A NEUTRAL SYSTEM APPROACH TO STABILITY OF SINGULAR TIME-VARYING DELAY SYSTEM

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Advisor and Dean of School of Science have considered the independent study entitled "A neutral system approach to stability of singular time-varying delay system" submitted in partial fulfillment of the requirements for the degree of Bachelor of Science Program in Mathematics is hereby approved.

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ชื่อเรื่อง การใช้วิธีทางระบบเป็นกลางสำหรับเสถียรของระบบเอกฐานที่มีตัวหน่วงแปรผันตามเวลา

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บทคัดย่อ

ในการศึกษาอิสระนี้ จะศึกษาปัญหาเกี่ยวกับเสถียรภาพของระบบเอกฐานที่มีตัวหน่วงแปรผันตามเวลา โดยวิธีการเปลี่ยนระบบเอกฐานให้อยู่ในรูประบบเป็นกลางแล้วใช้วิธีไลปูนอฟคราซอฟสกีและวิธีอสมการเวอทิงเจอร์ ทำให้เราได้เงื่อนไขเพียงพอใหม่สำหรับเสถียรภาพของระบบดังกล่าว โดยเงื่อนไขเพียงพอข้างต้นจะอยู่ในรูปสมการเมทริกซ์เชิงเส้น เงื่อนไขนี้สามารถนำไปทดสอบเสถียรภาพของระบบเอกฐานและระบบเป็นกลางที่มีตัวหน่วงแปรผันตามเวลาได้ในตอนท้ายของการศึกษาอิสระนี้ ได้ยกตัวอย่างเพื่อแสดงให้เห็นถึงประสิทธิภาพของวิธีการดังกล่าว

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ABSTRACT

This independent study is concerned with the problem of delay-dependent stability for a class of singular time-varying delay system. By representing the singular system as a neutral form, using an augmented Lyapunov-Krasovskii functional and the Wirtinger-based inequality method, we obtain a new stability criterion in terms of a linear matrix inequality (LMI). The criterion is applicable for the stability test of both singular time-varying delay systems and neutral systems with time-varying delays.

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CHAPTER 1

INTRODUCTION AND PRELIMINARIES

1.1 Introduction

Singular systems (also referred to as descriptor, generalized, differential-algebraic or semi-state systems) arise in a variety of physical systems such as economic systems, power systems and many other systems which can be modeled by dynamic equations and algebraic constraints [4],[6],[16].

It is well known that time-varying delay appears in many dynamic systems such as digital control systems, long transmission lines in pneumatic systems, manufacturing processes and remote control systems, which may cause poor performance and even instability [1],[3],[10-11],[14].

Recently, increasing attention has been paid to the study of singular time-varying delay systems due to the fact that such types of systems can better describe systems than normal ones and have extensive applications in various engineering systems, including flexible arm control of robots, large-scale electric network control and lossless transmission lines [7],[9],[17],[20],[23-27],[29].

Recent study of stability analysis for singular time-varying delay systems mainly focuses on delay-dependent criteria which are usually less conservative than delay-independent ones, see [9],[17],[20],[23],[25-27],[29] and the references therein, among which one main index of measuring the conservativeness is the maximum allowable upper bound on the time-varying delay. It is well known that the Lyapunov theory has been recognized as a powerful methodology for the stability analysis [13], and there are two key points to reduce the inherent conservativeness : the construction of appropriate Lyapunov-Krasovskii functional (LKF) and the way to estimate its derivative. The simple LKF is employed in [7],[23-25] to investigate the stability of the singular time-varying delay systems, and rich results have been reported. Several attempts towards the construction of the LKFs have been done such as adopting parameterized LKF [17], augmenting serval terms in LKF [20],[27], introducing the triple integral term [20], and using the delay-decomposition method [9]. Moreover the delay-decomposition method has been proved to be effective in reduction of the conservativeness, but it yields heavier computational burden with the number of delay-partitioning increasing. As for the way to bound some cross terms arisen when estimating the derivative of the LKF, there are some different methods, among which the free weighting matrix approach [27],[29] and Jensen's inequality method [20],[23],[26-27],[29] are the most popular methods. Recently a new Wirtinger-based integral inequality was introduced in [2] and it is shown effectively to reduce the conservativeness for testing stability of normal time-varying delay systems since it encompasses Jensen's inequality as the special case. At present, there are no obvious ways to obtain less conservative results and the singular time-varying delay systems still need more investigation. This motivates the study in this work.

In this independent study, the problem of stability analysis for a class of singular systems with time-varying delay is investigated. We aim at establishing a delay-dependent stability criterion in terms of LMIs which can produce allowable delay bounds as large as possible and the number of decision variables as few as possible. Firstly, the considered system is represented as a neutral system under a moderate assumption. Then a delay-dependent stability criterion is proposed by using Lyapunov method. The main idea is the transformation of singular time-varying delay systems into neutral systems and thus new delay-dependent stability test is derived by using appropriate LKF and the Wirtinger-based integral inequality, which forms the main contribution of our work. The present result can be used in testing stability of both singular time-varying delay system and neutral systems. Numerical examples show in Section 3. Section 4 give the conclusion of independent study.

Notation: In this paper, R^n denotes the n-dimensional real Euclidean space; $R^{n\times m}$ is the set of $n \times m$ real matrices; the superscript 'T' denotes the matrix transportation, for any square matrix $A \in R^{n\times m}$, we define $He\{A\} = A + A^T$; $\| \, . \, \|$ refers to the Euclidean norm of the vector and $diag\{...\}$ denotes the block diagonal matrix; $P > 0$ $(P \geq 0)$ means that P is a real, symmetric and positive definite (positive semi-definite) matrix; the symbol '$\star$' represents the symmetric elements in asymmetric matrix; I_n and 0_n are, respectively, the $n \times n$ dimensional identity matrix and the $n \times n$ dimensional zero matrix. Whereas no confusion is caused, we also use I and 0 to denote, respectively, the identity matrix and the zero matrix with compatible dimensions.

1.2 Problem formulation and Preliminaries

Consider the following linear singular time-varying delay system:

$$
\begin{aligned}
&E\dot{x}(t) = Ax(t) + A_d x(t - h(t)), \\
&x(t) = \phi(t), t \in [-h_2, 0],
\end{aligned} \tag{1}
$$

where $x(t) \in R^n$ is the state variable. $h(t)$ is a time-varying delay which satisfies $0 \leq h_1 \leq h(t) \leq h_2$, and $\dot{h}(t) \leq D$. $E, A, A_d \in R^{n\times n}$ are knows real constant matrices where E may be a singular matrix, so and we assume that $rank(E) = r \leq n$. $\phi(t)$ is a continuously real-valued initial function.

We now tranform system (1) into a neutral system under certain constraint. To proceed, the following Definitions and Lemmas are needed.

Definition 1 (Dai [1]). The pair (E, A) is said to be regular if $det(sE - A)$ is not identically zero. The pair (E, A) is said to be impulse free if $deg(det(\lambda E - A)) = rank(E)$.

Lemma 1 (Xu et al. [24]). If the pair (E, A) is regular and impulse free, then the solution to the singular time-varying delay system (1) exists and is impulse free and unique on $[0, \infty)$.

From Lemma 1, the following definition is naturally introduced.

Definition 2 (Xu et al. [24]). The singular time-varying delay system (1) is said to be regular and impulse free, if the pair (E, A) is regular and impulse free.

Lemma 2 (Dai [1]). If the pair (E, A) is regular and impulse free, then there exists two invertible matrices $M \in R^{n\times n}$ and $N \in R^{n\times n}$ such that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \ MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}. \tag{2}$$

According to Lemma 2, if the pair (E, A) is regular and impulse free, invertible matrices $M, N \in R^{n\times n}$ can always be found such that

$$MEN := \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = \bar{E}, \ MAN := \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix} = \bar{A}. \tag{3}$$

Let

$$MA_dN = \bar{A}_d = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix}, \ N^{-1}x(t) = \mu(t) = \begin{bmatrix} \mu_1(t) \\ \mu_2(t) \end{bmatrix}, \tag{4}$$

where the partitions are compatible with the structure of $\bar{E}$.

From, Lemma 2, we have

$$N^{-1}x(t) = \mu(t),$$

$$x(t) = N\mu(t),$$

$$\therefore \dot{x}(t) = N\dot{\mu}(t),$$

and

$$EN\dot{\mu}(t) = AN\mu(t) + A_dN\mu(t - h(t)),$$

$$MEN\dot{\mu}(t) = MAN\mu(t) + MA_dN\mu(t - h(t)),$$

$$\bar{E}\dot{\mu}(t) = \bar{A}\mu(t) + \bar{A}_d\mu(t - h(t)).$$

The following system :

$$\bar{E}\dot{\mu}(t) = \bar{A}\mu(t) + \bar{A}_d\mu(t - h(t)), \tag{5}$$

which can be written as follows :

$$\dot{\mu}_1(t) = A_1\mu(t) + A_{d1}\mu_1(t-h(t)) + A_{d2}\mu_2(t-h(t)), \tag{6}$$

$$0 = \mu_2(t) + A_{d3}\mu_1(t-h(t)) + A_{d4}\mu_2(t-h(t)). \tag{7}$$

Inspired by [9] and [13] , we rewrite the second equation. By differentiating (7) , we obtain

$$\frac{d}{dt}[\mu_2(t) + A_{d3}\mu_1(t-h(t)) + A_{d4}\mu_2(t-h(t))] = 0. \tag{8}$$

By combining (7) and (8) , we have

$$\begin{aligned}\dot{\mu}_2(t) = &- \mu_2(t) - A_{d3}\dot{\mu}_1(t-h(t))(1-\dot{h}(t)) - A_{d4}\dot{\mu}_2(t-h(t))(1-\dot{h}(t)) \\ &- A_{d3}\mu_1(t-h(t)) - A_{d4}\mu_2(t-h(t)).\end{aligned} \tag{9}$$

From (6) and (9) , we can have

$$\begin{aligned}\begin{bmatrix}\dot{\mu}_1(t)\\ \dot{\mu}_2(t)\end{bmatrix} = &\begin{bmatrix}A_1\mu_1(t) + A_{d1}\mu_1(t-h(t)) + A_{d2}\mu_2(t-h(t))\\ -\mu_2(t) - A_{d3}\mu_1(t-h(t)) - A_{d4}\mu_2(t-h(t))\end{bmatrix} \\ &+ [1-\dot{h}(t)]\begin{bmatrix}0 & 0\\ -A_{d3} & -A_{d4}\end{bmatrix}\begin{bmatrix}\dot{\mu}_1(t-h(t))\\ \dot{\mu}_2(t-h(t))\end{bmatrix}.\end{aligned} \tag{10}$$

Let

$$\hat{A} = \begin{bmatrix} A_1 & 0 \\ 0 & -I_{n-r} \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} A_{d1} & A_{d2} \\ -A_{d3} & -A_{d4} \end{bmatrix}, \ \hat{C} = \begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix}. \tag{11}$$

Then the system (10) is of the form of a neutral system ,

$$\begin{aligned} \dot{\mu}(t) &= \hat{A}\mu(t) + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\ \hat{C}\dot{\mu}(t-h(t)) \ , \\ \mu(t) &= \varphi(t) \ , \ t\epsilon[-h_2, 0] \end{aligned} \tag{12}$$

Definition 3 The equilibrium point $x = 0$ of system (12) is said to be asymptotically stable (A.S.) if it is stable and δ can be chosen such that

$$\| x(0) \| < \delta \Rightarrow \lim_{t\to\infty} x(t) = 0$$

Lemma 3 Let $x = 0$ be an equilibrium point for system (12) and $D \subset R^n$ be a domain containing $x = 0$.

1. If there exists a Lyapunov function V then the equilibrium point $x = 0$ of system (12) is stable.

2. If there exists a Lyapunov function V and $\dot{V}(x) < 0$ in $D - \{0\}$ the equilibrium point $x = 0$ of system (12) is asymptotically stable.

In this work, we will provide an alternative way to study the stability of singular time-varying delay system (1) via the neutral system (12). Although the dynamic responses for system (1) and (12) may be different, it is easy to see that the stability property for both systems remains the same. That is, the asymptotically stability of system (12) will guarantee the asymptotic stability of system (1), and vice versa. With respect to the stability analysis of system (12), it is always assumed that the spectral radius of $\hat{C}(1-D)$ is less than 1. So the following assumption is made before our main result.

Assumption 1 Assume that the pair (E, A) is regular and impulse free and all the eigenvalues of $\hat{C}(1-D)$ are inside the unit circle.

Lemma 4 (Seuret and Gouaisbaut [2]). For any constant matrix $R \in \mathbb{R}^{n\times n}$, $R = R^T \geq 0$, the following inequality holds for all continuously differentiable function x in $[a, b] \to \mathbb{R}^n$:

$$\int_a^b \dot{x}^T R\dot{x}(s)ds \geq \frac{1}{b-a}(x(b)-x(a))^T R(x(b)-x(a)) + \frac{3}{b-a}\Omega^T R\Omega, \qquad (13)$$

where $\Omega = x(a) + x(b) - (\frac{2}{b-a})\int_a^b x(s)ds.$

Proposition 2.1 (Schur complement lemma [8]). Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0 \textbf{ or } \begin{bmatrix} -Y & Z \\ Z^T & X \end{bmatrix} < 0.$$

Proposition 2.2 (Cauchy's inequality [5]). Let E and H be any constant matrices. For any $\varepsilon > 0$, we have

$$2EH \leq \varepsilon EF^T + \varepsilon^{-1} H^T H.$$

CHAPTER 2

MAIN RESULTS

In this section, we present our main result which is for both the singular time-varying delay system (1) and the neutral system (12).

