NEW CONDITIONS FOR STABILITY OF TIME-VARYING DELAY WITH NEUTRAL SYSTEMS

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ABSTRACT

In this work, new conditions for stability of time-varying delay with neutral systems is studied, a new inequality which is the modified version of free-matrix-based integral inequality is considered, and then by using of this new inequality, two novel lemmas which are relaxed conditions for some matrices in a Lyapunov function are proposed. Based on the lemmas, improved delay-dependent stability criteria which guarantee the asymptotic stability of the system are show in the form of linear matrix in equalities (LMIs).

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บทคัดย่อ

งานวิจัยนี้ ผู้วิจัยจะทำการศึกษาเงื่อนไขใหม่สำหรับเสถียรภาพของตัวหน่วงที่แปรผันตามเวลาของระบบเป็นกลาง โดยการใช้การประมาณของอสมการใหม่ ซึ่งปรับปรุงมาจาก free-matrix-based และการใช้บทตั้งใหม่ เพื่อทำให้เงื่อนไขที่ถูกปรับปรุงดีขึ้น โดยการใช้ Lyapunov function ในการพิสูจน์ เนื่องจากเงื่อนไขที่ได้จะแสดงอยู่ในรูปของอสมการเมทริกซ์เชิงเส้นที่ขึ้นอยู่กับตัวหน่วงที่แปรผันตามเวลา

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CHAPTER 1

Introduction and Preliminaries

1. Introduction

Time delay is a natural phenomenon in real world. It is ell known that the existence of time delay often causes the oscillation, deterioration of system performance, and even instability, so the stability analysis of time-delay systems strongly requires before experimental stage. As these reason, the stability analysis of time -delay system has formed a sturdy research field during the past years (Gu, Kharitonov, & Chen, 2003)

This work focuses on to develop relaxed conditions for time-varying delay systems because above commented works on relaxed conditions can be applied only integral terms with constant time-delay interva., $i, e, \int_{t-h}^{t} f(d)ds.$

To this end, a new inequality is derived based on free-matrix-based integral inequality, and then by utilizing this new inequality two new relaxed conditions are studied. More over are show a numerical example to show the effectiveness of the obtained result.

Throughout of this paper, the superscripts -1 and T stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^{n}$ denotes n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; $p > 0$ means that the matrix P is symmetric and positive definite; * denotes symmetric terms in a symmetric matrix; $Sym\,X = X + X^{T};$ I is the identity matrix; and 0 is a zero matrix. If the dimensions of a matrix are not explicitly stated, the matrix is assumed to have compatible dimensions.

2.Preliminaries

Consider the following time-varying delay systems.

$$\dot{x}(t) - C\dot{x}\big(t-\tau(t)\big) = Ax(t) + Bx(t-h(t)),$$

(1)

$$x(t) = \emptyset(t), t \in [-h, 0]$$

Where $x(t) \in \mathbb{R}^n$ is the state vector , $h(t)$ is the time-varying delay satisfying

$0 \le h(t) \le h$ *and* $-\mu \le \dot{h}(t) \le \mu < 1$, $0 \le \tau(t) \le 1$, $\emptyset(t)$

is initial function, and *A* , *B* and *C* are know real constant matrices whit appropriate dimensions.

Problem formulation

The following lemmas will play a key role to derive main results.

Lemma 1 Zeng et and $Z_1, Z_3 \in \mathbb{R}^{3n\times 3n}$, and any matrices $Z_2 \in \mathbb{R}^{3n\times 3n}$

and $N_1, N_2 \in \mathbb{R}^{3n\times 3n}$ satisfying

$$\begin{bmatrix} Z_1 & Z_2 & N_1 \\ * & Z_3 & N_2 \\ * & * & R \end{bmatrix} \ge 0,$$

Let x be a differentiable function : $[\alpha, \beta] \to \mathbb{R}^n$.

For $R \in$ symmetric $\mathbb{R}^{n\times n}$

The following inequality holds:

$$-\int_{\alpha}^{\beta} \dot{x}^T(s)R\dot{x}(s)ds \le \omega_1^T(\alpha,\beta)\varphi_1\omega_1(\alpha,\beta),$$

where

$$\omega_1(\alpha,\beta) = [x^T(\beta), x^T(\alpha), \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta} x^T(s)ds]^T,$$

$$\varphi_1 = (\beta-\alpha)\left(Z_1 + \frac{1}{3}Z_3\right) + Sym\{N_1[I,-I,0] + N_2[-I,I,2I]\}.$$

Lemma 2 Let $x \in \mathbb{R}^n$ be a continuous function and admits a continuous derivative differentiable function in $[\alpha, \beta]$. For symmetric matrices $R \in \mathbb{R}^{n \times n}$

and $Z_1 \in \mathbb{R}^{2n \times 2n}$,and any matrix $Z_2 \in \mathbb{R}^{2n \times n}$ satisfying

$$\begin{bmatrix} Z_1 & Z_2 \\ * & R \end{bmatrix} \geq 0,$$

The following inequality hold:

$$-\int_{\alpha}^{\beta} x^T(s)Rx(s)ds \leq \omega_2^T(x,\alpha,\beta)\varphi_2\omega_2(x,\alpha,\beta).$$

Where

$$\omega_2(x,\alpha,\beta) = [\int_{\alpha}^{\beta} x^T(s)ds, \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta}\int_{v}^{\beta} x^T(s)dsdv]^T,$$

$$\varphi_2 = \frac{(\beta-\alpha)}{3}Z_1 + Sym\{Z_2[-I, 2I]\}.$$

Lemma 3 For the system (1) with given a positive constant h, if there exist positive definite matrices $Q_i \in \mathbb{R}^{3n \times 3n}$ (i = 1,2) , symmetric matrices

$P \in \mathbb{R}^{5n \times 5n}, G_i \in \mathbb{R}^{6n \times 6n}$ (i = 1,2) , and any matrices $H_i \in \mathbb{R}^{6n \times 3n}$ (i = 1,2) satisfying the following LMIs:

$$\forall h(t) \in \{0, h\}, \forall h^2(t) \in \{0, h^2\}, \forall h^3(t) \in \{0, h^3\}$$

$$\Sigma_{[\chi]} > 0,$$

$$\begin{bmatrix} G_1 & H_1 \\ * & Q_1 \end{bmatrix} \geq 0, \quad \begin{bmatrix} G_2 & H_2 \\ * & Q_2 \end{bmatrix} \geq 0,$$

Lemma 4. For the system (1) with given a positive constant h, if there exist positive definite matrices $Q_i \in \mathbb{R}^{2n\times 2n}(i=1,2)$, sysmmetric matrices $P \in \mathbb{R}^{5n\times 5n}, \bar{G}_i \in \mathbb{R}^{4n\times 4n}(i=1,2)$,

and any matrices $\bar{H}_i \in \mathbb{R}^{4n\times 2n}(i=1,2)$

Satisfying the following LMIs: $\forall h(t) \in \{0, h\}$, $\forall h^2(t) \in \{0, h^2\}$,

$\forall h^3(t) \in \{0, h^3\}$

$$\bar{\Sigma}_{[\mathcal{H}]} > 0, \tag{8}$$

$$\begin{bmatrix} \bar{G}_1 & \bar{H}_1 \\ * & Q_1 \end{bmatrix} \geq 0, \qquad \begin{bmatrix} \bar{G}_1 & \bar{H}_1 \\ * & Q_1 \end{bmatrix} \geq 0, \tag{9}$$

Basic Concepts

1. Types of Matrix

Let $M \in \mathbb{R}^{n\times m}$, then we have the following definition.

