 -W1],1,B'*(1/2));                 % LMI #2: W1'*B'*(1/2)
lmiterm([2 2 2 R1],.5*B'*ha,B,'s');              % LMI #2: B'*ha*R1*B
(NON SYMMETRIC?)
lmiterm([2 2 2 R2],.5*B'*hb,B,'s');              % LMI #2: B'*hb*R2*B
(NON SYMMETRIC?)
lmiterm([2 2 2 R3],.5*B'*hb,B,'s');              % LMI #2: B'*hb*R3*B
(NON SYMMETRIC?)
lmiterm([2 2 2 R3],.5*B'*ha,-B,'s');             % LMI #2: -B'*ha*R3*B
(NON SYMMETRIC?)
lmiterm([2 2 2 W1],.5*B,(1/2),'s');              % LMI #2: B*W1*(1/2)
(NON SYMMETRIC?)
lmiterm([2 2 2 -W1],.5*1,B'*(1/2),'s');          % LMI #2: W1'*B'*(1/2)
(NON SYMMETRIC?)
lmiterm([2 2 2 T],1,-1);                         % LMI #2: -T
lmiterm([2 2 2 0],E2*(1/2)*S^2*I+I*E2'*(1/2)*S^2+I*E3);      % LMI #2:
E2*(1/2)*S^2*I+I*E2'*(1/2)*S^2+I*E3
lmiterm([2 3 1 P],F',1);                         % LMI #2: F'*P
lmiterm([2 3 1 R1],F'*ha,A);                     % LMI #2: F'*ha*R1*A
lmiterm([2 3 1 R2],F'*hb,A);                     % LMI #2: F'*hb*R2*A
lmiterm([2 3 1 R3],F'*hb,A);                     % LMI #2: F'*hb*R3*A
lmiterm([2 3 1 R3],F'*ha,-A);                    % LMI #2: -F'*ha*R3*A
lmiterm([2 3 1 -W1],1,F'*(1/2));                 % LMI #2: W1'*F'*(1/2)
lmiterm([2 3 2 R1],F'*ha,B);                     % LMI #2: F'*ha*R1*B
lmiterm([2 3 2 R2],F'*hb,B);                     % LMI #2: F'*hb*R2*B
lmiterm([2 3 2 R3],F'*hb,B);                     % LMI #2: F'*hb*R3*B
lmiterm([2 3 2 R3],F'*ha,-B);                    % LMI #2: -F'*ha*R3*B
lmiterm([2 3 2 -W1],1,F'*(1/2));                 % LMI #2: W1'*F'*(1/2)
lmiterm([2 3 3 R1],.5*F'*ha,F,'s');              % LMI #2: F'*ha*R1*F
(NON SYMMETRIC?)
lmiterm([2 3 3 R2],.5*F'*hb,F,'s');              % LMI #2: F'*hb*R2*F
(NON SYMMETRIC?)
lmiterm([2 3 3 R3],.5*F'*hb,F,'s');              % LMI #2: F'*hb*R3*F
(NON SYMMETRIC?)
lmiterm([2 3 3 R3],.5*F'*ha,-F,'s');             % LMI #2: -F'*ha*R3*F
(NON SYMMETRIC?)
lmiterm([2 3 3 0],-E1*I);                        % LMI #2: -E1*I
lmiterm([2 4 1 P],G',1);                         % LMI #2: G'*P
```

```
lmiterm([2 4 1 R1],G'*ha,A);                    % LMI #2: G'*ha*R1*A
lmiterm([2 4 1 R2],G'*hb,A);                    % LMI #2: G'*hb*R2*A
lmiterm([2 4 1 R3],G'*hb,A);                    % LMI #2: G'*hb*R3*A
lmiterm([2 4 1 R3],G'*ha,-A);                   % LMI #2: -G'*ha*R3*A
lmiterm([2 4 1 -W1],1,G'*(1/2));                % LMI #2: W1'*G'*(1/2)
lmiterm([2 4 2 R1],G'*ha,B);                    % LMI #2: G'*ha*R1*B
lmiterm([2 4 2 R2],G'*hb,B);                    % LMI #2: G'*hb*R2*B
lmiterm([2 4 2 R3],G'*hb,B);                    % LMI #2: G'*hb*R3*B
lmiterm([2 4 2 R3],G'*ha,-B);                   % LMI #2: -G'*ha*R3*B
lmiterm([2 4 2 -W1],1,G'*(1/2));                % LMI #2: W1'*G'*(1/2)
lmiterm([2 4 3 R1],G'*ha,F);                    % LMI #2: G'*ha*R1*F
lmiterm([2 4 3 R2],G'*hb,F);                    % LMI #2: G'*hb*R2*F
lmiterm([2 4 3 R3],G'*hb,F);                    % LMI #2: G'*hb*R3*F
lmiterm([2 4 3 R3],G'*ha,-F);                   % LMI #2: -G'*ha*R3*F
lmiterm([2 4 4 R1],.5*G'*ha,G,'s');             % LMI #2: G'*ha*R1*G
(NON SYMMETRIC?)