Theorem 1. Under Assumption 1, system (12) and thus system (1) are asymptotically stable, if there exist $5n \times 5n$ positive-definite matrix P and $n \times n$ positive-definite matrices $Q_1, Q_2, R_1, R_2, S_1, S_2, W, T, N$ and M such that the following LMI holds:

$$\omega = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & \Psi_{110} & 0 & 0 & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} & \Psi_{28} & \Psi_{29} & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} & \Psi_{38} & \Psi_{39} & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & \Psi_{46} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & \Psi_{56} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & \Psi_{68} & \Psi_{69} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} & \Psi_{79} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & \Psi_{89} & \Psi_{810} & \Psi_{811} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99} & \Psi_{910} & 0 & \Psi_{912} & 0 \\ * & * & * & * & * & * & * & * & * & \Psi_{1010} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Psi_{1111} & 0 & \Psi_{1113} \\ * & * & * & * & * & * & * & * & * & * & * & \Psi_{1212} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \Psi_{1313} \end{bmatrix} < 0 \, , \qquad (14)$$

where

$$\Psi_{11} = P_{11}\hat{A} + \hat{A}^T P_{11} + P_{14} + P_{14}^T + P_{15} + P_{15}^T + Q_1 + Q_2 + \hat{A}^T S_1 \hat{A}$$
$$+ \hat{A}^T S_2 \hat{A} + \hat{A}^T h_1^2 R_1 \hat{A} - 4R_1 + \hat{A}^T h_2^2 R_2 \hat{A} - 4R_2 \ ;$$
$$\Psi_{12} = -P_{14} + P_{12}\hat{A} + P_{24} + P_{25} - 2R_1 \ ;$$
$$\Psi_{13} = -P_{15} + P_{13}\hat{A} + P_{34} + P_{35} - 2R_2 \ ;$$

$\Psi_{14} = P_{12}$;

$\Psi_{15} = P_{13}$;

$\Psi_{16} = h_1 P_{14}\hat{A} + h_1 P_{44} + h_1 P_{45} + 6R_1$;

$\Psi_{17} = h_2 P_{15}\hat{A} + h_2 P_{45}^T + h_2 P_{55} + 6R_2$;

$\Psi_{18} = \hat{A}_d^{\ T} P_{11} + \hat{A}_d^{\ T} S_1 \hat{A} + \hat{A}_d^{\ T} S_2 \hat{A} + \hat{A}_d^{\ T} h_1^2 R_1 \hat{A} + \hat{A}_d^{\ T} h_2^2 R_2 \hat{A} - W\hat{A}$

$+T\alpha$;

$\Psi_{19} = \hat{C}^T P_{11} + \hat{C}^T S_1 \hat{A} + \hat{C}^T S_2 \hat{A} + \hat{C}^T h_1^2 R_1 \hat{A} + \hat{C}^T h_2^2 R_2 \hat{A} - W\hat{A}$;

$\Psi_{110} = W\hat{A}$;

$\Psi_{22} = -P_{24} - P_{24}^T - Q_1 - 4R_1$;

$\Psi_{23} = -P_{25} - P_{34}$;

$\Psi_{24} = P_{22}$;

$\Psi_{25} = P_{23}$;

$\Psi_{26} = -h_1 P_{44} + 6R_2$;

$\Psi_{27} = h_2 P_{25}^T - h_2 P_{45}^T$;

$\Psi_{28} = \hat{A}_d^{\ T} P_{12}^T$;

$\Psi_{29} = \hat{C}^T P_{12}^T$;

$\Psi_{33} = -P_{35} - P_{35}^T - Q_2 - 4R_2$;

$\Psi_{34} = P_{23}^T$;

$\Psi_{35} = P_{33}$;

$\Psi_{36} = -h_1 P_{45}$;

$\Psi_{37} = h_2 P_{35}^T - h_2 P_{55} + 6R_2$;

$\Psi_{38} = \hat{A}_d^{\ T} P_{13}^T$;

$\Psi_{39} = \hat{C}^T P_{13}^T$;

$\Psi_{44} = -S_1$;

$\Psi_{46} = h_1 P_{24}^T$;

$\Psi_{55} = -S_2$;

$\Psi_{56} = h_1 P_{34}^T$;

$\Psi_{66} = -12R_1$;

$\Psi_{68} = h_1 \hat{A}_d^{\ T} P_{14}^T$;

$\Psi_{69} = h_1 \hat{C}^T P_{14}^T$;

$\Psi_{77} = -12R_2$;

$\Psi_{78} = h_2 \hat{A}_d^{\ T} P_{15}^T$;

$\Psi_{79} = h_2 \hat{C}^T P_{15}^T$;

$$\Psi_{88} = \hat{A}_d^T S_1 \hat{A}_d + \hat{A}_d^T S_2 \hat{A}_d + \hat{A}_d^{\ T} h_1^2 R_1 \hat{A}_d + \hat{A}_d^{\ T} h_2^2 R_2 \hat{A}_d - W\hat{A}_d - \hat{A}_d^{\ T} W$$
$$-2T\alpha \ ;$$

$$\Psi_{89} = \hat{C}^T S_1 \hat{A}_d + \hat{C}^T S_2 \hat{A}_d + \hat{C}^T h_1^2 R_1 \hat{A}_d + \hat{C}^T h_2^2 R_2 \hat{A}_d - \hat{C}^T W - W\hat{A}_d \ ;$$

$\Psi_{810} = W\hat{A}_d + W$;

$\Psi_{811} = -T$;

$$\Psi_{99} = \hat{C}^T S_1 \hat{C} + \hat{C}^T S_2 \hat{C} + \hat{C}^T h_1^2 R_1 \hat{C} + \hat{C}^T h_2^2 R_2 \hat{C} - W\hat{C}$$
$$-\hat{C}^T W - 2N + \varepsilon_1 (1-D)^2 I \ ;$$

$\Psi_{910} = W\hat{C} + W$;

$\Psi_{912} = N^T$;

$\Psi_{1010} = -2W$;

$\Psi_{1111} = -2M + \varepsilon_2 \alpha^2 I$;

$\Psi_{1113} = M^T$;

$\Psi_{1212} = -\varepsilon_1 I$;

$\Psi_{1313} = -\varepsilon_2 I$.

Proof. For simplicity, we define

$$\zeta(t) = \begin{bmatrix} \mu^T(t) & \mu^T(t-h_1) & \mu^T(t-h_2) & \int_{t-h_1}^t \mu^T(s)ds & \int_{t-h_2}^t \mu^T(s)ds \end{bmatrix}^T,$$

$$\xi(t) = [\mu^T(t) \ \mu^T(t-h_1) \ \mu^T(t-h_2) \ \dot{\mu}^T(t-h_1) \ \dot{\mu}^T(t-h_2)$$
$$\tfrac{1}{h_1}\int_{t-h_1}^t \mu^T(s)ds \ \tfrac{1}{h_2}\int_{t-h_2}^t \mu^T(s)ds \ \mu^T(t-h(t)) \ (1-\dot{h}(t))\dot{\mu}^T(t-h(t))$$
$$\dot{\mu}^T(t) \ \alpha \int_{t-h(t)}^t \dot{\mu}^T(s)ds]^T.$$

We introduce the following LKF:

$$V(t) = V_1 + V_2 + V_3 + V_4 \ , \tag{15}$$

where

$$V_1 = \zeta^T(t) P \zeta(t),$$

$$V_2 = \int_{t-h_1}^{t} \mu^T(s) Q_1 \mu(s) ds + \int_{t-h_2}^{t} \mu^T(s) Q_2 \mu(s) ds,$$

$$V_3 = \int_{t-h_1}^{t} \dot{\mu}^T(s) S_1 \dot{\mu}^T(s) ds + \int_{t-h_2}^{t} \dot{\mu}^T(s) S_2 \dot{\mu}^T(s) ds,$$

$$V_4 = h_1 \int_{-h_1}^{0} \int_{t-\theta}^{t} \dot{\mu}^T(s) R_1 \dot{\mu}(s) ds d\theta + h_2 \int_{-h_2}^{0} \int_{t+\theta}^{t} \dot{\mu}^T(s) R_2 \dot{\mu}(s) ds d\theta.$$

Take derivative of $V(t)$ for along the trajectory solution of system (12) yields

$$\dot{V}(t) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4. \tag{16}$$

Where

$$\begin{aligned} \dot{V}_1 &= \dot{\zeta}^T(t) P \zeta(t) + \zeta^T(t) P \dot{\zeta}(t) \\ &= \zeta^T(t) P \dot{\zeta}(t) + \zeta^T(t) P \dot{\zeta}(t) \\ &= 2\zeta^T(t) P \dot{\zeta}(t) \end{aligned}$$

$$= 2\begin{bmatrix} \mu(t) \\ \mu(t-h_1) \\ \mu(t-h_2) \\ \int_{t-h_1}^{t}\mu(s)ds \\ \int_{t-h_2}^{t}\mu(s)ds \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ P_{12}^T & P_{22} & P_{23} & P_{24} & P_{25} \\ P_{13}^T & P_{23}^T & P_{33} & P_{34} & P_{35} \\ P_{14}^T & P_{24}^T & P_{34}^T & P_{44} & P_{45} \\ P_{15}^T & P_{25}^T & P_{35}^T & P_{45}^T & P_{55} \end{bmatrix} \begin{bmatrix} \dot{\mu}(t) \\ \dot{\mu}(t-h_1) \\ \dot{\mu}(t-h_2) \\ \mu(t)-\mu(t-h_1) \\ \mu(t)-\mu(t-h_2) \end{bmatrix}$$

$$\begin{aligned}
&= 2[\mu^T(t)P_{11}\dot{\mu}(t) + \mu^T(t)P_{12}\dot{\mu}(t-h_1) + \mu^T(t)P_{13}\dot{\mu}(t-h_2) \\
&\quad +\mu^T(t)P_{14}(\mu(t)-\mu(t-h_1)) + \mu^T(t)P_{15}(\mu(t)-\mu(t-h_2)) \\
&\quad +\mu^T(t-h_1)P_{12}^T\dot{\mu}(t) + \mu^T(t-h_1)P_{22}\dot{\mu}(t-h_1) \\
&\quad +\mu^T(t-h_1)P_{23}\dot{\mu}(t-h_2) + \mu^T(t-h_1)P_{24}(\mu(t)-\mu(t-h_1)) \\
&\quad +\mu^T(t-h_1)P_{25}(\mu(t)-\mu(t-h_2)) + \mu^T(t-h_2)P_{13}^T\dot{\mu}(t) \\
&\quad +\mu^T(t-h_2)P_{23}^T\dot{\mu}(t-h_1) + \mu^T(t-h_2)P_{33}\dot{\mu}(t-h_2) \\
&\quad +\mu^T(t-h_2)P_{34}(\mu(t)-\mu(t-h_1)) + \mu^T(t-h_2)P_{35}(\mu(t)-\mu(t-h_2)) \\
&\quad + \int_{t-h_1}^{t}\mu(s)dsP_{14}^T\dot{\mu}(t) + \int_{t-h_1}^{t}\mu(s)dsP_{24}^T\dot{\mu}(t-h_1) \\
&\quad + \int_{t-h_1}^{t}\mu(s)dsP_{34}^T\dot{\mu}(t-h_2) + \int_{t-h_1}^{t}\mu(s)dsP_{44}(\mu(t)-\mu(t-h_1)) \\
&\quad + \int_{t-h_1}^{t}\mu(s)dsP_{45}(\mu(t)-\mu(t-h_2)) + \int_{t-h_2}^{t}\mu(s)dsP_{15}^T\dot{\mu}(t) \\
&\quad + \int_{t-h_2}^{t}\mu(s)dsP_{25}^T\dot{\mu}(t-h_1) + \int_{t-h_2}^{t}\mu(s)dsP_{35}^T\dot{\mu}(t-h_2) \\
&\quad + \int_{t-h_2}^{t}\mu(s)dsP_{45}^T(\mu(t)-\mu(t-h_1)) + \int_{t-h_2}^{t}\mu(s)dsP_{55}(\mu(t)-\mu(t-h_2))] \\
&= 2[\mu^T(t)P_{11}(\hat{A}\mu(t) + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) \\
&\quad +\mu^T(t)P_{12}\dot{\mu}(t-h_1) + \mu^T(t)P_{13}\dot{\mu}(t-h_2) + \mu^T(t)P_{14}(\mu(t)-\mu(t-h_1)) \\
&\quad +\mu^T(t)P_{15}(\mu(t)-\mu(t-h_2)) + \mu^T(t-h_1)P_{12}^T(\hat{A}\mu(t) + \hat{A}_d\mu(t-h(t)) \\
&\quad +(1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) + \mu^T(t-h_1)P_{22}\dot{\mu}(t-h_1) \\
&\quad +\mu^T(t-h_1)P_{23}\dot{\mu}(t-h_2) + \mu^T(t-h_1)P_{24}(\mu(t)-\mu(t-h_1)) \\
&\quad +\mu^T(t-h_1)P_{25}(\mu(t)-\mu(t-h_2)) + \mu^T(t-h_2)P_{13}^T(\hat{A}\mu(t) \\
&\quad +\hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) + \mu^T(t-h_2)P_{23}^T\dot{\mu}(t-h_1) \\
&\quad +\mu^T(t-h_2)P_{33}\dot{\mu}(t-h_2) + \mu^T(t-h_2)P_{34}(\mu(t)-\mu(t-h_1)) \\
&\quad +\mu^T(t-h_2)P_{35}(\mu(t)-\mu(t-h_2)) + \int_{t-h_1}^{t}\mu(s)dsP_{14}^T(\hat{A}\mu(t) \\
&\quad +\hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) + \int_{t-h_1}^{t}\mu(s)dsP_{24}^T\dot{\mu}(t-h_1)
\end{aligned}$$

$$+\int_{t-h_1}^{t}\mu(s)dsP_{34}^T\dot{\mu}(t-h_2)+\int_{t-h_1}^{t}\mu(s)dsP_{44}(\mu(t)-\mu(t-h_1))$$
$$+\int_{t-h_1}^{t}\mu(s)dsP_{45}(\mu(t)-\mu(t-h_2))+\int_{t-h_2}^{t}\mu(s)dsP_{15}^T(\hat{A}\mu(t)$$
$$+\hat{A}_d\mu(t-h(t))+(1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))+\int_{t-h_2}^{t}\mu(s)dsP_{25}^T\dot{\mu}(t-h_1)$$
$$+\int_{t-h_2}^{t}\mu(s)dsP_{35}^T\dot{\mu}(t-h_2)+\int_{t-h_2}^{t}\mu(s)dsP_{45}^T(\mu(t)-\mu(t-h_1))$$
$$+\int_{t-h_2}^{t}\mu(s)dsP_{55}(\mu(t)-\mu(t-h_2))]. \tag{17}$$

From (15) , we have

$$\begin{aligned}\dot{V}_2 &= [\mu^T(t)Q_1\mu(t)-\mu^T(t-h_1)Q_1\mu(t-h_1)+\mu^T(t)Q_2\mu(t)\\ &\quad -\mu^T(t-h_2)Q_2\mu(t-h_2)]\\ &= [\mu^T(t)(Q_1+Q_2)\mu(t)-\mu^T(t-h_1)Q_1\mu(t-h_1)\\ &\quad -\mu^T(t-h_2)Q_2\mu(t-h_2)].\end{aligned} \tag{18}$$