Definition 1.1 Matrix M is semi-positive definite if $x^T M x \geq 0$ for all $x \in \mathbb{R}^n$.

Definition 1.2 Matrix M is positive definite $x^T M x > 0$ if for all $x \in \mathbb{R}^n$, $x \neq 0$.

Definition 1.3 Matrix M is semi-negative definite $x^T M x \leq 0$ if for all $x \in \mathbb{R}^n$.

Definition 1.4 Matrix M is negative definite $x^T M x < 0$ if for all $x \in \mathbb{R}, x \neq 0$.

2.Notations

We give some important notations will be used throughout this thesis:

$\mathbb{R}^+$denotes the set of all non-negative real number;

$\mathbb{R}^n$denotes the n-dimensional Euclidean space;

$M > 0$ ($M \geq 0$) denotes the square symmetric, M is positive (semi-) definite matrix;

$M < 0$ ($M \leq 0$) denotes the square symmetric, M is negative (semi-) definite matrix;

$M > N$ ($M \geq N$) denotes the M - N matrix is square symmetric positive (semi-) definite matrix;

$M < N$ ($M \leq N$) denotes the M - N matrix is square symmetric negative (semi-) definite matrix;

$\mathbb{R}^{n \times m}$ denotes the space of all $(n \times m)$ real matrices;

A^T denotes the transpose of the vector/matrix A;

A^{-1} denotes the inverse of a non-singular matrix A;

I denotes the identity matrix;

3. Stability of Ordinary Differential Equation

Consider a dynamical system described by

$\dot{x}(t) = f(t, x(t))$

where $x \in \mathbb{R}^n$ and f is a vector having components $f_i(t, x_1, \dots, x_n)$,

$i = 1,2, \dots, n$. We shall assume that the f_i are continuous and satisfy standard conditions, such as having continuous first partial derivatives so that the solution of (1) exists and is unique for the given initial conditions. If f_i do not depend explicitly on t, (1) is called autonomous (otherwise, nonautonomous).

If $f(t, c) = 0$ for all t,

where c is some constant vector, then it follows at once from (1) that if

$x(t_0) = c$ then $x(t) = c$ for all $t \geq t_0$.

Definition The equilibrium point $x = 0$ of the system (2) is

(i) Stable if, for each $\varepsilon > 0$, $\delta = \delta\,(\varepsilon, t_0) > 0$ *such that*

$\| x(t_0)\| < \delta \rightarrow \|x(t)\| < \varepsilon, \forall t \geq t_0 \geq 0$,

(ii) Unstable if not stable,

(iii) asymptotically stable if it is stable and there is $c = c(t_0) > 0$ *such that*

$x(t) \rightarrow 0$ *as* $t \rightarrow \infty$, for all $\|x(t_0)\| < c$,

Theorem Let $x = 0$ be an equilibrium point for (2.2) and $D \subset \mathbb{R}^n$ be a domain containing $x = 0$.

Let $V(x): D \rightarrow \mathbb{R}$ be continuously differentiable function, such that

$$V(x) = 0 \text{ and } V(x) > 0 \text{ in } D - \{0\},$$

$$\dot{V}(x) \leq 0 \text{ in } D.$$

Then, $x = 0$ is stable. Moreover, if

$$\dot{V}(x) < 0 \text{ in } D - \{0\}.$$

Then, $x = 0$ is asymptotically stable.

CHAPTER 2

Main Results

<u>Theorem 1</u>. For given a positive constant h, the system (1) is asymptotically stable, if there exist positive definite matrices $Q_i \in \boldsymbol{R}^{3n\times 3n}(i = 1,2)$,

$R \in \boldsymbol{R}^{n\times n}$,symmetric matrices $P \in \boldsymbol{R}^{5n\times 5n}, Z_i \in \boldsymbol{R}^{3n\times 3n}(i = 1, \dots ,4)$,

$G_i \in \boldsymbol{R}^{6n\times 6n}(i = 1,2)$, and any matrices $Y_i \in \boldsymbol{R}^{3n\times 3n}(i = 1,2)$,

$N_i \in \boldsymbol{R}^{3n\times n}(i = 1, \dots ,4), H_i \in \boldsymbol{R}^{6n\times 6n}(i = 1,2)$ satisfying LMIs (2) and (3) and the following LMIs: for $\forall h(t) \in \{0, h\}, h(t) \in \{-\mu, \mu\}$

$$(\mathcal{B}^{\perp})^T \Omega_{[h(t),h(t)]}(\mathcal{B}^{\perp}) < 0, \quad (5)$$

$$\begin{bmatrix} Z_1 & Y_1 & N_1 \\ * & Z_2 & N_2 \\ * & * & R \end{bmatrix} \geq 0, \qquad \begin{bmatrix} Z_3 & Y_2 & N_3 \\ * & Z_4 & N_4 \\ * & * & R \end{bmatrix} \geq 0, \quad (6)$$

where

$\mathcal{B} = [A,\ B,\ 0, -I,\ 0,\ 0,\ 0,\ 0]$,
$e_0 = 0_{8n\times n}, \quad h_d(t) = 1 - \dot{h}(t), \tau_d(t) = 1 - \tau^{\cdot}(t)$

$\boldsymbol{\Omega}_{[\boldsymbol{h(t)},\dot{h}(\boldsymbol{t})]} = Sym\left\{\Xi_{1[h(t)]} p \Xi_{2[\dot{h}(t)]}\right\} + \Omega_{1[h(t),\dot{h}(t)]} + \Omega_{2[h(t),\dot{h}(t)]} + he_4 R_1 e_4^T + \emptyset_{1[h(t)]} + \emptyset_{2[h(t)]} + e_4 R_2 e_4^T - \tau_d(t) e_{12} R_2 e_{12}^T + \tau^2[e_4 Z_1 e_4^T] + \emptyset_{3[h(t)]}$

$\boldsymbol{\Omega}_{\boldsymbol{1}[\boldsymbol{h(t)},\dot{h}(\boldsymbol{t})]} = \Xi_3 Q_1 \Xi_3^T - h_d(t)\, \Xi_4 Q_1 \Xi_4^T + Sym\{\Xi_{5[h(t)]} Q_1 \Xi_6^T\}$,