lmiterm([2 4 4 R2],.5*G'*hb,G,'s');             % LMI #2: G'*hb*R2*G
(NON SYMMETRIC?)
lmiterm([2 4 4 R3],.5*G'*hb,G,'s');             % LMI #2: G'*hb*R3*G
(NON SYMMETRIC?)
lmiterm([2 4 4 R3],.5*G'*ha,-G,'s');            % LMI #2: -G'*ha*R3*G
(NON SYMMETRIC?)
lmiterm([2 4 4 0],-E2*I);                       % LMI #2: -E2*I
lmiterm([2 5 1 -L21],1,1);                      % LMI #2: L21'
lmiterm([2 5 1 -L11],1,-1);                     % LMI #2: -L11'
lmiterm([2 5 1 -M11],1,-1);                     % LMI #2: -M11'
lmiterm([2 5 1 -M21],1,-1);                     % LMI #2: -M21'
lmiterm([2 5 1 -J12],ha,1);                     % LMI #2: ha*J12'
lmiterm([2 5 1 -K12],ha,(1/3));                 % LMI #2: ha*K12'*(1/3)
lmiterm([2 5 2 0],O);                           % LMI #2: O
lmiterm([2 5 3 0],O);                           % LMI #2: O
lmiterm([2 5 4 0],O);                           % LMI #2: O
lmiterm([2 5 5 Q3],1,1);                        % LMI #2: Q3
lmiterm([2 5 5 Q1],1,-1);                       % LMI #2: -Q1
lmiterm([2 5 5 L21],1,-1,'s');                  % LMI #2: -L21-L21'
lmiterm([2 5 5 M21],1,-1,'s');                  % LMI #2: -M21-M21'
lmiterm([2 5 5 J22],.5*ha,1,'s');               % LMI #2: ha*J22 (NON
SYMMETRIC?)
lmiterm([2 5 5 K22],.5*ha,(1/3),'s');           % LMI #2: ha*K22*(1/3)
(NON SYMMETRIC?)
lmiterm([2 5 5 L11],1,1,'s');                   % LMI #2: L11+L11'
lmiterm([2 5 5 M11],1,-1,'s');                  % LMI #2: -M11-M11'
lmiterm([2 5 5 J11],.5*hb,1,'s');               % LMI #2: hb*J11 (NON
SYMMETRIC?)
lmiterm([2 5 5 K11],.5*hb,(1/3),'s');           % LMI #2: hb*K11*(1/3)
(NON SYMMETRIC?)
lmiterm([2 5 5 J11],.5*ha,-1,'s');              % LMI #2: -ha*J11 (NON
SYMMETRIC?)
lmiterm([2 5 5 K11],.5*ha,-(1/3),'s');          % LMI #2: -ha*K11*(1/3)
(NON SYMMETRIC?)
lmiterm([2 6 1 -L21],1,1);                      % LMI #2: L21'
lmiterm([2 6 1 -L11],1,-1);                     % LMI #2: -L11'
lmiterm([2 6 1 -M11],1,-1);                     % LMI #2: -M11'
lmiterm([2 6 1 -M21],1,-1);                     % LMI #2: -M21'
```

```
lmiterm([2 6 1 -J12],hb,1);                    % LMI #2: hb*J12'
lmiterm([2 6 1 -K12],hb,(1/3));                % LMI #2: hb*K12'*(1/3)
lmiterm([2 6 2 0],O);                          % LMI #2: O
lmiterm([2 6 3 0],O);                          % LMI #2: O
lmiterm([2 6 4 0],O);                          % LMI #2: O
lmiterm([2 6 5 -L21],1,1);                     % LMI #2: L21'
lmiterm([2 6 5 -L11],1,-1);                    % LMI #2: -L11'
lmiterm([2 6 5 -M11],1,-1);                    % LMI #2: -M11'
lmiterm([2 6 5 -M21],1,-1);                    % LMI #2: -M21'
lmiterm([2 6 5 -J12],hb,1);                    % LMI #2: hb*J12'
lmiterm([2 6 5 -K12],hb,(1/3));                % LMI #2: hb*K12'*(1/3)
lmiterm([2 6 5 -J12],ha,-(1/2));               % LMI #2: -ha*J12'*(1/2)
lmiterm([2 6 5 -K12],ha,-(1/3));               % LMI #2: -ha*K12'*(1/3)
lmiterm([2 6 6 Q2],1,-1);                      % LMI #2: -Q2
lmiterm([2 6 6 Q3],1,-1);                      % LMI #2: -Q3
lmiterm([2 6 6 L21],1,-1,'s');                 % LMI #2: -L21-L21'
lmiterm([2 6 6 M21],1,-1,'s');                 % LMI #2: -M21-M21'
lmiterm([2 6 6 J22],.5*hb,1,'s');              % LMI #2: hb*J22 (NON
SYMMETRIC?)