From (15) , we obtain

$$\begin{aligned}\dot{V}_3 &= [\dot{\mu}^T(t)S_1\dot{\mu}(t)-\dot{\mu}^T(t-h_1)S_1\dot{\mu}(t-h_1)\\ &\quad +\dot{\mu}^T(t)S_2\dot{\mu}(t)-\dot{\mu}^T(t-h_1)S_2\dot{\mu}(t-h_2)]\\ &= [\dot{\mu}^T(t)(S_1+S_2)\dot{\mu}(t)-\dot{\mu}^T(t-h_1)S_1\dot{\mu}(t-h_1)-\dot{\mu}^T(t-h_2)S_2\dot{\mu}(t-h_2)]\\ &= [(\hat{A}\mu(t)+\hat{A}_d(t-h(t))+(1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))(S_1+S_2)(\hat{A}\mu(t)\\ &\quad +\hat{A}_d(t-h(t))+(1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))-\dot{\mu}^T(t-h_1)S_1\dot{\mu}(t-h_1)\\ &\quad -\dot{\mu}^T(t-h_2)S_2\dot{\mu}(t-h_2))].\end{aligned} \tag{19}$$

From (15) , we obtain

$$\begin{aligned}\dot{V}_4 &= h_1\int_{-h_1}^{0}\dot{\mu}^T(t)R_1\dot{\mu}(t)-\dot{\mu}^T(t+\theta)R_1\dot{\mu}(t-\theta)d\theta\\ &\quad +h_2\int_{-h_2}^{0}\dot{\mu}^T(t)R_2\dot{\mu}(t)-\dot{\mu}^T(t+\theta)R_2\dot{\mu}(t+\theta)d\theta\\ &= h_1^2(\dot{\mu}^T(t)R_1\dot{\mu}(t))-h_1\int_{-h_1}^{0}\dot{\mu}^T(t+\theta)R_1\dot{\mu}(t+\theta)d\theta\\ &\quad +h_2^2(\dot{\mu}^T(t)R_2\dot{\mu}(t))-h_2\int_{-h_1}^{0}\dot{\mu}^T(t+\theta)R_2\dot{\mu}(t+\theta)d\theta\end{aligned}$$

$$
\begin{aligned}
&= h_1^2(\hat{A}\mu(t) + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))^T R_1(\hat{A}\mu(t) \\
&\quad + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)) - h_1 \int_{t-h_1}^{t} \dot{\mu}^T(s)R_1\dot{\mu}(s)ds \\
&\quad + h_2^2(\hat{A}\mu(t) + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))^T R_2(\hat{A}\mu(t) \\
&\quad + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)) - h_2 \int_{t-h_2}^{t} \dot{\mu}^T(s)R_2\dot{\mu}(s)ds.
\end{aligned}
$$

By adopting Lemma 4 , we have

$$
\begin{aligned}
&-h_1 \int_{t-h_1}^{t} \dot{\mu}^T(s)R_1\dot{\mu}(s)ds \\
&\leq -h_1[\tfrac{1}{t-(t-h_1)}(\mu(t)-\mu(t-h_1))^T R_1(\mu(t)-\mu(t-h_1)) \\
&\quad + \tfrac{3}{t-(t-h_1)}(\mu(t-h_1)+\mu(t) - \tfrac{2}{t-(t-h_1)}\int_{t-h_1}^{t}\mu(s)ds)^T R_1(\mu(t-h_1) \\
&\quad + \mu(t) - \tfrac{2}{t-h(t)}\int_{t-h_1}^{t}\mu(s)ds)] \\
&= -h_1[\tfrac{1}{h_1}(\mu(t)-\mu(t-h_1))^T R_1(\mu(t)-(\mu(t-h_1)) + \tfrac{3}{h_1}(\mu(t-h_1) \\
&\quad + \mu(t) - \tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds)^T R_1(\mu(t-h_1)+\mu(t) - \tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds)] \\
&= -(\mu(t)-\mu(t-h_1))^T R_1(\mu(t)-\mu(t-h_1)) - (\mu(t)+\mu(t-h_1) \\
&\quad - \tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds)^T 3R_1(\mu(t)+\mu(t-h_1) - \tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds).
\end{aligned} \tag{20}
$$

Similarly , we have

$$
\begin{aligned}
&-h_2 \int_{t-h_2}^{t} \dot{\mu}^T(s)R_2\dot{\mu}(s)ds \\
&\leq -h_2[\tfrac{1}{t-(t-h_2)}(\mu(t)-\mu(t-h_2))^T R_2(\mu(t)-\mu(t-h_2)) + \tfrac{3}{t-(t-h_2)}(\mu(t-h_2) \\
&\quad + \mu(t) - \tfrac{2}{t-(t-h_2)}\int_{t-h_2}^{t}\mu(s)ds)^T R_2(\mu(t-h_2)+\mu(t) - \tfrac{2}{t-(t-h_2)}\int_{t-h_2}^{t}\mu(s)ds)] \\
&= -h_2[\tfrac{1}{h_2}(\mu(t)-\mu(t-h_2))^T R2(\mu(t)-\mu(t-h_2)) + \tfrac{3}{h_2}(\mu(t-h_2) \\
&\quad + \mu(t) - \tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds)^T R_2(\mu(t-h_2)+\mu(t) - \tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds)] \\
&= -(\mu(t)-\mu(t-h_2))^T R_2(\mu(t)-\mu(t-h_2)) - (\mu(t)+\mu(t-h_2) \\
&\quad - \tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds)^T 3R_1(\mu(t)+\mu(t-h_2) - \tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds.
\end{aligned} \tag{21}
$$

By using the following identity relation :

$$\dot{\mu}(t) - \hat{A}\mu(t) - \hat{A}_d\mu(t-h(t)) - (1-\dot{h}(t))\hat{C}\mu(t-h(t)) = 0, \tag{22}$$

we have

$$[-\mu^T(t-h(t)) + \dot{\mu}^T(t)][-2W]$$
$$[\dot{\mu}(t) - \hat{A}\mu(t) - \hat{A}_d\mu(t-h(t)) - (1-\dot{h}(t))\hat{C}\mu(t-h(t))] = 0, \tag{23}$$

where W is a positive definite matrix.

By using the following identity relation

$$\begin{aligned}\alpha \int_{t-h(t)}^{t} \dot{\mu}(s)ds &= \alpha(\mu(t) - \mu(t-h(t))) \\ &= \alpha\mu(t) - \alpha\mu(t-h(t)).\end{aligned} \tag{24}$$

From (24) we get

$$\alpha \int_{t-h(t)}^{t} \dot{\mu}(s)ds - \alpha\mu(t) + \alpha\mu(t-h(t)) = 0,$$

and we have

$$[-\mu(t-h(t))][-2T][\alpha \int_{t-h(t)}^{t} \dot{\mu}(s)ds - \alpha\mu(t) + \alpha\mu(t-h(t))]. \tag{25}$$

Where T is a positive definite matrix and α is a positive real number.

By using the following identity relation.

$$(1-\dot{h}(t))\dot{\mu}^T(t-h(t))2N(1-\dot{h}(t))\dot{\mu}(t-h(t))$$
$$-(1-\dot{h}(t))\dot{\mu}^T(t-h(t))2N(1-\dot{h}(t))\dot{\mu}(t-h(t))=0, \quad (26)$$

where $N>0$.

From Proposition 2.2 and $\dot{h}(t)\leq D$, we obtain

$$2(1-D)(\mu^T(t-h(t)))N(\mu(t-h(t)))\leq\varepsilon_1(1-D)^2\mu^T((t-h(t))\mu(t-h(t)))$$
$$+\varepsilon_1^{-1}N^T\mu^T(t-h(t))N(\mu(t-h(t))), \quad (27)$$

where $\varepsilon_1>0$.

Similarly , we have

$$\alpha\int_{t-h(t)}^{t}\dot{\mu}^T(s)2M\alpha\int_{t-h(t)}^{t}\dot{\mu}(s)ds$$
$$-\alpha\int_{t-h(t)}^{t}\dot{\mu}^T(s)ds2M\alpha\int_{t-h(t)}^{t}\dot{\mu}(s)ds=0, \quad (28)$$

where $M>0$.

From Proposition 2.2, we obtain

$$2\alpha(\int_{t-h(t)}^{t}\dot{\mu}^T(s)dsM\int_{t-h(t)}^{t}\dot{\mu}^T(s)ds)\leq\varepsilon_2\alpha^2\int_{t-h(t)}^{t}\dot{\mu}^T(s)ds\int_{t-h(t)}^{t}\dot{\mu}(s)ds$$
$$+\varepsilon_2^{-1}M^T\int_{t-h(t)}^{t}\dot{\mu}^T(s)dsM\int_{t-h(t)}^{t}\dot{\mu}(s)ds. \quad (29)$$

Combining system (12) with (15)-(29) , we arrive at

$$\dot{V}(t) \leq \xi^T(t)\omega\xi(t), \tag{30}$$

where

$$\begin{aligned}\xi(t) = [&\mu^T(t)\ \mu^T(t-h_1)\ \mu^T(t-h_2)\ \dot{\mu}^T(t-h_1)\ \dot{\mu}^T(t-h_2)\\ &\tfrac{1}{h_1}\int_{t-h_1}^{t}\mu^T(s)ds\ \tfrac{1}{h_2}\int_{t-h_2}^{t}\mu^T(s)ds\ \mu^T(t-h(t))\ (1-\dot{h}(t))\dot{\mu}^T(t-h(t))\\ &\dot{\mu}^T(t)\ \alpha\int_{t-h(t)}^{t}\dot{\mu}^T(s)ds]^T,\end{aligned}$$

$$\omega = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & \Psi_{110} & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} & \Psi_{28} & \Psi_{29} & 0 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} & \Psi_{38} & \Psi_{39} & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & \Psi_{46} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & \Psi_{56} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & \Psi_{68} & \Psi_{69} & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} & \Psi_{79} & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & \Psi_{89} & \Psi_{810} & \Psi_{811} \\ * & * & * & * & * & * & * & * & \Psi_{99} & \Psi_{910} & 0 \\ * & * & * & * & * & * & * & * & * & \Psi_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Psi_{1111} \end{bmatrix} < 0,$$

$$\begin{aligned}\Psi_{11} &= P_{11}\hat{A} + \hat{A}^T P_{11} + P_{14} + P_{14}^T + P_{15} + P_{15}^T + Q_1 + Q_2 + \hat{A}^T S_1 \hat{A}\\ &\quad + \hat{A}^T S_2 \hat{A} + \hat{A}^T h_1^2 R_1 \hat{A} - 4R_1 + \hat{A}^T h_2^2 R_2 \hat{A} - 4R_2\ ;\\ \Psi_{12} &= -P_{14} + P_{12}\hat{A} + P_{24} + P_{25} - 2R_1\ ;\\ \Psi_{13} &= -P_{15} + P_{13}\hat{A} + P_{34} + P_{35} - 2R_2\ ;\\ \Psi_{14} &= P_{12}\ ;\end{aligned}$$

$\Psi_{15} = P_{13}$;

$\Psi_{16} = h_1 P_{14} \hat{A} + h_1 P_{44} + h_1 P_{45} + 6R_1$;

$\Psi_{17} = h_2 P_{15} \hat{A} + h_2 P_{45}^T + h_2 P_{55} + 6R_2$;

$\Psi_{18} = \hat{A_d}^T P_{11} + \hat{A_d}^T S_1 \hat{A} + \hat{A_d}^T S_2 \hat{A} + \hat{A_d}^T h_1^2 R_1 \hat{A} + \hat{A_d}^T h_2^2 R_2 \hat{A} - W\hat{A}$

$+T\alpha$;

$\Psi_{19} = \hat{C}^T P_{11} + \hat{C}^T S_1 \hat{A} + \hat{C}^T S_2 \hat{A} + \hat{C}^T h_1^2 R_1 \hat{A} + \hat{C}^T h_2^2 R_2 \hat{A} - W\hat{A}$;

$\Psi_{110} = W\hat{A}$;

$\Psi_{22} = -P_{24} - P_{24}^T - Q_1 - 4R_1$;

$\Psi_{23} = -P_{25} - P_{34}$;

$\Psi_{24} = P_{22}$;

$\Psi_{25} = P_{23}$;

$\Psi_{26} = -h_1 P_{44} + 6R_2$;

$\Psi_{27} = h_2 P_{25}^T - h_2 P_{45}^T$;

$\Psi_{28} = \hat{A_d}^T P_{12}^T$;

$\Psi_{29} = \hat{C}^T P_{12}^T$;

$\Psi_{33} = -P_{35} - P_{35}^T - Q_2 - 4R_2$;

$\Psi_{34} = P_{23}^T$;

$\Psi_{35} = P_{33}$;

$\Psi_{36} = -h_1 P_{45}$;

$\Psi_{37} = h_2 P_{35}^T - h_2 P_{55} + 6R_2$;

$\Psi_{38} = \hat{A_d}^T P_{13}^T$;

$\Psi_{39} = \hat{C}^T P_{13}^T$;

$\Psi_{44} = -S_1$;

$\Psi_{46} = h_1 P_{24}^T$;

$\Psi_{55} = -S_2$;

$\Psi_{56} = h_1 P_{34}^T$;

$\Psi_{66} = -12R_1$;

$\Psi_{68} = h_1 \hat{A_d}^T P_{14}^T$;

$$\Psi_{69} = h_1 \hat{C}^T P_{14}^T \ ;$$

$$\Psi_{77} = -12R_2 \ ;$$

$$\Psi_{78} = h_2 \hat{A}_d^{\ T} P_{15}^T \ ;$$

$$\Psi_{79} = h_2 \hat{C}^T P_{15}^T \ ;$$

$$\Psi_{88} = \hat{A}_d^T S_1 \hat{A}_d + \hat{A}_d^T S_2 \hat{A}_d + \hat{A}_d^{\ T} h_1^2 R_1 \hat{A}_d + \hat{A}_d^{\ T} h_2^2 R_2 \hat{A}_d - W\hat{A}_d - \hat{A}_d^{\ T} W$$
$$-2T\alpha \ ;$$

$$\Psi_{89} = \hat{C}^T S_1 \hat{A}_d + \hat{C}^T S_2 \hat{A}_d + \hat{C}^T h_1^2 R_1 \hat{A}_d + \hat{C}^T h_2^2 R_2 \hat{A}_d - \hat{C}^T W - W\hat{A}_d \ ;$$

$$\Psi_{810} = W\hat{A}_d + W \ ;$$

$$\Psi_{811} = -T \ ;$$

$$\Psi_{99} = \hat{C}^T S_1 \hat{C} + \hat{C}^T S_2 \hat{C} + \hat{C}^T h_1^2 R_1 \hat{C} + \hat{C}^T h_2^2 R_2 \hat{C} - W\hat{C}$$
$$-\hat{C}^T W - 2N + \varepsilon_1 (1-D)^2 I + \varepsilon_1^{-1} N^T N \ ;$$

$$\Psi_{910} = W\hat{C} + W \ ;$$

$$\Psi_{1010} = -2W \ ;$$

$$\Psi_{1111} = -2M + \varepsilon_2 \alpha^2 I + \varepsilon_2^{-1} M^T M \ .$$

From (29) and Proposition 2.1 , it is easy to see that $\dot{V}(t) < 0$. From Lemma 3, we conclude that system (12) and hence , system (1) are asymptotically stable.