$\boldsymbol{\Omega}_{\boldsymbol{2}[\boldsymbol{h(t)},\dot{h}(\boldsymbol{t})]} = h_d(t) \Xi_7 Q_2 \Xi_7^T - \Xi_8 Q_2 \Xi_8^T + Sym\left\{\Xi_{9[h(t)]} Q_2 \Xi_{10[\dot{h}(t)]}^T\right\}$,

$\Xi_{1[h(t)]} = [e_1,\ \ e_2,\ \ e_3,\ \ h(t)e_7,\ \ (h - h(t)e_8]$,

$\Xi_{2[h(t)]}n = [e_1, h_d(t)e_2, e_6, e_1 - h_d(t)e_2,\ h_d(t)e_2 - e_3]$,

$\Xi_3 = [e_1,\ e_4,\ e_0], \quad \Xi_4 = [e_2,\ e_5, e_1 - e_2]$,

$\Xi_{5[h(t)]} = [h(t)e_7,\ e_1 - e_2,\ h(t)(e_1 - e_7)]$,

$\Xi_6 = [e_0,\ e_0,\ e_4],\ \Xi_7 = [e_2,\ e_5,\ e_0]$,

$\Xi_8 = [e_3,\ e_6,\ e_2 - e_3],\ \Xi_{9[h(t)]} = [(h - h(t))e_8,\ \ e_2 - e_3,\ (h - h(t))(e_2 - e_8)]$,

$\Xi_{10[\dot{h}(t)]} = [e_0,\ \ e_0,\ \ h_d(t)e_5], \quad \Xi_{11} = [e_1,\ e_2,\ e_7]$,

$\Xi_{12} = [e_2,\ e_3,\ e_8]$, $\quad \Xi_{13} = [e_1,\ e_9,\ e_{13}]$,

$$\emptyset_{1[h(t)]} = h(t)\Xi_{11}\left(Z_1 + \frac{1}{3}Z_2\right)\Xi_{11}^T$$
$$+\mathrm{Sym}\{\Xi_{11}(N_1(e_1 - e_2)^T + N_2(2e_7 - e_1 - e_2)^T)\},$$

$$\emptyset_{2[h(t)]} = (h - h(t)\Xi_{12}\left(Z_3 + \frac{1}{3}Z_4\right)\Xi_{12}^T$$
$$+\mathrm{Sym}\{\Xi_{12}(N_3(e_2 - e_3)^T + N_4(2e_8 - e_2 - e_3)^T)\},$$

$$\emptyset_{3[h(t)]} = -\tau\Xi_{13[h(t)]}\left(Z_5 + \frac{1}{3}Z_6\right)\Xi_{13[h(t)]}^T$$
$$+\mathrm{Sym}\{\Xi_{13}(N_5(e_1 - e_9)^T + N_6(2e_{13} - e_1 - e_9)^T)\},$$

$$\zeta^T(t) = [x^T(t),\ x^T(t-h(t)), x^T(t-h), \dot{x}^T(t),\ \dot{x}^T(t-h(t)),$$
$$\dot{x}^T(t-h),\ \frac{1}{h(t)}\int_{t-h(t)}^{t} x^T(s)ds,\ \frac{1}{h-h(t)}\int_{t-h}^{t-h(t)} x^T(s)ds, x^T(t-\tau), -\frac{1}{\tau}\int_{t-\tau}^{t} x^T(s)ds],$$

<u>Proof</u>. Consider the following Lyapunov functional:

$$V(t) = V_a(t) + \int_{t-h}^{t}\int_{v}^{t}\dot{x}^T(s)R\dot{x}(s)dsdv + \int_{t-\tau(h)}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds$$
$$+\tau\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)d\theta ds$$

Where $V_a(t)$ is defined in Lemma 3. The time derivative of *V(t)* can be computed as follows:

$$V(t) = V_a(t) + \int_{t-h}^{t}\int_{v}^{t}\dot{x}^T(s)R\dot{x}(s)dsdv + \int_{t-\tau(h)}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds$$
$$+\tau\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)d\theta ds$$

$$= \eta_1^T(t)P\eta_1(t) + \int_{t-h(t)}^{t}\eta_2^T(v)Q_1\eta_2(v)dv + \int_{t-h}^{t-h(t)}\eta_3^T(v)Q_2\eta_3(v)dv$$
$$+\int_{t-h}^{t}\int_{v}^{t}\dot{x}^T(s)R\dot{x}(s)dsdv + \int_{t-\tau(h)}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds$$
$$+\tau\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)d\theta ds$$

$$= 2\dot{\eta}_1^T P\eta_1(t) + \eta_2^T(t)Q_1\eta_2(t) - \left(1 - \dot{h}(t)\right)\eta_2^T\left(t - h(t)\right)Q_1\eta_2\left(t - h(t)\right) + \left(1 - \dot{h}(t)\right)\eta_3^T\left(t - h(t)\right)Q_2\eta_3\left(t - h(t)\right) - \eta_3^T(t-h)Q_2\eta_3(t-h) + \int_{t-h}^{t}[\dot{x}^T(t)R\dot{x}(t) - \dot{x}^T(v)R\dot{x}^T(v)]dv + \int_{t-\tau(h)}^{t}\dot{x}^T(s)R_2\dot{x}(s)ds + \tau\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Z_1\dot{x}(s)d\theta ds$$

$$\begin{aligned} = 2\dot{\eta}_1^T P\eta_1(t) &+ \eta_2^T(t)Q_1\eta_2(t) - \left(1 - \dot{h}(t)\right)\eta_2^T\left(t - h(t)\right)Q_1\eta_2\left(t - h(t)\right) \\ &+ \left(1 - \dot{h}(t)\right)\eta_3^T\left(t - h(t)\right)Q_2\eta_3\left(t - h(t)\right) - \eta_3^T(t-h)Q_2\eta_3(t-h) \\ &+ \int_{t-h}^{t}\dot{x}^T(t)R\dot{x}(t)\,dv - \int_{t-h}^{t}\dot{x}^T(v)R\dot{x}^T(v)dv + \dot{x}^T(t)R_2\dot{x}(t) \\ &-(1 - \dot{\tau}(t))\dot{x}^T\left(t - \tau(t)\right)R_2\dot{x}\left(t - \tau(t)\right) \\ &+\tau\int_{-\tau}^{0}\dot{x}^T(s)Z_1(t) - \dot{x}^T(t+\theta)Z_1\dot{x}(t+\theta)ds \end{aligned}$$