lmiterm([2 6 6 K22],.5*hb,(1/3),'s');          % LMI #2: hb*K22*(1/3)
(NON SYMMETRIC?)
lmiterm([2 6 6 L21],1,-1,'s');                 % LMI #2: -L21-L21'
lmiterm([2 6 6 M21],1,-1,'s');                 % LMI #2: -M21-M21'
lmiterm([2 6 6 J22],.5*hb,1,'s');              % LMI #2: hb*J22 (NON
SYMMETRIC?)
lmiterm([2 6 6 K22],.5*hb,(1/3),'s');          % LMI #2: hb*K22*(1/3)
(NON SYMMETRIC?)
lmiterm([2 6 6 J22],.5*ha,-1,'s');             % LMI #2: -ha*J22 (NON
SYMMETRIC?)
lmiterm([2 6 6 K22],.5*ha,-(1/3),'s');         % LMI #2: -ha*K22*(1/3)
(NON SYMMETRIC?)
lmiterm([2 7 1 -L31],1,1);                     % LMI #2: L31'
lmiterm([2 7 1 -M31],1,-1);                    % LMI #2: -M31'
lmiterm([2 7 1 -M11],2,1);                     % LMI #2: 2*M11'
lmiterm([2 7 1 -J13],ha,1);                    % LMI #2: ha*J13'
lmiterm([2 7 1 -K13],ha,(1/3));                % LMI #2: ha*K13'*(1/3)
lmiterm([2 7 2 0],O);                          % LMI #2: O
lmiterm([2 7 3 0],O);                          % LMI #2: O
lmiterm([2 7 4 0],O);                          % LMI #2: O
lmiterm([2 7 5 -L31],1,-1);                    % LMI #2: -L31'
lmiterm([2 7 5 -M31],1,-1);                    % LMI #2: -M31'
lmiterm([2 7 5 -M21],2,1);                     % LMI #2: 2*M21'
lmiterm([2 7 5 -J23],ha,1);                    % LMI #2: ha*J23'
lmiterm([2 7 5 -K23],ha,(1/3));                % LMI #2: ha*K23'*(1/3)
lmiterm([2 7 6 0],O);                          % LMI #2: O
lmiterm([2 7 7 M31],.5*2,1,'s');               % LMI #2: 2*M31 (NON
SYMMETRIC?)
lmiterm([2 7 7 -M31],.5*2,1,'s');              % LMI #2: 2*M31' (NON
SYMMETRIC?)
lmiterm([2 7 7 J33],.5*ha,1,'s');              % LMI #2: ha*J33 (NON
SYMMETRIC?)
lmiterm([2 7 7 K33],.5*ha,(1/3),'s');          % LMI #2: ha*K33*(1/3)
(NON SYMMETRIC?)
lmiterm([2 8 1 -L31],1,1);                     % LMI #2: L31'
```

```
lmiterm([2 8 1 -M31],1,-1);                              % LMI #2: -M31'
lmiterm([2 8 1 -M11],2,1);                               % LMI #2: 2*M11'
lmiterm([2 8 1 -J13],hb,1);                              % LMI #2: hb*J13'
lmiterm([2 8 1 -K13],hb,(1/3));                          % LMI #2: hb*K13'*(1/3)
lmiterm([2 8 2 0],O);                                    % LMI #2: O
lmiterm([2 8 3 0],O);                                    % LMI #2: O
lmiterm([2 8 4 0],O);                                    % LMI #2: O
lmiterm([2 8 5 0],O);                                    % LMI #2: O
lmiterm([2 8 6 -L31],1,-1);                              % LMI #2: -L31'
lmiterm([2 8 6 -M31],1,-1);                              % LMI #2: -M31'
lmiterm([2 8 6 -M21],2,1);                               % LMI #2: 2*M21'
lmiterm([2 8 6 -J23],hb,1);                              % LMI #2: hb*J23'
lmiterm([2 8 6 -K23],hb,(1/3));                          % LMI #2: hb*K23'*(1/3)
lmiterm([2 8 7 0],O);                                    % LMI #2: O
lmiterm([2 8 8 M31],.5*2,1,'s');                         % LMI #2: 2*M31 (NON
SYMMETRIC?)