CHAPTER 3

NUMERICAL EXAMPLES

In this section, three numerical examples are given to illustrate the validity and superiority of the proposed scheme.

Example 1. Consider the following singular time-varying system with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \ A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \ E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Obviously, the pair (E, A) is regular and impulse free. Let

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \textbf{ and } N = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix},$$

such that

$$MEN = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ MAN = \begin{bmatrix} -2 & 0 \\ 0.9 & 0.9 \end{bmatrix}, \ MA_dN = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

According to Theorem 1, let

$$\hat{C} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \ \hat{A} = \begin{bmatrix} -2 & 0 \\ 0.9 & 0.9 \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

We assume that with satisfy

$$h(t) = sin^2(t)$$

and $0 \le h_1 \le h(t) \le h_2$.

By taking parameters $D = 0.5, n = 0.9, m = 0.5$ and $a = 0.2$, we get Example 1. remains feasible for any delay time $h_1 = 0$ and $h_2 = 1$. Theorem 1. yields the following set of feasible solutions :

$$Q_1 = \begin{bmatrix} 3.9601 \times 10^3 & -0.0000 \times 10^3 \\ -0.0000 \times 10^3 & 3.9601 \times 10^3 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 19.2506 & 1.4550 \\ 1.4550 & 12.2446 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 416.7206 & 1.2125 \\ 1.2125 & 405.3354 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.0033 & -0.0002 \\ -0.0002 & 0.0061 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.0040 & 0.0001 \\ 0.0001 & 0.0050 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.0041 & 0.0001 \\ 0.0001 & 0.0051 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.0024 & 0.0000 \\ 0.0000 & 0.0140 \end{bmatrix}, \quad N = \begin{bmatrix} 1.1390 & 0.0014 \\ 0.0014 & 1.1369 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.5105 & -0.0007 \\ -0.0007 & 0.5176 \end{bmatrix}, \quad T = \begin{bmatrix} 0.1003 & -0.0005 \\ -0.0005 & 0.1024 \end{bmatrix},$$

$$P_{11} = \begin{bmatrix} 1.7268 & 0.0016 \\ 0.0016 & 1.3562 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} -0.6723 & -0.0077 \\ 0.0171 & -0.4829 \end{bmatrix},$$

$$P_{13} = \begin{bmatrix} -1.0055 & 0.0033 \\ -0.0204 & -0.9085 \end{bmatrix}, \quad P_{14} = \begin{bmatrix} -4.1883 \times 10^3 & -0.0235 \times 10^3 \\ 0.0268 \times 10^3 & -4.4274 \times 10^3 \end{bmatrix},$$

$$P_{15} = \begin{bmatrix} -0.0244 \times 10^3 & -0.4054 \times 10^3 \\ -0.5297 \times 10^3 & -0.0135 \times 10^3 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 2.0373 & 0.0368 \\ 0.0368 & 2.2302 \end{bmatrix},$$

$$P_{23} = \begin{bmatrix} -1.3690 & -0.0544 \\ -0.0305 & -1.7511 \end{bmatrix}, \quad P_{24} = \begin{bmatrix} 331.7989 & -10.1762 \\ 49.3883 & 378.7585 \end{bmatrix},$$

$$P_{25} = \begin{bmatrix} -3.2653 \times 10^3 & -0.0358 \times 10^3 \\ -0.0361 \times 10^3 & -3.4496 \times 10^3 \end{bmatrix}, \quad P_{33} = \begin{bmatrix} 2.3817 & 0.0520 \\ 0.0520 & 2.6648 \end{bmatrix},$$

$$P_{34} = \begin{bmatrix} -113.7437 & -19.1402 \\ -28.4193 & 80.2111 \end{bmatrix}, \qquad P_{35} = \begin{bmatrix} 3.2653 \times 10^3 & 0.0358 \times 10^3 \\ 0.0361 \times 10^3 & 3.4495 \times 10^3 \end{bmatrix},$$

$$P_{44} = \begin{bmatrix} 3.9601 \times 10^3 & 0 \\ 0 & 3.9601 \times 10^3 \end{bmatrix}, \qquad P_{45} = \begin{bmatrix} -3.2653 \times 10^3 & -0.0359 \times 10^3 \\ -0.0360 \times 10^3 & -3.4496 \times 10^3 \end{bmatrix},$$

$$P_{55} = \begin{bmatrix} 3.2653 \times 10^3 & 0.0359 \times 10^3 \\ 0.0359 \times 10^3 & 3.4496 \times 10^3 \end{bmatrix}.$$

From Theorem 1 , we conclude that system (1) is asymptotically stable.

Example 2. Consider the system (1) with the following parameters :

$$\hat{A} = \begin{bmatrix} -0.2 & 0 \\ 0 & 1 \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$\hat{C} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

By taking parameters $D = 0.5, n = 0.9, m = 0.5$ and $a = 0.2$, we get Example 2. remains feasible for any delay time $h_1 = 0$ and $h_2 = 1$. Theorem 1. yields the following set of feasible solutions :

$$Q_1 = \begin{bmatrix} 54.6121 & 0 \\ 0 & 54.6121 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 14.7851 & 0 \\ 0 & 17.3196 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 5.7487 & 0 \\ 0 & 5.7423 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 0.4765 & 0 \\ 0 & 0.4554 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.3456 & 0 \\ 0 & 0.3655 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0.3524 & 0 \\ 0 & 0.3739 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.0058 & 0 \\ 0 & 0.0095 \end{bmatrix}, \qquad N = \begin{bmatrix} 1.1109 & 0 \\ 0 & 1.1299 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.5138 & 0 \\ 0 & 0.5169 \end{bmatrix}, \qquad T = \begin{bmatrix} 0.0991 & 0 \\ 0 & 0.0995 \end{bmatrix},$$

$$P_{11} = \begin{bmatrix} 2.2300 & 0 \\ 0 & 1.3229 \end{bmatrix}, \qquad P_{12} = \begin{bmatrix} -0.8572 & 0 \\ 0 & -1.5350 \end{bmatrix},$$

$$P_{13} = \begin{bmatrix} -0.9577 & 0 \\ 0 & -1.6762 \end{bmatrix}, \qquad P_{14} = \begin{bmatrix} -59.9172 & 0 \\ 0 & -82.1298 \end{bmatrix},$$

$$P_{15} = \begin{bmatrix} -0.0502 & 0 \\ 0 & -0.1172 \end{bmatrix}, \qquad P_{22} = \begin{bmatrix} 2.3548 & 0 \\ 0 & 2.6176 \end{bmatrix},$$

$$P_{23} = \begin{bmatrix} -1.5363 & 0 \\ 0 & -0.9575 \end{bmatrix}, \qquad P_{24} = \begin{bmatrix} 1.9360 & 0 \\ 0 & 6.2648 \end{bmatrix},$$

$$P_{25} = \begin{bmatrix} -41.5093 & 0 \\ 0 & -43.3520 \end{bmatrix}, \qquad P_{33} = \begin{bmatrix} 2.8955 & 0 \\ 0 & 3.1551 \end{bmatrix},$$

$$P_{34} = \begin{bmatrix} 0.6773 & 0 \\ 0 & 22.8484 \end{bmatrix}, \qquad P_{35} = \begin{bmatrix} 34.9494 & 0 \\ 0 & 37.1122 \end{bmatrix},$$

$$P_{44} = \begin{bmatrix} 54.6121 & 0 \\ 0 & 54.6121 \end{bmatrix}, \qquad P_{45} = \begin{bmatrix} -41.8184 & 0 \\ 0 & -42.8570 \end{bmatrix},$$

$$P_{55} = \begin{bmatrix} 39.9496 & 0 \\ 0 & 42.0913 \end{bmatrix}.$$

From Theorem 1 , we conclude that system (1) is asymptotically stable.

Example 3. Consider the system (1) with the following parameters :

$$\hat{A} = \begin{bmatrix} -4.5 & 0 \\ 0 & 6 \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix},$$
$$\hat{C} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

By taking parameters $D = 0.5, n = 0.9, m = 0.5$ and $a = 0.2$, we get Example 3. remains feasible for any delay time $h_1 = 0$ and $h_2 = 1$. Theorem 1. yields the following set of feasible solutions :

$$Q_1 = \begin{bmatrix} 15.5362 & 0 \\ 0 & 15.5362 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 2.2288 & 0 \\ 0 & 1.5451 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 1.6352 & 0 \\ 0 & 1.6353 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.0040 & 0 \\ 0 & 0.0010 \end{bmatrix},$$
$$S_1 = \begin{bmatrix} 0.0045 & 0 \\ 0 & 0.0012 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.0047 & 0 \\ 0 & 0.0012 \end{bmatrix},$$
$$W = \begin{bmatrix} 0.0018 & 0 \\ 0 & 0.0068 \end{bmatrix}, \quad N = \begin{bmatrix} 1.1026 & 0 \\ 0 & 1.1006 \end{bmatrix},$$
$$M = \begin{bmatrix} 0.5148 & 0 \\ 0 & 0.5174 \end{bmatrix}, \quad T = \begin{bmatrix} 0.1019 & 0 \\ 0 & 0.1004 \end{bmatrix},$$
$$P_{11} = \begin{bmatrix} 0.1802 & 0 \\ 0 & 0.0579 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} -0.0370 & 0 \\ 0 & -0.0467 \end{bmatrix},$$
$$P_{13} = \begin{bmatrix} -0.0486 & 0 \\ 0 & -0.0573 \end{bmatrix}, \quad P_{14} = \begin{bmatrix} -15.0881 & 0 \\ 0 & -21.7708 \end{bmatrix},$$
$$P_{15} = \begin{bmatrix} -0.7682 \times 10^3 & 0 \\ 0 & 0.3741 \times 10^3 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 0.1443 & 0 \\ 0 & 0.0815 \end{bmatrix},$$

$$P_{23} = \begin{bmatrix} -0.1136 & 0 \\ 0 & -0.0366 \end{bmatrix}, \quad P_{24} = \begin{bmatrix} 0.8564 & 0 \\ 0 & 1.6070 \end{bmatrix},$$

$$P_{25} = \begin{bmatrix} -11.7344 & 0 \\ 0 & -12.9741 \end{bmatrix}, \quad P_{33} = \begin{bmatrix} 0.1785 & 0 \\ 0 & 0.0994 \end{bmatrix},$$

$$P_{34} = \begin{bmatrix} -2.2540 & 0 \\ 0 & 4.1814 \end{bmatrix}, \quad P_{35} = \begin{bmatrix} 11.6815 & 0 \\ 0 & 12.9609 \end{bmatrix},$$

$$P_{44} = \begin{bmatrix} 15.5362 & 0 \\ 0 & 15.5362 \end{bmatrix}, \quad P_{45} = \begin{bmatrix} -11.7400 & 0 \\ 0 & -12.9757 \end{bmatrix},$$

$$P_{55} = \begin{bmatrix} 11.7258 & 0 \\ 0 & 12.9713 \end{bmatrix}.$$

From Theorem 1 , we conclude that system (1) is asymptotically stable.

CHAPTER 4

CONCLUSION

We obtained a new criteria for asymptotical stability for system (1) as follow :

Theorem 1. Under Assumption 1, system (12) and thus system (1) are asymptotically stable, if there exist $5n \times 5n$ positive-definite matrix P and $n \times n$ positive-definite matrices $Q_1, Q_2, R_1, R_2, S_1, S_2, W, T, N$ and M such that the following LMI holds:

$$\omega = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & \Psi_{110} & 0 & 0 & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} & \Psi_{28} & \Psi_{29} & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} & \Psi_{38} & \Psi_{39} & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & \Psi_{46} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & \Psi_{56} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & \Psi_{68} & \Psi_{69} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} & \Psi_{79} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & \Psi_{89} & \Psi_{810} & \Psi_{811} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99} & \Psi_{910} & 0 & \Psi_{912} & 0 \\ * & * & * & * & * & * & * & * & * & \Psi_{1010} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Psi_{1111} & 0 & \Psi_{1113} \\ * & * & * & * & * & * & * & * & * & * & * & \Psi_{1212} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \Psi_{1313} \end{bmatrix} < 0, \tag{14}$$

where

$$\begin{aligned} \Psi_{11} &= P_{11}\hat{A} + \hat{A}^T P_{11} + P_{14} + P_{14}^T + P_{15} + P_{15}^T + Q_1 + Q_2 + \hat{A}^T S_1 \hat{A} \\ &\quad + \hat{A}^T S_2 \hat{A} + \hat{A}^T h_1^2 R_1 \hat{A} - 4R_1 + \hat{A}^T h_2^2 R_2 \hat{A} - 4R_2 \ ; \\ \Psi_{12} &= -P_{14} + P_{12}\hat{A} + P_{24} + P_{25} - 2R_1 \ ; \\ \Psi_{13} &= -P_{15} + P_{13}\hat{A} + P_{34} + P_{35} - 2R_2 \ ; \\ \Psi_{14} &= P_{12} \ ; \\ \Psi_{15} &= P_{13} \ ; \end{aligned}$$

$\Psi_{16} = h_1 P_{14}\hat{A} + h_1 P_{44} + h_1 P_{45} + 6R_1$;

$\Psi_{17} = h_2 P_{15}\hat{A} + h_2 P_{45}^T + h_2 P_{55} + 6R_2$;

$\Psi_{18} = \hat{A_d}^T P_{11} + \hat{A_d}^T S_1\hat{A} + \hat{A_d}^T S_2\hat{A} + \hat{A_d}^T h_1^2 R_1\hat{A} + \hat{A_d}^T h_2^2 R_2\hat{A} - W\hat{A}$
$+T\alpha$;

$\Psi_{19} = \hat{C}^T P_{11} + \hat{C}^T S_1\hat{A} + \hat{C}^T S_2\hat{A} + \hat{C}^T h_1^2 R_1\hat{A} + \hat{C}^T h_2^2 R_2\hat{A} - W\hat{A}$;

$\Psi_{110} = W\hat{A}$;

$\Psi_{22} = -P_{24} - P_{24}^T - Q_1 - 4R_1$;

$\Psi_{23} = -P_{25} - P_{34}$;

$\Psi_{24} = P_{22}$;

$\Psi_{25} = P_{23}$;

$\Psi_{26} = -h_1 P_{44} + 6R_2$;

$\Psi_{27} = h_2 P_{25}^T - h_2 P_{45}^T$;