$$\begin{aligned} = 2\dot{\eta}_1^T P\eta_1(t) &+ \eta_2^T(t)Q_1\eta_2(t) - \left(1 - \dot{h}(t)\right)\eta_2^T\left(t - h(t)\right)Q_1\eta_2\left(t - h(t)\right) \\ &+ \left(1 - \dot{h}(t)\right)\eta_3^T\left(t - h(t)\right)Q_2\eta_3\left(t - h(t)\right) - \eta_3^T(t-h)Q_2\eta_3(t-h) \\ &+ \dot{x}^T(t)R\dot{x}(t)\left(\int_{t-h}^{t}dv\right) - \int_{t-h}^{t}\dot{x}^T(v)R\dot{x}^T(v)dv \\ &+\dot{x}^T(t)R_2\dot{x}(t) - (1 - \dot{\tau}(t))\dot{x}^T(t - \tau(t))R_2\dot{x}(t - \tau(t)) \\ &+\int_{-\tau}^{0}\dot{x}(t)Z_1\dot{x}(t)ds - \int_{-\tau}^{0}\dot{x}(t-\theta)Z_1\dot{x}(t-\theta)ds \end{aligned}$$

$$\begin{aligned} = 2\dot{\eta}_1^T P\eta_1(t) &+ \eta_2^T(t)Q_1\eta_2(t) - \left(1 - \dot{h}(t)\right)\eta_2^T\left(t - h(t)\right)Q_1\eta_2\left(t - h(t)\right) \\ &+ \left(1 - \dot{h}(t)\right)\eta_3^T\left(t - h(t)\right)Q_2\eta_3\left(t - h(t)\right) \\ &-\eta_3^T(t-h)Q_2\eta_3(t-h) + \dot{x}^T(t)R\dot{x}(t)(t - t + h) - \int_{t-h}^{t}\dot{x}^T(v)R\dot{x}^T(v)dv \\ &+ \dot{x}^T(t)R_2\dot{x}(t) - (1 - \dot{\tau}(t))\dot{x}^T(t - \tau(t))R_2\dot{x}(t - \tau(t)) \\ &+\tau[\dot{x}^T(t)Z_1\dot{x}(t)]\int_{\tau}^{0}ds - \tau\int_{t-\tau}^{t}\dot{x}^T(s)Z_1\dot{x}(s)ds \end{aligned}$$

$$\begin{aligned} = 2\dot{\eta}_1^T P\eta_1(t) &+ \eta_2^T(t)Q_1\eta_2(t) - \left(1 - \dot{h}(t)\right)\eta_2^T\left(t - h(t)\right)Q_1\eta_2\left(t - h(t)\right) \\ &+ \left(1 - \dot{h}(t)\right)\eta_3^T\left(t - h(t)\right)Q_2\eta_3\left(t - h(t)\right) - \eta_3^T(t-h)Q_2\eta_3(t-h) \\ &+ h\dot{x}^T(t)R\dot{x}(t) - \int_{t-h}^{t}\dot{x}^T(v)R\dot{x}^T(v)dv \\ &+\dot{x}^T(t)R_2\dot{x}(t) - (1 - \dot{\tau}(t))\dot{x}^T(t - \tau(t))R_2\dot{x}(t - \tau(t)) \end{aligned}$$

$$+\tau^2[\dot{x}^T(t)Z_1\dot{x}(t)] - \tau\int_{t-\tau}^{t}\dot{x}^T(s)Z_1\dot{x}(s)ds$$

$$\begin{aligned}
= 2\dot{\eta}_1^T P\eta_1(t) + \eta_2^T(t)Q_1\eta_2(t) &- \left(1-\dot{h}(t)\right)\eta_2^T\big(t-h(t)\big)Q_1\eta_2\big(t-h(t)\big) \\
&+ \left(1-\dot{h}(t)\right)\eta_3^T\big(t-h(t)\big)Q_2\eta_3\big(t-h(t)\big) - \eta_3^T(t-h)Q_2\eta_3(t-h) \\
&+ h\dot{x}^T(t)R\dot{x}(t) - \int_{-h}^{-h(t)}\dot{x}^T(v-t)R\dot{x}^T(v-t)d(v-t) \\
&\qquad - \int_{-h(t)}^{0}\dot{x}^T(v-t)R\dot{x}^T(v-t)d(v-t) \\
&\qquad + \dot{x}^T(t)R_2\dot{x}(t) - (1-\dot{\tau}(t))\dot{x}^T(t-\tau(t))R_2\dot{x}(t-\tau(t)) \\
&\qquad + \tau^2[\dot{x}^T(t)Z_1\dot{x}(t)] + \omega_1^T(t-\tau,t)\varphi_1\omega_1(t-\tau,t)
\end{aligned}$$

$$\begin{aligned}
= 2\dot{\eta}_1^T P\eta_1(t) + \eta_2^T(t)Q_1\eta_2(t) &- \left(1-\dot{h}(t)\right)\eta_2^T\big(t-h(t)\big)Q_1\eta_2\big(t-h(t)\big) \\
&+ \left(1-\dot{h}(t)\right)\eta_3^T\big(t-h(t)\big)Q_2\eta_3\big(t-h(t)\big) - \eta_3^T(t-h)Q_2\eta_3(t-h) \\
&+ \dot{x}^T(t)R_2\dot{x}(t) + \omega_1^T(-h,-h(t))\varphi_1\omega_1(-h,-h(t)) \\
&+ \omega_1^T(-h(t),0)\varphi_1\omega_1(-h(t),0)
\end{aligned}$$

$$+\dot{x}^T(t)R_2\dot{x}(t) - \big(1-\dot{\tau}(t)\big)\dot{x}^T\big(t-\tau(t)\big)R_2\dot{x}\big(t-\tau(t)\big)$$

$$+\tau^2[\dot{x}^T(t)Z_1\dot{x}(t)] + \omega_1^T(t-\tau,t)\varphi_1\omega_1(t-\tau,t)$$

$$\begin{aligned}
= 2\dot{\eta}_1^T P\eta_1(t) + \eta_2^T(t)Q_1\eta_2(t) &- \left(1-\dot{h}(t)\right)\eta_2^T\big(t-h(t)\big)Q_1\eta_2\big(t-h(t)\big) \\
&+ \left(1-\dot{h}(t)\right)\eta_3^T\big(t-h(t)\big)Q_2\eta_3\big(t-h(t)\big) - \eta_3^T(t-h)Q_2\eta_3(t-h) \\
&+ [x^T\big(t-h(t)\big),\ x^T(t-h), \frac{1}{h-h(t)}\int_{t-h}^{t-h(t)}x^T(s)ds]\begin{bmatrix} W_{11} & W_{12} & W_{13} \\ W_{12}^T & W_{22} & W_{32} \\ W_{13}^T & W_{23}^T & W_{33}\end{bmatrix} \\
&+ \frac{1}{3}\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{12}^T & G_{22} & G_{32} \\ G_{13}^T & G_{23}^T & G_{33}\end{bmatrix} \\
&+ \left\{\begin{bmatrix} N_1 & 0 & 0 \\ 0 & -N_1 & 0 \\ 0 & 0 & 0\end{bmatrix}, \begin{bmatrix} -N_2 & 0 & 0 \\ 0 & -N_2 & 0 \\ 0 & 0 & 2N_2\end{bmatrix}\right\}^T \times\ [x^T\big(t-h(t)\big), x^T(t \\
&- h), \frac{1}{h-h(t)}\int_{t-h}^{t-h(t)}x^T(s)ds]^T
\end{aligned}$$