lmiterm([2 8 8 -M31],.5*2,1,'s');                        % LMI #2: 2*M31' (NON
SYMMETRIC?)
lmiterm([2 8 8 J33],.5*hb,1,'s');                        % LMI #2: hb*J33 (NON
SYMMETRIC?)
lmiterm([2 8 8 K33],.5*hb,(1/3),'s');                    % LMI #2: hb*K33*(1/3)
(NON SYMMETRIC?)
lmiterm([2 9 1 0],O);                                    % LMI #2: O
lmiterm([2 9 2 0],O);                                    % LMI #2: O
lmiterm([2 9 3 0],O);                                    % LMI #2: O
lmiterm([2 9 4 0],O);                                    % LMI #2: O
lmiterm([2 9 5 -L31],1,1);                               % LMI #2: L31'
lmiterm([2 9 5 -M31],1,-1);                              % LMI #2: -M31'
lmiterm([2 9 5 -M11],2,1);                               % LMI #2: 2*M11'
lmiterm([2 9 5 -J13],hb,1);                              % LMI #2: hb*J13'
lmiterm([2 9 5 -K13],hb,(1/3));                          % LMI #2: hb*K13'*(1/3)
lmiterm([2 9 5 -J13],ha,-1);                             % LMI #2: -ha*J13'
lmiterm([2 9 5 -K13],ha,-(1/3));                         % LMI #2: -ha*K13'*(1/3)
lmiterm([2 9 6 -L31],1,-1);                              % LMI #2: -L31'
lmiterm([2 9 6 -M31],1,-1);                              % LMI #2: -M31'
lmiterm([2 9 6 -M21],2,1);                               % LMI #2: 2*M21'
lmiterm([2 9 6 -J23],hb,1);                              % LMI #2: hb*J23'
lmiterm([2 9 6 -K23],hb,(1/3));                          % LMI #2: hb*K23'*(1/3)
lmiterm([2 9 6 -J23],ha,-1);                             % LMI #2: -ha*J23'
lmiterm([2 9 6 -K23],ha,-(1/3));                         % LMI #2: -ha*K23'*(1/3)
lmiterm([2 9 7 0],O);                                    % LMI #2: O
lmiterm([2 9 8 0],O);                                    % LMI #2: O
lmiterm([2 9 9 M31],.5*2,1,'s');                         % LMI #2: 2*M31 (NON
SYMMETRIC?)
lmiterm([2 9 9 -M31],.5*2,1,'s');                        % LMI #2: 2*M31' (NON
SYMMETRIC?)
lmiterm([2 9 9 J33],.5*hb,1,'s');                        % LMI #2: hb*J33 (NON
SYMMETRIC?)
lmiterm([2 9 9 K33],.5*hb,(1/3),'s');                    % LMI #2: hb*K33*(1/3)
(NON SYMMETRIC?)
lmiterm([2 9 9 J33],.5*ha,-1,'s');                       % LMI #2: -ha*J33 (NON
SYMMETRIC?)
lmiterm([2 9 9 K33],.5*ha,-(1/3),'s');                   % LMI #2: -ha*K33*(1/3)
(NON SYMMETRIC?)
```

```
lmiterm([2 10 1 -W1],1,A'*(1/2));                   % LMI #2: W1'*A'*(1/2)
lmiterm([2 10 1 -W1],1,-(1/2));                     % LMI #2: -W1'*(1/2)
lmiterm([2 10 2 -W1],1,B'*(1/2));                   % LMI #2: W1'*B'*(1/2)
lmiterm([2 10 2 -W1],1,-(1/2));                     % LMI #2: -W1'*(1/2)
lmiterm([2 10 3 -W1],1,F'*(1/2));                   % LMI #2: W1'*F'*(1/2)
lmiterm([2 10 4 -W1],1,G'*(1/2));                   % LMI #2: W1'*G'*(1/2)
lmiterm([2 10 5 0],O);                              % LMI #2: O
lmiterm([2 10 6 0],O);                              % LMI #2: O
lmiterm([2 10 7 0],O);                              % LMI #2: O
lmiterm([2 10 8 0],O);                              % LMI #2: O
lmiterm([2 10 9 0],O);                              % LMI #2: O
lmiterm([2 10 10 W1],.5*1,-(1/2),'s');              % LMI #2: -W1*(1/2) (NON
SYMMETRIC?)