$\Psi_{28} = \hat{A_d}^T P_{12}^T$;

$\Psi_{29} = \hat{C}^T P_{12}^T$;

$\Psi_{33} = -P_{35} - P_{35}^T - Q_2 - 4R_2$;

$\Psi_{34} = P_{23}^T$;

$\Psi_{35} = P_{33}$;

$\Psi_{36} = -h_1 P_{45}$;

$\Psi_{37} = h_2 P_{35}^T - h_2 P_{55} + 6R_2$;

$\Psi_{38} = \hat{A_d}^T P_{13}^T$;

$\Psi_{39} = \hat{C}^T P_{13}^T$;

$\Psi_{44} = -S_1$;

$\Psi_{46} = h_1 P_{24}^T$;

$\Psi_{55} = -S_2$;

$\Psi_{56} = h_1 P_{34}^T$;

$\Psi_{66} = -12R_1$;

$\Psi_{68} = h_1\hat{A_d}^T P_{14}^T$;

$\Psi_{69} = h_1\hat{C}^T P_{14}^T$;

$\Psi_{77} = -12R_2$;

$\Psi_{78} = h_2\hat{A}_d{}^T P_{15}^T$;

$\Psi_{79} = h_2\hat{C}^T P_{15}^T$;

$\Psi_{88} = \hat{A}_d^T S_1\hat{A}_d + \hat{A}_d^T S_2\hat{A}_d + \hat{A}_d{}^T h_1^2 R_1\hat{A}_d + \hat{A}_d{}^T h_2^2 R_2\hat{A}_d - W\hat{A}_d - \hat{A}_d{}^T W$
$\quad -2T\alpha$;

$\Psi_{89} = \hat{C}^T S_1\hat{A}_d + \hat{C}^T S_2\hat{A}_d + \hat{C}^T h_1^2 R_1\hat{A}_d + \hat{C}^T h_2^2 R_2\hat{A}_d - \hat{C}^T W - W\hat{A}_d$;

$\Psi_{810} = W\hat{A}_d + W$;

$\Psi_{811} = -T$;

$\Psi_{99} = \hat{C}^T S_1\hat{C} + \hat{C}^T S_2\hat{C} + \hat{C}^T h_1^2 R_1\hat{C} + \hat{C}^T h_2^2 R_2\hat{C} - W\hat{C}$
$\quad -\hat{C}^T W - 2N + \varepsilon_1(1-D)^2 I$;

$\Psi_{910} = W\hat{C} + W$;

$\Psi_{912} = N^T$;

$\Psi_{1010} = -2W$;

$\Psi_{1111} = -2M + \varepsilon_2\alpha^2 I$;

$\Psi_{1113} = M^T$;

$\Psi_{1212} = -\varepsilon_1 I$;

$\Psi_{1313} = -\varepsilon_2 I$.

The stability problem for a class of singular time-varying delay systems has been investigated. Firstly, the singular system is represented as a neutral system. Then an augmented LKF and the Wirtinger-based integral inequality method are used to derive a new delay-dependent stability criterion in terms of LMIs. Three numerical examples are given to illustrate the reduced conservatism of the proposed method. Note that our method is proposed for dealing with time-varying delay. In the sense of finite-time stability (FTS), the proposed neutral system approach is not applicable and this has to be opened problem future works. For we expected that our present method is useful and may be effective method for FTS of singular systems.

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APPENDIX

A neutral system approach to stability of singular time-varying delay system

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Abstract

This independent study is concerned with the problem of delay-dependent stability for a class of singular time-varying delay system. By representing the singular system as a neutral form, using an augmented Lyapunov-Krasovskii functional and the Wirtinger-based inequality method, we obtain a new stability criterion in terms of a linear matrix inequality (LMI). The criterion is applicable for the stability test of both singular time-varying delay systems and neutral systems with time-varying delays.

1 Introduction

Singular systems (also referred to as descriptor, generalized, differential-algebraic or semi-state systems) arise in a variety of physical systems such as economic systems, power systems and many other systems which can be modeled by dynamic equations and algebraic constraints [4],[6],[16].

It is well known that time-varying delay appears in many dynamic systems such as digital control systems, long transmission lines in pneumatic systems, manufacturing processes and remote control systems, which may cause poor performance and even instability [1],[3],[10-11],[14].

Recently, increasing attention has been paid to the study of singular time-varying delay systems due to the fact that such types of systems can better describe systems than normal ones and have extensive applications in various engineering systems, including flexible arm control of robots, large-scale electric network control and lossless transmission lines [7],[9],[17],[20],[23-27],[29].

Recent study of stability analysis for singular time-varying delay systems mainly focuses on delay-dependent criteria which are usually less conservative than delay-independent ones, see [9],[17],[20],[23],[25-27],[29] and the references therein, among which one main index of measuring the conservativeness is the maximum allowable upper bound on the time-varying delay. It is well known that the Lyapunov theory has been recognized as a powerful methodology for the stability analysis [13], and there are two key points to reduce the inherent conservativeness : the construction of appropriate Lyapunov-Krasovskii functional (LKF) and the way to estimate its derivative. The simple LKF is employed in [7],[23-25] to investigate the stability of the singular time-varying delay systems, and rich results have been reported. Several attempts towards the construction of the LKFs have been done such as adopting parameterized LKF [17], augmenting serval terms in LKF [20],[27], introducing the triple integral term [20], and using the delay-decomposition method [9]. Moreover the delay-decomposition method has been proved to be effective in reduction of the conservativeness, but it yields heavier computational burden with the number of delay-partitioning increasing. As for the way to bound some cross terms arisen when estimating the derivative of the LKF, there are some different methods, among which the free weighting matrix approach [27],[29] and Jensen's inequality method [20],[23],[26-27],[29] are the most popular methods. Recently a new Wirtinger-based integral inequality was introduced in [2] and it is shown effectively to reduce the conservativeness for testing stability of normal time-varying delay systems since it encompasses Jensen's inequality as the special case. At present, there are no obvious ways to obtain less conservative results and the singular time-varying delay systems still need more investigation. This motivates the study in this work.

In this independent study, the problem of stability analysis for a class of singular systems with time-varying delay is investigated. We aim at establishing a delay-dependent stability criterion in terms of LMIs which can produce allowable delay bounds as large as possible and the number of decision variables as few as possible. Firstly, the considered system is represented as a neutral system under a moderate assumption. Then a delay-dependent stability criterion is proposed by using Lyapunov method. The main idea is the transformation of singular time-varying delay systems into neutral systems and thus new delay-dependent stability test is derived by using appropriate LKF and the Wirtinger-based integral inequality, which forms the main contribution of our work. The present result can be used in testing stability of both singular time-varying delay system and neutral systems. Numerical examples show in Section 3. Section 4 give the conclusion of independent study.

Notation: In this paper, R^n denotes the n-dimensional real Euclidean space; $R^{n\times m}$ is the set of $n \times m$ real matrices; the superscript 'T' denotes the matrix transportation, for any square matrix $A \in R^{n\times m}$, we define $He\{A\} = A + A^T$; $\| \, . \, \|$ refers to the Euclidean norm of the vector and $diag\{...\}$ denotes the block diagonal matrix; $P > 0$ ($P \geq 0$) means that P is a real, symmetric and positive definite (positive semi-definite) matrix; the symbol '$\star$' represents the symmetric elements in asymmetric matrix; I_n and 0_n are, respectively, the $n \times n$ dimensional identity matrix and the $n \times n$ dimensional zero matrix. Whereas no confusion is caused, we also use I and 0 to denote, respectively, the identity matrix and the zero matrix with compatible dimensions.

2 Problem formulation and preliminaries

Consider the following linear singular time-varying delay system:

$$E\dot{x}(t) = Ax(t) + A_d x(t - h(t)),$$
$$x(t) = \phi(t), t \in [-h_2, 0], \tag{1}$$

where $x(t) \in R^n$ is the state variable. $h(t)$ is a time-varying delay which satisfies $0 \leq h_1 \leq h(t) \leq h_2$, and $\dot{h}(t) \leq D$. $E, A, A_d \in R^{n\times n}$ are knows real constant matrices where E may be a singular matrix, so and we assume that $rank(E) = r \leq n$. $\phi(t)$ is a continuously real-valued initial function.

We now tranform system (1) into a neutral system under certain constraint. To proceed, the following Definitions and Lemmas are needed.

Definition 1 (Dai [1]). The pair (E, A) is said to be regular if $det(sE - A)$ is not identically zero. The pair (E, A) is said to be impulse free if $deg(det(\lambda E - A)) = rank(E)$.

Lemma 1 (Xu et al. [24]). If the pair (E, A) is regular and impulse free, then the solution to the singular time-varying delay system (1) exists and is impulse free and unique on $[0, \infty)$.

From Lemma 1, the following definition is naturally introduced.

Definition 2 (Xu et al. [24]). The singular time-varying delay system (1) is said to be regular and impulse free, if the pair (E, A) is regular and impulse free.

Lemma 2 (Dai [1]). If the pair (E, A) is regular and impulse free, then there exists two invertible matrices $M \in R^{n\times n}$ and $N \in R^{n\times n}$ such that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \; MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}. \tag{2}$$

According to Lemma 2, if the pair (E, A) is regular and impulse free, invertible matrices $M, N \in R^{n\times n}$ can always be found such that

$$MEN := \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} = \bar{E}, \; MAN := \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix} = \bar{A}. \tag{3}$$

Let

$$MA_dN = \bar{A}_d = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix}, \; N^{-1}x(t) = \mu(t) = \begin{bmatrix} \mu_1(t) \\ \mu_2(t) \end{bmatrix}, \tag{4}$$

The following system :

$$\bar{E}\dot{\mu}(t) = \bar{A}\mu(t) + \bar{A}_d\mu(t - h(t)), \tag{5}$$

which can be written as follows :

$$\dot{\mu_1}(t) = A_1\mu(t) + A_{d1}\mu_1(t - h(t)) + A_{d2}\mu_2(t - h(t)), \tag{6}$$

$$0 = \mu_2(t) + A_{d3}\mu_1(t - h(t)) + A_{d4}\mu_2(t - h(t)). \tag{7}$$

Inspired by [9] and [13] , we rewrite the second equation. By differentiating (7) , we obtain

$$\frac{d}{dt}[\mu_2(t) + A_{d3}\mu_1(t - h(t)) + A_{d4}\mu_2(t - h(t))] = 0. \tag{8}$$

By combining (7) and (8) , we have

$$\begin{aligned} \dot{\mu_2}(t) = &- \mu_2(t) - A_{d3}\dot{\mu_1}(t - h(t))(1 - \dot{h}(t)) - A_{d4}\dot{\mu_2}(t - h(t))(1 - \dot{h}(t)) \\ &- A_{d3}\mu_1(t - h(t)) - A_{d4}\mu_2(t - h(t)). \end{aligned} \tag{9}$$

From (6) and (9) , we can have

$$\begin{aligned} \begin{bmatrix} \dot{\mu_1}(t) \\ \dot{\mu_2}(t) \end{bmatrix} &= \begin{bmatrix} A_1\mu_1(t) + A_{d1}\mu_1(t - h(t)) + A_{d2}\mu_2(t - h(t)) \\ -\mu_2(t) - A_{d3}\mu_1(t - h(t)) - A_{d4}\mu_2(t - h(t)) \end{bmatrix} \\ &+ [1 - \dot{h}(t)] \begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix} \begin{bmatrix} \dot{\mu_1}(t - h(t)) \\ \dot{\mu_2}(t - h(t)) \end{bmatrix}. \end{aligned} \tag{10}$$

Let

$$\hat{A} = \begin{bmatrix} A_1 & 0 \\ 0 & -I_{n-r} \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} A_{d1} & A_{d2} \\ -A_{d3} & -A_{d4} \end{bmatrix}, \ \hat{C} = \begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix}. \tag{11}$$

Then the system (10) is of the form of a neutral system ,

$$\begin{aligned} &\dot{\mu}(t) = \hat{A}\mu(t) + \hat{A}_d\mu(t - h(t)) + (1 - \dot{h}(t))\ \hat{C}\dot{\mu}(t - h(t))\ , \\ &\mu(t) = \varphi(t)\ ,\ t\epsilon[-h_2, 0] \end{aligned} \tag{12}$$

Definition 3 The equilibrium point $x = 0$ of system (12) is said to be asymptotically stable (A.S.) if it is stable and δ can be chosen such that

$$\parallel x(0) \parallel < \delta \Rightarrow \lim_{t\to\infty} x(t) = 0$$

Lemma 3 Let $x = 0$ be an equilibrium point for system (12) and $D \subset R^n$ be a domain containing $x = 0$.

1. If there exists a Lyapunov function V then the equilibrium point $x = 0$ of system (12) is stable.

2. If there exists a Lyapunov function V and $\dot{V}(x) < 0$ in $D - \{0\}$ the equilibrium point $x = 0$ of system (12) is asymptotically stable.

In this work, we will provide an alternative way to study the stability of singular time-varying delay system (1) via the neutral system (12). Although the dynamic responses for system (1) and (12) may be different, it is easy to see that the stability property for both systems remains the same. That is, the asymptotically stability of system (12) will guarantee the asymptotic stability of system (1), and vice versa. With respect to the stability analysis of system (12), it is always assumed that the spectral radius of $\hat{C}(1 - D)$ is less than 1. So the following assumption is made before our main result.

Assumption 1 Assume that the pair (E, A) is regular and impulse free and all the eigenvalues of $\hat{C}(1 - D)$ are inside the unit circle.

Lemma 4 (Seuret and Gouaisbaut [2]). For any constant matrix $R \in \mathbb{R}^{n\times n}$, $R = R^T \geq 0$, the following inequality holds for all continuously differentiable function x in $[a, b] \to \mathbb{R}^n$:

$$\int_a^b \dot{x}^T R\dot{x}(s)ds \geq \frac{1}{b-a}(x(b)-x(a))^T R(x(b)-x(a)) + \frac{3}{b-a}\Omega^T R\Omega, \tag{13}$$

where $\Omega = x(a) + x(b) - (\frac{2}{b-a})\int_a^b x(s)ds$.

Proposition 2.1 (Schur complement lemma [8]). Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$ if and only if

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0 \textbf{ or } \begin{bmatrix} -Y & Z \\ Z^T & X \end{bmatrix} < 0.$$

Proposition 2.2 (Cauchy's inequality [5]). Let E and H be any constant matrices. For any $\varepsilon > 0$, we have

$$2EH \leq \varepsilon EF^T + \varepsilon^{-1} H^T H.$$

3 Main results

In this section, we present our main result which is for both the singular time-varying delay system (1) and the neutral system (12).