$$+[x^T(t), x^T\big(t-h(t)\big), \frac{1}{h(t)}\int_{t-h(t)}^{t}x^T(s)ds]\big(h(t)\big)\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{12}^T & Z_{22} & Z_{32} \\ Z_{13}^T & Z_{23}^T & Z_{33}\end{bmatrix}$$

$$+\frac{1}{3}\begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12}^T & Q_{22} & Q_{32} \\ Q_{13}^T & Q_{23}^T & Q_{33}\end{bmatrix} + \left\{\begin{bmatrix} N_3 & 0 & 0 \\ 0 & -N_3 & 0 \\ 0 & 0 & 0\end{bmatrix}, \begin{bmatrix} -N_4 & 0 & 0 \\ 0 & -N_4 & 0 \\ 0 & 0 & 2N_4\end{bmatrix}\right\}^T$$

$$\times [x^T(t), x^T(t-h(t)), \frac{1}{h(t)}\int_{t-h(t)}^{t} x^T(s)ds]^T$$

$$+\dot{x}^T(t)R_2\dot{x}(t) - (1-\dot{\tau}(t))\dot{x}^T(t-\tau(t))R_2\dot{x}(t-\tau(t))$$

$$+\tau^2[\dot{x}^T(t)Q_3\dot{x}(t)] + [x^T(t), x^T(t-\tau), -\frac{1}{\tau}\int_{t-}^{t} \tau x^T(s)ds] \times$$

$$(-\tau)(Z_5 + \frac{1}{3}Z_6) + \left\{\begin{bmatrix} N_5 & 0 & 0 \\ 0 & -N_5 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} -N_6 & 0 & 0 \\ 0 & -N_6 & 0 \\ 0 & 0 & 2N_6 \end{bmatrix}\right\}^T$$

$$\times [x^T(t), x^T(t-\tau), -\frac{1}{\tau}\int_{t-\tau}^{t} x^T(s)ds]^T$$

It should be mentioned that from **Lemma3.** With **LMIs (2) , (3)** and the fact , R > 0 , V(t) is a positive definite function.

By applying **Lemma2**. With **LMIs (6),** the upper bound of the derivative of V(t) can be estimated as: $\dot{V}(t) \leq \zeta^T \Omega_{[h(t),\dot{h}(t)]}\zeta(t)$

Where $\Omega_{[h(t),\dot{h}(t)]}$ and $\zeta(t)$ and defined in **Theorem1**. And the beginning of this section , respectively.

Thus , **by Finsler's Lemma (Skelton, Iwaki, and Grigoradis,1997)**

It is clear that if **LMIs (5)** holds, then system (1) is asymptotically stable, which completes the proof.

□

CHAPTER 3

Numerical Example

3.1 Numerical example

<u>Example 3.1</u> Consider the system (1) with

$$\dot{x}(t) - C\dot{x}\big(t - \tau(t)\big) = Ax(t) + Bx(t - h(t)),$$

Where $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ $C = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$

<u>Solution:</u> Theorem 1 by using Matlab LMI toolbox. From the condition (1) are feasible with solutions given by,

$q_{11} = q_{22} = g_{11} = g_{22} = g_{33} = R_1 = z_{11} = z_{22} = z_{33} = w_{11} = w_{22} = w_{33} = s_{11} = s_{22} = v_{11} = v_{22} = v_{33} = 1 \times 10^8 \begin{bmatrix} 1.5462 & 0 \\ 0 & 1.5462 \end{bmatrix}$

and the rest of metrices are equal to $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

CHAPTER 4

Conclusion

4.1 Conclusion

Theorem 1. For given a positive constant h, the system(1) is asymptotically stable, if there exist positive definite matrices $Q_i \in \boldsymbol{R}^{3n\times 3n}(i = 1,2),\ R \in \boldsymbol{R}^{n\times n}$,symmetric matrices $P \in \boldsymbol{R}^{5n\times 5n}, Z_i \in \boldsymbol{R}^{3n\times 3n}(i = 1, \dots, 4)$,

$G_i \in \boldsymbol{R}^{6n\times 6n}(i = 1,2)$, and any matrices $Y_i \in \boldsymbol{R}^{3n\times 3n}(i = 1,2)$,

$N_i \in \boldsymbol{R}^{3n\times n}(i = 1, \dots, 4), H_i \in \boldsymbol{R}^{6n\times 6n}(i = 1,2)$ satisfying LMIs (2) and (3) and the following LMIs: for $\forall h(t) \in \{0, h\}, h(t) \in \{-\mu, \mu\}$

$$(\mathcal{B}^{\perp})^T \Omega_{[h(t),h(t)]}(\mathcal{B}^{\perp}) < 0, \tag{5}$$

$$\begin{bmatrix} Z_1 & Y_1 & N_1 \\ * & Z_2 & N_2 \\ * & * & R \end{bmatrix} \geq 0, \qquad \begin{bmatrix} Z_3 & Y_2 & N_3 \\ * & Z_4 & N_4 \\ * & * & R \end{bmatrix} \geq 0, \tag{6}$$

where

$\mathcal{B} = [A,\ B,\ 0, -I,\ 0,\ 0,\ 0,\ 0]$,
$e_0 = 0_{8n\times n}, \quad h_d(t) = 1 - \dot{h}(t), \tau_d(t) = 1 - \dot{\tau}(t)$

$\boldsymbol{\Omega}_{[\boldsymbol{h(t)},\dot{h}(t)]} = Sym\left\{\Xi_{1[h(t)]} p \Xi_{2[\dot{h}(t)]}\right\} + \Omega_{1[h(t),\dot{h}(t)]} + \Omega_{2[h(t),\dot{h}(t)]} + he_4 R_1 e_4^T + \emptyset_{1[h(t)]} + \emptyset_{2[h(t)]} + e_4 R_2 e_4^T - \tau_d(t) e_{12} R_2 e_{12}^T + \tau^2 [e_4 Z_1 e_4^T] + \emptyset_{3[h(t)]}$

$\boldsymbol{\Omega}_{\boldsymbol{1}[\boldsymbol{h(t)},\dot{h}(t)]} = \Xi_3 Q_1 \Xi_3^T - h_d(t)\, \Xi_4 Q_1 \Xi_4^T + Sym\{\Xi_{5[h(t)]} Q_1 \Xi_6^T\}$,

$\boldsymbol{\Omega}_{\boldsymbol{2}[\boldsymbol{h(t)},\dot{h}(t)]} = h_d(t) \Xi_7 Q_2 \Xi_7^T - \Xi_8 Q_2 \Xi_8^T + Sym\left\{\Xi_{9[h(t)]} Q_2 \Xi_{10[\dot{h}(t)]}^T\right\}$,