lmiterm([2 10 10 -W1],.5*1,-(1/2),'s');             % LMI #2: -W1'*(1/2)
(NON SYMMETRIC?)
lmiterm([2 11 1 T],1,1);                            % LMI #2: T
lmiterm([2 11 2 0],O);                              % LMI #2: O
lmiterm([2 11 3 0],O);                              % LMI #2: O
lmiterm([2 11 4 0],O);                              % LMI #2: O
lmiterm([2 11 5 0],O);                              % LMI #2: O
lmiterm([2 11 6 0],O);                              % LMI #2: O
lmiterm([2 11 7 0],O);                              % LMI #2: O
lmiterm([2 11 8 0],O);                              % LMI #2: O
lmiterm([2 11 9 0],O);                              % LMI #2: O
lmiterm([2 11 10 0],O);                             % LMI #2: O
lmiterm([2 11 11 0],-I*(1/E3));                     % LMI #2: -I*(1/E3)
lmiterm([-3 1 1 P],1,1);                            % LMI #3: P
lmiterm([-4 1 1 Q1],1,1);                           % LMI #4: Q1
lmiterm([-5 1 1 Q2],1,1);                           % LMI #5: Q2
lmiterm([-6 1 1 Q3],1,1);                           % LMI #6: Q3
lmiterm([-7 1 1 R1],1,1);                           % LMI #7: R1
lmiterm([-8 1 1 R2],1,1);                           % LMI #8: R2
lmiterm([-9 1 1 R3],1,1);                           % LMI #9: R3
lmiterm([-10 1 1 T],1,1);                           % LMI #10: T
lmiterm([-11 1 1 J11],1,1);                         % LMI #11: J11
lmiterm([-12 1 1 J22],1,1);                         % LMI #12: J22
lmiterm([-13 1 1 J33],1,1);                         % LMI #13: J33
lmiterm([-14 1 1 H11],1,1);                         % LMI #14: H11
lmiterm([-15 1 1 H22],1,1);                         % LMI #15: H22
lmiterm([-16 1 1 H33],1,1);                         % LMI #16: H33
lmiterm([-17 1 1 K11],1,1);                         % LMI #17: K11
lmiterm([-18 1 1 K22],1,1);                         % LMI #18: K22
lmiterm([-19 1 1 K33],1,1);                         % LMI #19: K33
lmiterm([-20 1 1 R],1,1);                           % LMI #20: R
lmiterm([-21 1 1 W1],1,1);                          % LMI #21: W1
      Nl=getlmis;[tmin,xfeas]=feasp(Nl)
P=dec2mat(Nl , xfeas , P) ;
Q1=dec2mat(Nl , xfeas ,Q1) ;
Q2=dec2mat(Nl , xfeas , Q2) ;
Q3=dec2mat(Nl , xfeas , Q3) ;
R1=dec2mat(Nl , xfeas , R1) ;
R2=dec2mat(Nl , xfeas , R2) ;
R3=dec2mat(Nl , xfeas , R3) ;
T=dec2mat(Nl , xfeas , T) ;
```

```
J11=dec2mat(Nl , xfeas , J11) ;
J22=dec2mat(Nl , xfeas , J22) ;
J33=dec2mat(Nl , xfeas , J33) ;
H11=dec2mat(Nl , xfeas , H11) ;
H22=dec2mat(Nl , xfeas , H22) ;
H33=dec2mat(Nl , xfeas , H33) ;
K11=dec2mat(Nl , xfeas , K11) ;
K22=dec2mat(Nl , xfeas , K22) ;
K33=dec2mat(Nl , xfeas , K33) ;
R=dec2mat(Nl , xfeas , R) ;
W1=dec2mat(Nl , xfeas , W1) ;
J12=dec2mat(Nl , xfeas , J12) ;
J13=dec2mat(Nl , xfeas , J13) ;
J23=dec2mat(Nl , xfeas , J23) ;
H12=dec2mat(Nl , xfeas , H12) ;
H13=dec2mat(Nl , xfeas , H13) ;
H23=dec2mat(Nl , xfeas , H23) ;
K12=dec2mat(Nl , xfeas , K12) ;
K13=dec2mat(Nl , xfeas , K13) ;
K23=dec2mat(Nl , xfeas , K23) ;
L11=dec2mat(Nl , xfeas , L11) ;
L21=dec2mat(Nl , xfeas , L21) ;
L31=dec2mat(Nl , xfeas , L31) ;
M11=dec2mat(Nl , xfeas , M11) ;
M21=dec2mat(Nl , xfeas , M21) ;
M31=dec2mat(Nl , xfeas , M31)
tmin
```

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