Theorem 1. Under Assumption 1, system (12) and thus system (1) are asymptotically stable, if there exist $5n \times 5n$ positive-definite matrix P and $n \times n$ positive-definite matrices $Q_1, Q_2, R_1, R_2, S_1, S_2, W, T, N$ and M such that the following LMI holds:

$$\omega = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & \Psi_{110} & 0 & 0 & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} & \Psi_{28} & \Psi_{29} & 0 & 0 & 0 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} & \Psi_{38} & \Psi_{39} & 0 & 0 & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & \Psi_{46} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & \Psi_{56} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & \Psi_{68} & \Psi_{69} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} & \Psi_{79} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & \Psi_{89} & \Psi_{810} & \Psi_{811} & 0 & 0 \\ * & * & * & * & * & * & * & * & \Psi_{99} & \Psi_{910} & 0 & \Psi_{912} & 0 \\ * & * & * & * & * & * & * & * & * & \Psi_{1010} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Psi_{1111} & 0 & \Psi_{1113} \\ * & * & * & * & * & * & * & * & * & * & * & \Psi_{1212} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & \Psi_{1313} \end{bmatrix} < 0 \quad (14)$$

Where

$$\Psi_{11} = P_{11}\hat{A} + \hat{A}^T P_{11} + P_{14} + P_{14}^T + P_{15} + P_{15}^T + Q_1 + Q_2 + \hat{A}^T S_1 \hat{A}$$
$$+\hat{A}^T S_2 \hat{A} + \hat{A}^T h_1^2 R_1 \hat{A} - 4R_1 + \hat{A}^T h_2^2 R_2 \hat{A} - 4R_2 \ ;$$
$$\Psi_{12} = -P_{14} + P_{12}\hat{A} + P_{24} + P_{25} - 2R_1 \ ;$$
$$\Psi_{13} = -P_{15} + P_{13}\hat{A} + P_{34} + P_{35} - 2R_2 \ ;$$
$$\Psi_{14} = P_{12} \ ;$$
$$\Psi_{15} = P_{13} \ ;$$
$$\Psi_{16} = h_1 P_{14}\hat{A} + h_1 P_{44} + h_1 P_{45} + 6R_1 \ ;$$
$$\Psi_{17} = h_2 P_{15}\hat{A} + h_2 P_{45}^T + h_2 P_{55} + 6R_2 \ ;$$
$$\Psi_{18} = \hat{A_d}^T P_{11} + \hat{A_d}^T S_1 \hat{A} + \hat{A_d}^T S_2 \hat{A} + \hat{A_d}^T h_1^2 R_1 \hat{A} + \hat{A_d}^T h_2^2 R_2 \hat{A} - W\hat{A}$$
$$+T\alpha \ ;$$
$$\Psi_{19} = \hat{C}^T P_{11} + \hat{C}^T S_1 \hat{A} + \hat{C}^T S_2 \hat{A} + \hat{C}^T h_1^2 R_1 \hat{A} + \hat{C}^T h_2^2 R_2 \hat{A} - W\hat{A} \ ;$$
$$\Psi_{110} = W\hat{A} \ ;$$
$$\Psi_{22} = -P_{24} - P_{24}^T - Q_1 - 4R_1 \ ;$$
$$\Psi_{23} = -P_{25} - P_{34} \ ;$$
$$\Psi_{24} = P_{22} \ ;$$
$$\Psi_{25} = P_{23} \ ;$$

$$\Psi_{26} = -h_1 P_{44} + 6R_2 \ ;$$

$$\Psi_{27} = h_2 P_{25}^T - h_2 P_{45}^T \ ;$$

$$\Psi_{28} = \hat{A}_d{}^T P_{12}^T \ ;$$

$$\Psi_{29} = \hat{C}^T P_{12}^T \ ;$$

$$\Psi_{33} = -P_{35} - P_{35}^T - Q_2 - 4R_2 \ ;$$

$$\Psi_{34} = P_{23}^T \ ;$$

$$\Psi_{35} = P_{33} \ ;$$

$$\Psi_{36} = -h_1 P_{45} \ ;$$

$$\Psi_{37} = h_2 P_{35}^T - h_2 P_{55} + 6R_2 \ ;$$

$$\Psi_{38} = \hat{A}_d{}^T P_{13}^T \ ;$$

$$\Psi_{39} = \hat{C}^T P_{13}^T \ ;$$

$$\Psi_{44} = -S_1 \ ;$$

$$\Psi_{46} = h_1 P_{24}^T \ ;$$

$$\Psi_{55} = -S_2 \ ;$$

$$\Psi_{56} = h_1 P_{34}^T \ ;$$

$$\Psi_{66} = -12R_1 \ ;$$

$$\Psi_{68} = h_1 \hat{A}_d{}^T P_{14}^T \ ;$$

$$\Psi_{69} = h_1 \hat{C}^T P_{14}^T \ ;$$

$$\Psi_{77} = -12R_2 \ ;$$

$$\Psi_{78} = h_2 \hat{A}_d{}^T P_{15}^T \ ;$$

$$\Psi_{79} = h_2 \hat{C}^T P_{15}^T \ ;$$

$$\Psi_{88} = \hat{A}_d^T S_1 \hat{A}_d + \hat{A}_d^T S_2 \hat{A}_d + \hat{A}_d{}^T h_1^2 R_1 \hat{A}_d + \hat{A}_d{}^T h_2^2 R_2 \hat{A}_d - W\hat{A}_d - \hat{A}_d{}^T W$$
$$-2T\alpha \ ;$$

$$\Psi_{89} = \hat{C}^T S_1 \hat{A}_d + \hat{C}^T S_2 \hat{A}_d + \hat{C}^T h_1^2 R_1 \hat{A}_d + \hat{C}^T h_2^2 R_2 \hat{A}_d - \hat{C}^T W - W\hat{A}_d \ ;$$

$$\Psi_{810} = W\hat{A}_d + W \ ;$$

$$\Psi_{811} = -T \ ;$$

$$\Psi_{99} = \hat{C}^T S_1 \hat{C} + \hat{C}^T S_2 \hat{C} + \hat{C}^T h_1^2 R_1 \hat{C} + \hat{C}^T h_2^2 R_2 \hat{C} - W\hat{C}$$
$$-\hat{C}^T W - 2N + \varepsilon_1 (1-D)^2 I \ ;$$

$$\Psi_{910} = W\hat{C} + W \ ;$$

$$\Psi_{912} = N^T \ ;$$

$$\Psi_{1010} = -2W \ ;$$

$\Psi_{1111} = -2M + \varepsilon_2 \alpha^2 I$;
$\Psi_{1113} = M^T$;
$\Psi_{1212} = -\varepsilon_1 I$;
$\Psi_{1313} = -\varepsilon_2 I$.

Proof. For simplicity, we define

$$\zeta(t) = \Big[\mu^T(t) \quad \mu^T(t-h_1) \quad \mu^T(t-h_2) \quad \int_{t-h_1}^{t} \mu^T(s)ds \quad \int_{t-h_2}^{t} \mu^T(s)ds\Big]^T ,$$

$$\xi(t) = [\mu^T(t)\ \mu^T(t-h_1)\ \mu^T(t-h_2)\ \dot{\mu}^T(t-h_1)\ \dot{\mu}^T(t-h_2) \\ \tfrac{1}{h_1}\int_{t-h_1}^{t} \mu^T(s)ds\ \tfrac{1}{h_2}\int_{t-h_2}^{t} \mu^T(s)ds\ \mu^T(t-h(t))\ (1-\dot{h}(t))\dot{\mu}^T(t-h(t)) \\ \dot{\mu}^T(t)\ \alpha \int_{t-h(t)}^{t} \dot{\mu}^T(s)ds]^T.$$

We introduce the following LKF:

$$V(t) = V_1 + V_2 + V_3 + V_4 \ , \tag{15}$$

where

$$V_1 = \zeta^T(t) P \zeta(t) \ ,$$
$$V_2 = \int_{t-h_1}^{t} \mu^T(s) Q_1 \mu(s) ds + \int_{t-h_2}^{t} \mu^T(s) Q_2 \mu(s) ds \ ,$$
$$V_3 = \int_{t-h_1}^{t} \dot{\mu}^T(s) S_1 \dot{\mu}^T(s) ds + \int_{t-h_2}^{t} \dot{\mu}^T(s) S_2 \dot{\mu}^T(s) ds \ ,$$
$$V_4 = h_1 \int_{-h_1}^{0} \int_{t-\theta}^{t} \dot{\mu}^T(s) R_1 \dot{\mu}(s) ds d\theta + h_2 \int_{-h_2}^{0} \int_{t+\theta}^{t} \dot{\mu}^T(s) R_2 \dot{\mu}(s) ds d\theta \ .$$

Differentiating (15) with respect to t, we have

$$\begin{aligned}
\dot{V}_1 &= 2\zeta^T(t) P \dot{\zeta}(t) \\
&= 2 \begin{bmatrix} \mu(t) \\ \mu(t-h_1) \\ \mu(t-h_2) \\ \int_{t-h_1}^{t} \mu(s)ds \\ \int_{t-h_2}^{t} \mu(s)ds \end{bmatrix}^T P \begin{bmatrix} \dot{\mu}(t) \\ \dot{\mu}(t-h_1) \\ \dot{\mu}(t-h_2) \\ \mu(t)-\mu(t-h_1) \\ \mu(t)-\mu(t-h_2) \end{bmatrix} \\
&= 2[\mu^T(t) P_{11}(\hat{A}\mu(t) + \hat{A}_d \mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) \\
&\quad + \mu^T(t) P_{12} \dot{\mu}(t-h_1) + \mu^T(t) P_{13} \dot{\mu}(t-h_2) + \mu^T(t) P_{14}(\mu(t)-\mu(t-h_1)) \\
&\quad + \mu^T(t) P_{15}(\mu(t)-\mu(t-h_2)) + \mu^T(t-h_1) P_{12}^T(\hat{A}\mu(t) + \hat{A}_d \mu(t-h(t))
\end{aligned}$$

$$+(1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) + \mu^T(t-h_1)P_{22}\dot{\mu}(t-h_1)$$
$$+\mu^T(t-h_1)P_{23}\dot{\mu}(t-h_2) + \mu^T(t-h_1)P_{24}(\mu(t)-\mu(t-h_1))$$
$$+\mu^T(t-h_1)P_{25}(\mu(t)-\mu(t-h_2)) + \mu^T(t-h_2)P_{13}^T(\hat{A}\mu(t)$$
$$+\hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) + \mu^T(t-h_2)P_{23}^T\dot{\mu}(t-h_1)$$
$$+\mu^T(t-h_2)P_{33}\dot{\mu}(t-h_2) + \mu^T(t-h_2)P_{34}(\mu(t)-\mu(t-h_1))$$
$$+\mu^T(t-h_2)P_{35}(\mu(t)-\mu(t-h_2)) + \int_{t-h_1}^t \mu(s)dsP_{14}^T(\hat{A}\mu(t)$$
$$+\hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) + \int_{t-h_1}^t \mu(s)dsP_{24}^T\dot{\mu}(t-h_1)$$
$$+\int_{t-h_1}^t \mu(s)dsP_{34}^T\dot{\mu}(t-h_2) + \int_{t-h_1}^t \mu(s)dsP_{44}(\mu(t)-\mu(t-h_1))$$
$$+\int_{t-h_1}^t \mu(s)dsP_{45}(\mu(t)-\mu(t-h_2)) + \int_{t-h_2}^t \mu(s)dsP_{15}^T(\hat{A}\mu(t)$$
$$+\hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) + \int_{t-h_2}^t \mu(s)dsP_{25}^T\dot{\mu}(t-h_1)$$
$$+\int_{t-h_2}^t \mu(s)dsP_{35}^T\dot{\mu}(t-h_2) + \int_{t-h_2}^t \mu(s)dsP_{45}^T(\mu(t)-\mu(t-h_1))$$
$$+\int_{t-h_2}^t \mu(s)dsP_{55}(\mu(t)-\mu(t-h_2))]. \quad (16)$$

$$\dot{V}_2 = [\mu^T(t)(Q_1+Q_2)\mu(t) - \mu^T(t-h_1)Q_1\mu(t-h_1)$$
$$-\mu^T(t-h_2)Q_2\mu(t-h_2)]. \quad (17)$$

$$\dot{V}_3 = [(\hat{A}\mu(t) + \hat{A}_d(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))(S_1+S_2)(\hat{A}\mu(t)$$
$$+\hat{A}_d(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t))) - \dot{\mu}^T(t-h_1)S_1\dot{\mu}(t-h_1)$$
$$-\dot{\mu}^T(t-h_2)S_2\dot{\mu}(t-h_2))]. \quad (18)$$

$$\dot{V}_4 = h_1^2(\hat{A}\mu(t) + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))^T R_1(\hat{A}\mu(t)$$
$$+\hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)) - h_1\int_{t-h_1}^t \dot{\mu}^T(s)R_1\dot{\mu}(s)ds$$
$$+h_2^2(\hat{A}\mu(t) + \hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)))^T R_2(\hat{A}\mu(t)$$
$$+\hat{A}_d\mu(t-h(t)) + (1-\dot{h}(t))\hat{C}\dot{\mu}(t-h(t)) - h_2\int_{t-h_2}^t \dot{\mu}^T(s)R_2\dot{\mu}(s)ds.$$

By adopting Lemma 4 , we have

$$-h_1\int_{t-h_1}^t \dot{\mu}^T(s)R_1\dot{\mu}(s)ds$$
$$\leq -h_1[\tfrac{1}{t-(t-h_1)}(\mu(t)-\mu(t-h_1))^T R_1(\mu(t)-\mu(t-h_1))$$

$$
\begin{aligned}
&+\tfrac{3}{t-(t-h_1)}(\mu(t-h_1)+\mu(t)-\tfrac{2}{t-(t-h_1)}\int_{t-h_1}^{t}\mu(s)ds)^T R_1(\mu(t-h_1)\\
&+\mu(t)-\tfrac{2}{t-h(t)}\int_{t-h_1}^{t}\mu(s)ds)]\\
&=-h_1[\tfrac{1}{h_1}(\mu(t)-\mu(t-h_1))^T R_1(\mu(t)-(\mu(t-h_1))+\tfrac{3}{h_1}(\mu(t-h_1)\\
&+\mu(t)-\tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds)^T R_1(\mu(t-h_1)+\mu(t)-\tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds)]\\
&=-(\mu(t)-\mu(t-h_1))^T R_1(\mu(t)-\mu(t-h_1))-(\mu(t)+\mu(t-h_1)\\
&-\tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds)^T 3R_1(\mu(t)+\mu(t-h_1)-\tfrac{2}{h_1}\int_{t-h_1}^{t}\mu(s)ds). \qquad (19)
\end{aligned}
$$

Similarly , we have

$$
\begin{aligned}
&-h_2\int_{t-h_2}^{t}\dot{\mu}^T(s)R_2\dot{\mu}(s)ds\\
&\leq -h_2[\tfrac{1}{t-(t-h_2)}(\mu(t)-\mu(t-h_2))^T R_2(\mu(t)-\mu(t-h_2))+\tfrac{3}{t-(t-h_2)}(\mu(t-h_2)\\
&+\mu(t)-\tfrac{2}{t-(t-h_2)}\int_{t-h_2}^{t}\mu(s)ds)^T R_2(\mu(t-h_2)+\mu(t)-\tfrac{2}{t-(t-h_2)}\int_{t-h_2}^{t}\mu(s)ds)]\\
&=-h_2[\tfrac{1}{h_2}(\mu(t)-\mu(t-h_2))^T R2(\mu(t)-\mu(t-h_2))+\tfrac{3}{h_2}(\mu(t-h_2)\\
&+\mu(t)-\tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds)^T R_2(\mu(t-h_2)+\mu(t)-\tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds)]\\
&=-(\mu(t)-\mu(t-h_2))^T R_2(\mu(t)-\mu(t-h_2))-(\mu(t)+\mu(t-h_2)\\
&-\tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds)^T 3R_1(\mu(t)+\mu(t-h_2)-\tfrac{2}{h_2}\int_{t-h_2}^{t}\mu(s)ds. \qquad (20)
\end{aligned}
$$

By using the following identity relation :

$$\dot{\mu}(t)-\hat{A}\mu(t)-\hat{A}_d\mu(t-h(t))-(1-\dot{h}(t))\hat{C}\mu(t-h(t))=0,$$

we have

$$[-\mu^T(t-h(t))+\dot{\mu}^T(t)][-2W][\dot{\mu}(t)-\hat{A}\mu(t)-\hat{A}_d\mu(t-h(t))-(1-\dot{h}(t))\hat{C}\mu(t-h(t))]=0 \quad (21)$$

where W is a positive definite matrix.