$\Xi_{1[h(t)]} = [e_1,\ e_2,\ e_3,\ h(t)e_7,\ (h - h(t)e_8]$,

$\Xi_{2[h(t)]}n = [e_1, h_d(t)e_2, e_6, e_1 - h_d(t)e_2,\ h_d(t)e_2 - e_3]$,

$\Xi_3 = [e_1,\ e_4,\ e_0], \quad \Xi_4 = [e_2,\ e_5,\ e_1 - e_2]$,

$\Xi_{5[h(t)]} = [h(t)e_7,\ e_1 - e_2,\ h(t)(e_1 - e_7)]$,

$\Xi_6 = [e_0,\ e_0,\ e_4],\ \Xi_7 = [e_2,\ e_5,\ e_0]$,

$\Xi_8 = [e_3,\ e_6,\ e_2 - e_3]$,

$\Xi_{9[h(t)]} = [(h - h(t))e_8,\ e_2 - e_3,\ (h - h(t))(e_2 - e_8)]$,

$\Xi_{10[\dot{h}(t)]} = [e_0, \quad e_0, \quad h_d(t)e_5], \ \Xi_{11} = [e_1, \quad e_2, \quad e_7],$

$\Xi_{12} = [e_2, \quad e_3, \quad e_8],], \quad \Xi_{13} = [e_1, \quad e_9, \quad e_{13}],$

$$\emptyset_{1[h(t)]} = h(t)\Xi_{11}\left(Z_1 + \frac{1}{3}Z_2\right)\Xi_{11}^T$$

$$+Sym\{\Xi_{11}(N_1(e_1 - e_2)^T + N_2(2e_7 - e_1 - e_2)^T)\},$$

$$\emptyset_{2[h(t)]} = (h - h(t)\Xi_{12}\left(Z_3 + \frac{1}{3}Z_4\right)\Xi_{12}^T$$

$$+Sym\{\Xi_{12}(N_3(e_2 - e_3)^T + N_4(2e_8 - e_2 - e_3)^T)\}$$

$$\emptyset_{3[h(t)]} = -\tau\Xi_{13[h(t)]}\left(Z_5 + \frac{1}{3}Z_6\right)\Xi_{13[h(t)]}^T$$

$$+Sym\{\Xi_{13}(N_5(e_1 - e_9)^T + N_6(2e_{13} - e_1 - e_9)^T)\},$$

$$\zeta^T(t) = [x^T(t), \ x^T(t-h(t)), x^T(t-h), \dot{x}^T(t), \quad \dot{x}^T(t-h(t)),$$

$$\dot{x}^T(t-h), \quad \frac{1}{h(t)}\int_{t-h(t)}^{t} x^T(s)ds, \quad \frac{1}{h-h(t)}\int_{t-h}^{t-h(t)} x^T(s)ds, x^T(t-\tau), -\frac{1}{\tau}\int_{t-\tau}^{t} x^T(s)ds],$$

BIBLIOGRAPHY

BIBLIOGRAPHY

[1] Kwon, O. M., J., Park, J. H., Lee, S. M., & Cha, E. J. (2014). Improved results on stability of linear systems with time-varying delay via wirtinger-based integral inequality. Journal of the franklin Institute, 351, 5386-5398

[2] Seuret, A., & Gouaisbaut, F. (2013). Wirtinger-based integral inequality : application to time-delay systems. Automatica, 49, 2860-2866

[3] Skelton, R. E., Iwasaki, T., & Grigoradis, K. M.(1997). A unified algebraic approachto linear control desing. New York: Taylor and Francis.

[4] Xu, S., Lam, J., Zhang, B., & Zou, Y.(2015). New insight into delay-dependent stability of time-delay systems. International Journal of Robust and Nonlinear Control, 25, 961-970

[5] Zeng, H. B., He, Y., Wu, M., & She, J. (2015a). Free-matrix-based integral integral inequality for stability analysis of systems with time-varying delay. IEEE Transactions on Automatic Control, 60(10). 2768-2772.

[6] Zeng, H. -B., He. Y., Wu, M., & She, J. (2015b). New results on stability analysis for systems with discrete distributed delay. Automatica, 6, 189-192.

[7] Zhang, B., Lam, J., & Xu, S. (2015a). Relaxed results on reachable set estimation of time-delay systems with bounded peak inputs. International Journal of Robust and Nonlinear Control, http;//dx.doi.org/10.1002/rnc.3395.

[8] Zhang, B., Lam, J., & Xu, S. (2015b). Stability analysis of distributed delay neural networks based on relaxed Lyapunov-krasovskii functionals. IEEE Transactions of Neural Networks and Leaning Systems, 26(7), 1480-1492.

[9] Zhang, X. M., Wu, M., She, J., & He, Y, (2005). Delay-dependent stabilization of linear systems with time-varying state and input delays. Automatica, 41(8), 1405-1412.

MATHLAB CODE

MATHLAB CODE

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A=[0 1;-1 -2];
B=[0 0;-1 1];
C=[0.05 0;0 0.05];

setlmis([]);
q11=lmivar(1,[2,1]);
q12=lmivar(1,[2,1]);
q13=lmivar(1,[2,1]);
q22=lmivar(1,[2,1]);
q23=lmivar(1,[2,1]);
q33=lmivar(1,[2,1]);
g11=lmivar(1,[2,1]);
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g22=lmivar(1,[2,1]);
g23=lmivar(1,[2,1]);
g33=lmivar(1,[2,1]);
R1=lmivar(1,[2,1]);
R2=lmivar(1,[2,1]);
p11=lmivar(1,[2,1]);
p12=lmivar(1,[2,1]);
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p15=lmivar(1,[2,1]);
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p23=lmivar(1,[2,1]);
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p55=lmivar(1,[2,1]);
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w22=lmivar(1,[2,1]);
w23=lmivar(1,[2,1]);
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y33=lmivar(2,[2,2]);
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k12=lmivar(2,[2,2]);
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```

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m22=lmivar(2,[2,2]);
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m51=lmivar(2,[2,2]);
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m61=lmivar(2,[2,2]);
m62=lmivar(2,[2,2]);
m63=lmivar(2,[2,2]);



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lmiterm([-2 2 1 -z12],1,1);         % LMI #2: z12'
lmiterm([-2 2 2 z22],1,1);          % LMI #2: z22
lmiterm([-2 3 1 -z13],1,1);         % LMI #2: z13'
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lmiterm([-2 5 4 -w12],1,1);         % LMI #2: w12'
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lmiterm([-2 6 2 y23],1,1);          % LMI #2: y23
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lmiterm([-2 6 4 -w13],1,1);         % LMI #2: w13'
```

```
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lmiterm([-2 7 2 n2],1,1);               % LMI #2: n2
lmiterm([-2 7 3 n3],1,1);               % LMI #2: n3
lmiterm([-2 7 4 n4],1,1);               % LMI #2: n4
lmiterm([-2 7 5 n5],1,1);               % LMI #2: n5
lmiterm([-2 7 6 n6],1,1);               % LMI #2: n6
lmiterm([-2 7 7 R1],1,1);               % LMI #2: R1