By using the following identity relation

$$
\begin{aligned}
\alpha\int_{t-h(t)}^{t}\dot{\mu}(s)ds &= \alpha(\mu(t)-\mu(t-h(t)))\\
&= \alpha\mu(t)-\alpha\mu(t-h(t)).
\end{aligned}
$$

From we get

$\alpha \int_{t-h(t)}^{t} \dot{\mu}(s)ds - \alpha\mu(t) + \alpha\mu(t-h(t)) = 0.$

and we have

$$[-\mu(t-h(t))][-2T][\alpha \int_{t-h(t)}^{t} \dot{\mu}(s)ds - \alpha\mu(t) + \alpha\mu(t-h(t))]. \tag{22}$$

Where T is a positive definite matrix and α is a positive real number.

By using the following identity relation.

$$\begin{aligned}&(1-\dot{h}(t))\dot{\mu}^T(t-h(t))2N(1-\dot{h}(t))\dot{\mu}(t-h(t))\\&-(1-\dot{h}(t))\dot{\mu}^T(t-h(t))2N(1-\dot{h}(t))\dot{\mu}(t-h(t)) = 0.\end{aligned} \tag{23}$$

where $N > 0$.

From Proposition 2.2 and $\dot{h}(t) \leq D$, we obtain

$$\begin{aligned}2(1-D)(\mu^T(t-h(t)))N(\mu(t-h(t))) \leq{}& \varepsilon_1(1-D)^2\mu^T((t-h(t))\mu(t-h(t)))\\&+\varepsilon_1^{-1}N^T\mu^T(t-h(t))N(\mu(t-h(t))).\end{aligned} \tag{24}$$

where $\varepsilon_1 > 0$.

Similarly , we have

$$\begin{aligned}&\alpha \int_{t-h(t)}^{t} \dot{\mu}^T(s)2M\alpha \int_{t-h(t)}^{t} \dot{\mu}(s)ds\\&-\alpha \int_{t-h(t)}^{t} \dot{\mu}^T(s)ds2M\alpha \int_{t-h(t)}^{t} \dot{\mu}(s)ds = 0.\end{aligned} \tag{25}$$

where $M > 0$.

From Proposition 2.2, we obtain

$$\begin{aligned}2\alpha(\int_{t-h(t)}^{t} \dot{\mu}^T(s)dsM \int_{t-h(t)}^{t} \dot{\mu}^T(s)ds) \leq{}& \varepsilon_2\alpha^2 \int_{t-h(t)}^{t} \dot{\mu}^T(s)ds \int_{t-h(t)}^{t} \dot{\mu}(s)ds\\&+\varepsilon_2^{-1}M^T \int_{t-h(t)}^{t} \dot{\mu}^T(s)dsM \int_{t-h(t)}^{t} \dot{\mu}(s)ds.\end{aligned} \tag{26}$$

Combining system (12) with (15)-(29) , we arrive at

$$\dot{V}(t) \leq \xi^T(t)\omega\xi(t) \;, \tag{27}$$

where

$$\xi(t) = [\mu^T(t)\ \mu^T(t-h_1)\ \mu^T(t-h_2)\ \dot{\mu}^T(t-h_1)\ \dot{\mu}^T(t-h_2)$$
$$\tfrac{1}{h_1}\int_{t-h_1}^{t}\mu^T(s)ds\ \tfrac{1}{h_2}\int_{t-h_2}^{t}\mu^T(s)ds\ \mu^T(t-h(t))\ (1-\dot{h}(t))\dot{\mu}^T(t-h(t))$$
$$\dot{\mu}^T(t)\ \alpha\int_{t-h(t)}^{t}\dot{\mu}^T(s)ds]^T,$$

$$\omega = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} & \Psi_{15} & \Psi_{16} & \Psi_{17} & \Psi_{18} & \Psi_{19} & \Psi_{110} & 0 \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & \Psi_{25} & \Psi_{26} & \Psi_{27} & \Psi_{28} & \Psi_{29} & 0 & 0 \\ * & * & \Psi_{33} & \Psi_{34} & \Psi_{35} & \Psi_{36} & \Psi_{37} & \Psi_{38} & \Psi_{39} & 0 & 0 \\ * & * & * & \Psi_{44} & 0 & \Psi_{46} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Psi_{55} & \Psi_{56} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \Psi_{66} & 0 & \Psi_{68} & \Psi_{69} & 0 & 0 \\ * & * & * & * & * & * & \Psi_{77} & \Psi_{78} & \Psi_{79} & 0 & 0 \\ * & * & * & * & * & * & * & \Psi_{88} & \Psi_{89} & \Psi_{810} & \Psi_{811} \\ * & * & * & * & * & * & * & * & \Psi_{99} & \Psi_{910} & 0 \\ * & * & * & * & * & * & * & * & * & \Psi_{1010} & 0 \\ * & * & * & * & * & * & * & * & * & * & \Psi_{1111} \end{bmatrix} < 0,$$

$$\Psi_{11} = P_{11}\hat{A} + \hat{A}^T P_{11} + P_{14} + P_{14}^T + P_{15} + P_{15}^T + Q_1 + Q_2 + \hat{A}^T S_1 \hat{A}$$
$$+\hat{A}^T S_2 \hat{A} + \hat{A}^T h_1^2 R_1 \hat{A} - 4R_1 + \hat{A}^T h_2^2 R_2 \hat{A} - 4R_2 \;;$$
$$\Psi_{12} = -P_{14} + P_{12}\hat{A} + P_{24} + P_{25} - 2R_1 \;;$$
$$\Psi_{13} = -P_{15} + P_{13}\hat{A} + P_{34} + P_{35} - 2R_2 \;;$$
$$\Psi_{14} = P_{12} \;;$$
$$\Psi_{15} = P_{13} \;;$$
$$\Psi_{16} = h_1 P_{14}\hat{A} + h_1 P_{44} + h_1 P_{45} + 6R_1 \;;$$
$$\Psi_{17} = h_2 P_{15}\hat{A} + h_2 P_{45}^T + h_2 P_{55} + 6R_2 \;;$$
$$\Psi_{18} = \hat{A}_d^{\,T} P_{11} + \hat{A}_d^{\,T} S_1 \hat{A} + \hat{A}_d^{\,T} S_2 \hat{A} + \hat{A}_d^{\,T} h_1^2 R_1 \hat{A} + \hat{A}_d^{\,T} h_2^2 R_2 \hat{A} - W\hat{A}$$
$$+T\alpha \;;$$
$$\Psi_{19} = \hat{C}^T P_{11} + \hat{C}^T S_1 \hat{A} + \hat{C}^T S_2 \hat{A} + \hat{C}^T h_1^2 R_1 \hat{A} + \hat{C}^T h_2^2 R_2 \hat{A} - W\hat{A} \;;$$
$$\Psi_{110} = W\hat{A} \;;$$
$$\Psi_{22} = -P_{24} - P_{24}^T - Q_1 - 4R_1 \;;$$

$\Psi_{23} = -P_{25} - P_{34}$;

$\Psi_{24} = P_{22}$;

$\Psi_{25} = P_{23}$;

$\Psi_{26} = -h_1 P_{44} + 6R_2$;

$\Psi_{27} = h_2 P_{25}^T - h_2 P_{45}^T$;

$\Psi_{28} = \hat{A}_d^{\ T} P_{12}^T$;

$\Psi_{29} = \hat{C}^T P_{12}^T$;

$\Psi_{33} = -P_{35} - P_{35}^T - Q_2 - 4R_2$;

$\Psi_{34} = P_{23}^T$;

$\Psi_{35} = P_{33}$;

$\Psi_{36} = -h_1 P_{45}$;

$\Psi_{37} = h_2 P_{35}^T - h_2 P_{55} + 6R_2$;

$\Psi_{38} = \hat{A}_d^{\ T} P_{13}^T$;

$\Psi_{39} = \hat{C}^T P_{13}^T$;

$\Psi_{44} = -S_1$;

$\Psi_{46} = h_1 P_{24}^T$;

$\Psi_{55} = -S_2$;

$\Psi_{56} = h_1 P_{34}^T$;

$\Psi_{66} = -12R_1$;

$\Psi_{68} = h_1 \hat{A}_d^{\ T} P_{14}^T$;

$\Psi_{69} = h_1 \hat{C}^T P_{14}^T$;

$\Psi_{77} = -12R_2$;

$\Psi_{78} = h_2 \hat{A}_d^{\ T} P_{15}^T$;

$\Psi_{79} = h_2 \hat{C}^T P_{15}^T$;

$$\Psi_{88} = \hat{A}_d^T S_1 \hat{A}_d + \hat{A}_d^T S_2 \hat{A}_d + \hat{A}_d^{\ T} h_1^2 R_1 \hat{A}_d + \hat{A}_d^{\ T} h_2^2 R_2 \hat{A}_d - W\hat{A}_d - \hat{A}_d^{\ T} W \\ -2T\alpha \ ;$$

$$\Psi_{89} = \hat{C}^T S_1 \hat{A}_d + \hat{C}^T S_2 \hat{A}_d + \hat{C}^T h_1^2 R_1 \hat{A}_d + \hat{C}^T h_2^2 R_2 \hat{A}_d - \hat{C}^T W - W\hat{A}_d \ ;$$

$\Psi_{810} = W\hat{A}_d + W$;

$\Psi_{811} = -T$;

$$\Psi_{99} = \hat{C}^T S_1 \hat{C} + \hat{C}^T S_2 \hat{C} + \hat{C}^T h_1^2 R_1 \hat{C} + \hat{C}^T h_2^2 R_2 \hat{C} - W\hat{C} \\ -\hat{C}^T W - 2N + \varepsilon_1 (1-D)^2 I + \varepsilon_1^{-1} N^T N \ ;$$

$\Psi_{910} = W\hat{C} + W$;
$\Psi_{1010} = -2W$;
$\Psi_{1111} = -2M + \varepsilon_2\alpha^2 I + \varepsilon_2^{-1} M^T M$.

From (29) and Proposition 2.1 , it is easy to see that $\dot{V}(t) < 0$. From Lemma 3, we conclude that system (12) and hence , system (1) are asymptotically stable.

4 Numerical Examples

In this section, three numerical examples are given to illustrate the validity and superiority of the proposed scheme.

Example 1. Consider the following singular time-varying system with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \ A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \ E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

Obviously, the pair (E, A) is regular and impulse free. Let

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \textbf{ and } N = \begin{bmatrix} 1 & 0 \\ -1 & -1 \end{bmatrix},$$

such that

$$MEN = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ MAN = \begin{bmatrix} -2 & 0 \\ 0.9 & 0.9 \end{bmatrix}, \ MA_dN = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

According to Theorem 1, let

$$\hat{C} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \ \hat{A} = \begin{bmatrix} -2 & 0 \\ 0.9 & 0.9 \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