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lmiterm([-3 2 1 -s12],1,1);             % LMI #3: s12'
lmiterm([-3 2 2 s22],1,1);              % LMI #3: s22
lmiterm([-3 3 1 -s13],1,1);             % LMI #3: s13'
lmiterm([-3 3 2 -s23],1,1);             % LMI #3: s23'
lmiterm([-3 3 3 s33],1,1);              % LMI #3: s33
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lmiterm([-3 4 2 k21],1,1);              % LMI #3: k21
lmiterm([-3 4 3 k31],1,1);              % LMI #3: k31
lmiterm([-3 4 4 v11],1,1);              % LMI #3: v11
lmiterm([-3 5 1 k12],1,1);              % LMI #3: k12
lmiterm([-3 5 2 k22],1,1);              % LMI #3: k22
lmiterm([-3 5 3 k32],1,1);              % LMI #3: k32
lmiterm([-3 5 4 -v12],1,1);             % LMI #3: v12'
lmiterm([-3 5 5 v22],1,1);              % LMI #3: v22
lmiterm([-3 6 1 k13],1,1);              % LMI #3: k13
lmiterm([-3 6 2 k23],1,1);              % LMI #3: k23
```

```
lmiterm([-3 6 3 k33],1,1);          % LMI #3: k33
lmiterm([-3 6 4 -v13],1,1);         % LMI #3: v13'
lmiterm([-3 6 5 -v23],1,1);         % LMI #3: v23'
lmiterm([-3 6 6 v33],1,1);          % LMI #3: v33
lmiterm([-3 7 1 n7],1,1);           % LMI #3: n7
lmiterm([-3 7 2 n8],1,1);           % LMI #3: n8
lmiterm([-3 7 3 n9],1,1);           % LMI #3: n9
lmiterm([-3 7 4 n10],1,1);          % LMI #3: n10
lmiterm([-3 7 5 n11],1,1);          % LMI #3: n11
lmiterm([-3 7 6 n12],1,1);          % LMI #3: n12
lmiterm([-3 7 7 R2],1,1);           % LMI #3: R2


lmiterm([-4 1 1 q11],1,1);          % LMI #4: q11
lmiterm([-4 2 1 -q12],1,1);         % LMI #4: q12'
lmiterm([-4 2 2 q22],1,1);          % LMI #4: q22
lmiterm([-4 3 1 -q13],1,1);         % LMI #4: q13'
lmiterm([-4 3 2 -q23],1,1);         % LMI #4: q23'
lmiterm([-4 3 3 q33],1,1);          % LMI #4: q33


lmiterm([-5 1 1 g11],1,1);          % LMI #5: g11
lmiterm([-5 2 1 -g12],1,1);         % LMI #5: g12'
lmiterm([-5 2 2 g22],1,1);          % LMI #5: g22
lmiterm([-5 3 1 -g13],1,1);         % LMI #5: g13'
lmiterm([-5 3 2 -g23],1,1);         % LMI #5: g23'
lmiterm([-5 3 3 g33],1,1);          % LMI #5: g33
```

```
lmiterm([-6 1 1 R1],1,1);                % LMI #6: R1

lmiterm([-7 1 1 R2],1,1);                % LMI #7: R2

Apisara=getlmis;

[tmin,xfeas]=feasp(Apisara)
q11=dec2mat(Apisara,xfeas,g11)
q12=dec2mat(Apisara,xfeas,g12)
q13=dec2mat(Apisara,xfeas,g13)
q22=dec2mat(Apisara,xfeas,g22)
q23=dec2mat(Apisara,xfeas,g23)
q33=dec2mat(Apisara,xfeas,g33)
g11=dec2mat(Apisara,xfeas,g11)
g12=dec2mat(Apisara,xfeas,g12)
g13=dec2mat(Apisara,xfeas,g13)
g22=dec2mat(Apisara,xfeas,g22)
g23=dec2mat(Apisara,xfeas,g23)
g33=dec2mat(Apisara,xfeas,g33)
R1=dec2mat(Apisara,xfeas,R1)
R2=dec2mat(Apisara,xfeas,R2)
p11=dec2mat(Apisara,xfeas,p11)
p12=dec2mat(Apisara,xfeas,p12)
p13=dec2mat(Apisara,xfeas,p13)
p14=dec2mat(Apisara,xfeas,p14)
p15=dec2mat(Apisara,xfeas,p15)
p22=dec2mat(Apisara,xfeas,p22)
```

```
p23=dec2mat(Apisara,xfeas,p23)
p24=dec2mat(Apisara,xfeas,p24)
p25=dec2mat(Apisara,xfeas,p25)
p33=dec2mat(Apisara,xfeas,p33)
p34=dec2mat(Apisara,xfeas,p34)
p35=dec2mat(Apisara,xfeas,p35)
p44=dec2mat(Apisara,xfeas,p44)
p45=dec2mat(Apisara,xfeas,p45)
p55=dec2mat(Apisara,xfeas,p55)
z11=dec2mat(Apisara,xfeas,z11)
z12=dec2mat(Apisara,xfeas,z12)
z13=dec2mat(Apisara,xfeas,z13)
z22=dec2mat(Apisara,xfeas,z22)
Z23=dec2mat(Apisara,xfeas,z23)
Z33=dec2mat(Apisara,xfeas,z33)
w11=dec2mat(Apisara,xfeas,w11)
w12=dec2mat(Apisara,xfeas,w12)
w13=dec2mat(Apisara,xfeas,w13)
w22=dec2mat(Apisara,xfeas,w22)
w23=dec2mat(Apisara,xfeas,w23)
w33=dec2mat(Apisara,xfeas,w33)
s11=dec2mat(Apisara,xfeas,s11)
s12=dec2mat(Apisara,xfeas,s12)
s13=dec2mat(Apisara,xfeas,s13)
s22=dec2mat(Apisara,xfeas,s22)
s23=dec2mat(Apisara,xfeas,s23)
v11=dec2mat(Apisara,xfeas,v11)
v12=dec2mat(Apisara,xfeas,v12)
```

```
v13=dec2mat(Apisara,xfeas,v13)
v22=dec2mat(Apisara,xfeas,v22)
v23=dec2mat(Apisara,xfeas,v23)
v33=dec2mat(Apisara,xfeas,v33)
r11=dec2mat(Apisara,xfeas,r11)
r12=dec2mat(Apisara,xfeas,r12)
r13=dec2mat(Apisara,xfeas,r13)
r14=dec2mat(Apisara,xfeas,r14)
r15=dec2mat(Apisara,xfeas,r15)
r16=dec2mat(Apisara,xfeas,r16)
r22=dec2mat(Apisara,xfeas,r22)
r23=dec2mat(Apisara,xfeas,r23)
r24=dec2mat(Apisara,xfeas,r24)
r25=dec2mat(Apisara,xfeas,r25)
r26=dec2mat(Apisara,xfeas,r26)
r33=dec2mat(Apisara,xfeas,r33)
r34=dec2mat(Apisara,xfeas,r34)
r35=dec2mat(Apisara,xfeas,r35)
r36=dec2mat(Apisara,xfeas,r36)
r44=dec2mat(Apisara,xfeas,r44)
r45=dec2mat(Apisara,xfeas,r45)
r46=dec2mat(Apisara,xfeas,r46)
r55=dec2mat(Apisara,xfeas,r55)
l11=dec2mat(Apisara,xfeas,l11)
l12=dec2mat(Apisara,xfeas,l12)
l13=dec2mat(Apisara,xfeas,l13)
l14=dec2mat(Apisara,xfeas,l14)
l15=dec2mat(Apisara,xfeas,l15)
```

```
l16=dec2mat(Apisara,xfeas,l16)
l22=dec2mat(Apisara,xfeas,l22)
l23=dec2mat(Apisara,xfeas,l23)
l24=dec2mat(Apisara,xfeas,l24)
l25=dec2mat(Apisara,xfeas,l25)
l26=dec2mat(Apisara,xfeas,l26)
l33=dec2mat(Apisara,xfeas,l33)
l34=dec2mat(Apisara,xfeas,l34)
l35=dec2mat(Apisara,xfeas,l35)
l36=dec2mat(Apisara,xfeas,l36)
l44=dec2mat(Apisara,xfeas,l44)
l45=dec2mat(Apisara,xfeas,l45)
l46=dec2mat(Apisara,xfeas,l46)
l55=dec2mat(Apisara,xfeas,l55)
l56=dec2mat(Apisara,xfeas,l56)
l66=dec2mat(Apisara,xfeas,l66)
y11=dec2mat(Apisara,xfeas,y11)
y12=dec2mat(Apisara,xfeas,y12)
y13=dec2mat(Apisara,xfeas,y13)
y21=dec2mat(Apisara,xfeas,y21)
y22=dec2mat(Apisara,xfeas,y22)
y23=dec2mat(Apisara,xfeas,y23)
y31=dec2mat(Apisara,xfeas,y31)
y32=dec2mat(Apisara,xfeas,y32)
y33=dec2mat(Apisara,xfeas,y33)
k11=dec2mat(Apisara,xfeas,k11)
k12=dec2mat(Apisara,xfeas,k12)
k13=dec2mat(Apisara,xfeas,k13)
```

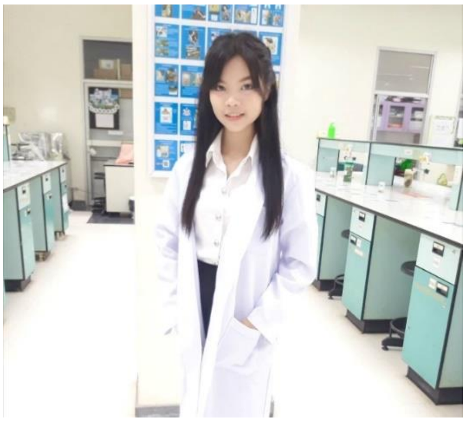
```
k21=dec2mat(Apisara,xfeas,k21)
k22=dec2mat(Apisara,xfeas,k22)
k23=dec2mat(Apisara,xfeas,k23)
k31=dec2mat(Apisara,xfeas,k31)
k32=dec2mat(Apisara,xfeas,k32)
k33=dec2mat(Apisara,xfeas,k33)
n1=dec2mat(Apisara,xfeas,n1)
n2=dec2mat(Apisara,xfeas,n2)
n3=dec2mat(Apisara,xfeas,n3)
n4=dec2mat(Apisara,xfeas,n4)
n5=dec2mat(Apisara,xfeas,n5)
n6=dec2mat(Apisara,xfeas,n6)
n7=dec2mat(Apisara,xfeas,n7)
n8=dec2mat(Apisara,xfeas,n8)
n9=dec2mat(Apisara,xfeas,n9)
n10=dec2mat(Apisara,xfeas,n10)
n11=dec2mat(Apisara,xfeas,n11)
n12=dec2mat(Apisara,xfeas,n12)
n13=dec2mat(Apisara,xfeas,n13)
n14=dec2mat(Apisara,xfeas,n14)
n15=dec2mat(Apisara,xfeas,n15)
n16=dec2mat(Apisara,xfeas,n16)
n17=dec2mat(Apisara,xfeas,n17)
n18=dec2mat(Apisara,xfeas,n18)
h11=dec2mat(Apisara,xfeas,h11)
h12=dec2mat(Apisara,xfeas,h12)
h13=dec2mat(Apisara,xfeas,h13)
h21=dec2mat(Apisara,xfeas,h21)
```

```
h22=dec2mat(Apisara,xfeas,h22)
h23=dec2mat(Apisara,xfeas,h23)
h31=dec2mat(Apisara,xfeas,h31)
h32=dec2mat(Apisara,xfeas,h32)
h33=dec2mat(Apisara,xfeas,h33)
h41=dec2mat(Apisara,xfeas,h41)
h42=dec2mat(Apisara,xfeas,h42)
h43=dec2mat(Apisara,xfeas,h43)
h51=dec2mat(Apisara,xfeas,h51)
h52=dec2mat(Apisara,xfeas,h52)
h53=dec2mat(Apisara,xfeas,h53)
h61=dec2mat(Apisara,xfeas,h61)
h62=dec2mat(Apisara,xfeas,h62)
h63=dec2mat(Apisara,xfeas,h63)
m11=dec2mat(Apisara,xfeas,m11)
m12=dec2mat(Apisara,xfeas,m12)
m13=dec2mat(Apisara,xfeas,m13)
m21=dec2mat(Apisara,xfeas,m21)
m22=dec2mat(Apisara,xfeas,m22)
m23=dec2mat(Apisara,xfeas,m23)
m31=dec2mat(Apisara,xfeas,m31)
m32=dec2mat(Apisara,xfeas,m32)
m33=dec2mat(Apisara,xfeas,m33)
m41=dec2mat(Apisara,xfeas,m41)
m42=dec2mat(Apisara,xfeas,m42)
m43=dec2mat(Apisara,xfeas,m43)
m51=dec2mat(Apisara,xfeas,m51)
m52=dec2mat(Apisara,xfeas,m52)
```

```
m53=dec2mat(Apisara,xfeas,m53)
m61=dec2mat(Apisara,xfeas,m61)
m62=dec2mat(Apisara,xfeas,m62)
m63=dec2mat(Apisara,xfeas,m63)
```

BIOGRAPHY

BIOGRAPHY



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Education Background

Year 2016 **Junior High School**

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Sukhothai, Thailand

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Date of Birth	**13 February 1998**
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Year 2016	**Junior High School** **and Senior High School from** **Banmaijaroenpon Pittayakhom School,** **Sukhothai, Thailand**