We assume that with satisfy

$$h(t) = sin^2(t)$$

and $0 \le h_1 \le h(t) \le h_2$

By taking parameters $D = 0.5, n = 0.9, m = 0.5$ and $a = 0.2$, we get Example 1. remains feasible for any delay time $h_1 = 0$ and $h_2 = 1$. Theorem 1. yields the following set of feasible solutions :

$$Q_1 = \begin{bmatrix} 3.9601 \times 10^3 & -0.0000 \times 10^3 \\ -0.0000 \times 10^3 & 3.9601 \times 10^3 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 19.2506 & 1.4550 \\ 1.4550 & 12.2446 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 416.7206 & 1.2125 \\ 1.2125 & 405.3354 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} 0.0033 & -0.0002 \\ -0.0002 & 0.0061 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.004 & 0.0001 \\ 0.0001 & 0.005 \end{bmatrix}, \qquad S_2 = \begin{bmatrix} 0.0041 & 0.0001 \\ 0.0001 & 0.0051 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.0024 & 0.0000 \\ 0.0000 & 0.014 \end{bmatrix}, \qquad N = \begin{bmatrix} 1.1390 & 0.0014 \\ 0.0014 & 1.1369 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.5103 & -0.0007 \\ -0.0007 & 0.5176 \end{bmatrix}, \qquad T = \begin{bmatrix} 0.1003 & -0.0005 \\ -0.0005 & 0.1024 \end{bmatrix},$$

$$P_{11} = \begin{bmatrix} 1.7268 & 0.0016 \\ 0.0016 & 1.3562 \end{bmatrix}, \qquad P_{12} = \begin{bmatrix} -0.6723 & -0.0077 \\ 0.0171 & -0.4829 \end{bmatrix},$$

$$P_{13} = \begin{bmatrix} -1.0055 & 0.0033 \\ -0.0204 & -0.9085 \end{bmatrix}, \qquad P_{14} = \begin{bmatrix} -4.1883 \times 10^3 & -0.0235 \times 10^3 \\ 0.0268 \times 10^3 & -4.4274 \times 10^3 \end{bmatrix},$$

$$P_{15} = \begin{bmatrix} -0.0244 \times 10^3 & -0.4054 \times 10^3 \\ -0.5297 \times 10^3 & -0.0135 \times 10^3 \end{bmatrix}, \qquad P_{22} = \begin{bmatrix} 2.0373 & 0.0368 \\ 0.0368 & 2.2302 \end{bmatrix},$$

$$P_{23} = \begin{bmatrix} -1.3690 & -0.0544 \\ -0.0305 & -1.7511 \end{bmatrix}, \qquad P_{24} = \begin{bmatrix} 331.7989 & -10.1762 \\ 49.3883 & 378.7585 \end{bmatrix},$$

$$P_{25} = \begin{bmatrix} -3.2653 \times 10^3 & -0.0358 \times 10^3 \\ -0.0361 \times 10^3 & -3.4496 \times 10^3 \end{bmatrix}, \qquad P_{33} = \begin{bmatrix} 2.3817 & 0.0520 \\ 0.0520 & 2.6648 \end{bmatrix},$$

$$P_{34} = \begin{bmatrix} -113.7437 & -19.1402 \\ -28.4193 & 80.2111 \end{bmatrix}, \qquad P_{35} = \begin{bmatrix} 3.2653 \times 10^3 & 0.0358 \times 10^3 \\ 0.0361 \times 10^3 & 3.4495 \times 10^3 \end{bmatrix},$$

$$P_{44} = \begin{bmatrix} 3.9601 \times 10^3 & 0 \\ 0 & 3.9601 \times 10^3 \end{bmatrix}, \qquad P_{45} = \begin{bmatrix} -3.2653 \times 10^3 & -0.0359 \times 10^3 \\ -0.0360 \times 10^3 & -3.4496 \times 10^3 \end{bmatrix},$$

$$P_{55} = \begin{bmatrix} 3.2653 \times 10^3 & 0.0359 \times 10^3 \\ 0.0359 \times 10^3 & 3.4496 \times 10^3 \end{bmatrix}.$$

From Theorem 1 , we conclude that system (1) is asymptotically stable.

Example 2. Consider the system (1) with the following parameters :

$$\hat{A} = \begin{bmatrix} -0.2 & 0 \\ 0 & 1 \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\hat{C} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

By taking parameters $D = 0.5, n = 0.9, m = 0.5$ and $a = 0.2$, we get Example 2. remains feasible for any delay time $h_1 = 0$ and $h_2 = 1$. Theorem 1. yields the following set of feasible solutions :

$$Q_1 = \begin{bmatrix} 54.6121 & 0 \\ 0 & 54.6121 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 14.7851 & 0 \\ 0 & 17.3196 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 5.7487 & 0 \\ 0 & 5.7423 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.4765 & 0 \\ 0 & 0.4554 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.3456 & 0 \\ 0 & 0.3655 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.3524 & 0 \\ 0 & 0.3739 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.0058 & 0 \\ 0 & 0.0095 \end{bmatrix}, \quad N = \begin{bmatrix} 1.1109 & 0 \\ 0 & 1.1299 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.5138 & 0 \\ 0 & 0.5169 \end{bmatrix}, \quad T = \begin{bmatrix} 0.0991 & 0 \\ 0 & 0.0995 \end{bmatrix},$$

$$P_{11} = \begin{bmatrix} 2.2300 & 0 \\ 0 & 1.3229 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} -0.8572 & 0 \\ 0 & -1.5350 \end{bmatrix},$$

$$P_{13} = \begin{bmatrix} -0.9577 & 0 \\ 0 & -1.6762 \end{bmatrix}, \quad P_{14} = \begin{bmatrix} -59.9172 & 0 \\ 0 & -82.1298 \end{bmatrix},$$

$$P_{15} = \begin{bmatrix} -0.0502 & 0 \\ 0 & -0.1172 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 2.3548 & 0 \\ 0 & 2.6176 \end{bmatrix},$$

$$P_{23} = \begin{bmatrix} -1.5363 & 0 \\ 0 & -0.9575 \end{bmatrix}, \quad P_{24} = \begin{bmatrix} 1.9360 & 0 \\ 0 & 6.2648 \end{bmatrix},$$

$$P_{25} = \begin{bmatrix} -41.5093 & 0 \\ 0 & -43.3520 \end{bmatrix}, \quad P_{33} = \begin{bmatrix} 2.8955 & 0 \\ 0 & 3.1551 \end{bmatrix},$$

$$P_{34} = \begin{bmatrix} 0.6773 & 0 \\ 0 & 22.8484 \end{bmatrix}, \quad P_{35} = \begin{bmatrix} 34.9494 & 0 \\ 0 & 37.1122 \end{bmatrix},$$

$$P_{44} = \begin{bmatrix} 54.6121 & 0 \\ 0 & 54.6121 \end{bmatrix}, \quad P_{45} = \begin{bmatrix} -41.8184 & 0 \\ 0 & -42.8570 \end{bmatrix},$$

$$P_{55} = \begin{bmatrix} 39.9496 & 0 \\ 0 & 42.0913 \end{bmatrix}.$$

From Theorem 1 , we conclude that system (1) is asymptotically stable.

Example 3. Consider the system (1) with the following parameters :

$$\hat{A} = \begin{bmatrix} -4.5 & 0 \\ 0 & 6 \end{bmatrix}, \ \hat{A}_d = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix},$$

$$\hat{C} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \ I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

By taking parameters $D = 0.5, n = 0.9, m = 0.5$ and $a = 0.2$, we get Example 3. remains feasible for any delay time $h_1 = 0$ and $h_2 = 1$. Theorem 1. yields the following set of feasible solutions :

$$Q_1 = \begin{bmatrix} 15.5362 & 0 \\ 0 & 15.5362 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 2.2288 & 0 \\ 0 & 1.5451 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 1.6352 & 0 \\ 0 & 1.6353 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 0.0040 & 0 \\ 0 & 0.0010 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.0045 & 0 \\ 0 & 0.0012 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0.0047 & 0 \\ 0 & 0.0012 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.0018 & 0 \\ 0 & 0.0068 \end{bmatrix}, \quad N = \begin{bmatrix} 1.1026 & 0 \\ 0 & 1.1006 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.5148 & 0 \\ 0 & 0.5174 \end{bmatrix}, \quad T = \begin{bmatrix} 0.1019 & 0 \\ 0 & 0.1004 \end{bmatrix},$$

$$P_{11} = \begin{bmatrix} 0.1802 & 0 \\ 0 & 0.0579 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} -0.0370 & 0 \\ 0 & -0.0467 \end{bmatrix},$$

$$P_{13} = \begin{bmatrix} -0.0486 & 0 \\ 0 & -0.0573 \end{bmatrix}, \quad P_{14} = \begin{bmatrix} -15.0881 & 0 \\ 0 & -21.7708 \end{bmatrix},$$

$$P_{15} = \begin{bmatrix} -0.7682 \times 10^3 & 0 \\ 0 & 0.3741 \times 10^3 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 0.1443 & 0 \\ 0 & 0.0815 \end{bmatrix},$$

$$P_{23} = \begin{bmatrix} -0.1136 & 0 \\ 0 & -0.0366 \end{bmatrix}, \quad P_{24} = \begin{bmatrix} 0.8564 & 0 \\ 0 & 1.6070 \end{bmatrix},$$

$$P_{25} = \begin{bmatrix} -11.7344 & 0 \\ 0 & -12.9741 \end{bmatrix}, \quad P_{33} = \begin{bmatrix} 0.1785 & 0 \\ 0 & 0.0994 \end{bmatrix},$$

$$P_{34} = \begin{bmatrix} -2.2540 & 0 \\ 0 & 4.1814 \end{bmatrix}, \qquad P_{35} = \begin{bmatrix} 11.6815 & 0 \\ 0 & 12.9609 \end{bmatrix},$$

$$P_{44} = \begin{bmatrix} 15.5362 & 0 \\ 0 & 15.5362 \end{bmatrix}, \qquad P_{45} = \begin{bmatrix} -11.7400 & 0 \\ 0 & -12.9757 \end{bmatrix},$$

$$P_{55} = \begin{bmatrix} 11.7258 & 0 \\ 0 & 12.9713 \end{bmatrix}.$$

From Theorem 1 , we conclude that system (1) is asymptotically stable.

5 Conclusion

The stability problem for a class of singular time-varying delay systems has been investigated. Firstly, the singular system is represented as a neutral system. Then an augmented LKF and the Wirtinger-based integral inequality method are used to derive a new delay-dependent stability criterion in terms of LMIs. Three numerical examples are given to illustrate the reduced conservatism of the proposed method. Note that our method is proposed for dealing with time-varying delay. In the sense of finite-time stability (FTS), the proposed neutral system approach is not applicable and this has to be opened problem future works. For we expected that our present method is useful and may be effective method for FTS of singular systems.

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MATLAB CODE

MATLAB CODE

1. MATLAB CODE for finding solution of examples 1

```
A=[-2,0;0.9,0.9]
Ad=[-1,0;0,1]
C=[0.1,0;0,-0.1]
I=[1,0;0,1]
h1=0
h2=1
D=0.5
n=0.9
m=0.5
a=0.2
    setlmis([]);
 S1=lmivar(1,[2,1]);
 S2=lmivar(1,[2,1]);
 Q1=lmivar(1,[2,1]);
 Q2=lmivar(1,[2,1]);
 R1=lmivar(1,[2,1]);
 R2=lmivar(1,[2,1]);
 W=lmivar(1,[2,1]);
 P11=lmivar(1,[2,1]);
 P12=lmivar(2,[2,2]);
 P13=lmivar(2,[2,2]);
 P14=lmivar(2,[2,2]);
 P15=lmivar(2,[2,2]);
 P22=lmivar(1,[2,1]);
 P23=lmivar(2,[2,2]);
```

```
P24=lmivar(2,[2,2]);
P25=lmivar(2,[2,2]);
P33=lmivar(1,[2,1]);
P34=lmivar(2,[2,2]);
P35=lmivar(2,[2,2]);
P44=lmivar(1,[2,1]);
P45=lmivar(2,[2,2]);
P55=lmivar(1,[2,1]);
N=lmivar(1,[2,1]);
M=lmivar(1,[2,1]);
T=lmivar(1,[2,1]);
lmiterm([1 1 1 P11],1,A,'s');                % LMI #1: P11*A+A'*P11
lmiterm([1 1 1 P14],1,1,'s');                % LMI #1: P14+P14'
lmiterm([1 1 1 P15],1,1,'s');                % LMI #1: P15+P15'
lmiterm([1 1 1 Q1],1,1);                     % LMI #1: Q1
lmiterm([1 1 1 Q2],1,1);                     % LMI #1: Q2
lmiterm([1 1 1 S1],A',A);                    % LMI #1: A'*S1*A
lmiterm([1 1 1 S2],A',A);                    % LMI #1: A'*S2*A
lmiterm([1 1 1 R1],.5*h1^(2)*A',A,'s');      % LMI #1: h1^(2)*A'*R1*A (NON SYMMETRIC?)
lmiterm([1 1 1 R1],.5*4,-1,'s');             % LMI #1: -4*R1 (NON SYMMETRIC?)
lmiterm([1 1 1 R2],.5*h2^(2)*A',A,'s');      % LMI #1: h2^(2)*A'*R2*A (NON SYMMETRIC?)
lmiterm([1 1 1 R2],.5*4,-1,'s');             % LMI #1: -4*R2 (NON SYMMETRIC?)
lmiterm([1 2 1 -P14],1,-1);                  % LMI #1: -P14'
lmiterm([1 2 1 -P12],A',1);                  % LMI #1: A'*P12'
lmiterm([1 2 1 -P24],1,1);                   % LMI #1: P24'
lmiterm([1 2 1 -P25],1,1);                   % LMI #1: P25'
lmiterm([1 2 1 R1],2,-1);                    % LMI #1: -2*R1
lmiterm([1 2 2 P24],1,-1,'s');               % LMI #1: -P24-P24'
lmiterm([1 2 2 Q1],1,-1);                    % LMI #1: -Q1
```

```
lmiterm([1 2 2 R1],.5*4,-1,'s');          % LMI #1: -4*R1 (NON SYMMETRIC?)
lmiterm([1 3 1 -P15],1,-1);               % LMI #1: -P15'
lmiterm([1 3 1 -P13],A',1);               % LMI #1: A'*P13'
lmiterm([1 3 1 -P34],1,1);                % LMI #1: P34'
lmiterm([1 3 1 -P35],1,1);                % LMI #1: P35'
lmiterm([1 3 1 R2],2,-1);                 % LMI #1: -2*R2
lmiterm([1 3 2 -P25],1,-1);               % LMI #1: -P25'
lmiterm([1 3 2 -P34],1,-1);               % LMI #1: -P34'
lmiterm([1 3 3 P35],1,-1,'s');            % LMI #1: -P35-P35'
lmiterm([1 3 3 Q2],1,-1);                 % LMI #1: -Q2
lmiterm([1 3 3 R2],.5*4,-1,'s');          % LMI #1: -4*R2 (NON SYMMETRIC?)
lmiterm([1 4 1 -P12],1,1);                % LMI #1: P12'
lmiterm([1 4 2 -P22],1,1);                % LMI #1: P22'
lmiterm([1 4 3 P23],1,1);                 % LMI #1: P23
lmiterm([1 4 4 S1],1,-1);                 % LMI #1: -S1
lmiterm([1 5 1 -P13],1,1);                % LMI #1: P13'
lmiterm([1 5 2 -P23],1,1);                % LMI #1: P23'
lmiterm([1 5 3 -P33],1,1);                % LMI #1: P33'
lmiterm([1 5 5 S2],1,-1);                 % LMI #1: -S2
lmiterm([1 6 1 -P14],h1*A',1);            % LMI #1: h1*A'*P14'
lmiterm([1 6 1 -P44],h1,1);               % LMI #1: h1*P44'
lmiterm([1 6 1 -P45],h1,1);               % LMI #1: h1*P45'
lmiterm([1 6 1 R1],6,1);                  % LMI #1: 6*R1
lmiterm([1 6 2 -P44],h1,-1);              % LMI #1: -h1*P44'
lmiterm([1 6 2 R1],6,1);                  % LMI #1: 6*R1
lmiterm([1 6 3 -P45],h1,-1);              % LMI #1: -h1*P45'
lmiterm([1 6 4 P24],h1,1);                % LMI #1: h1*P24
lmiterm([1 6 5 P34],h1,1);                % LMI #1: h1*P34
lmiterm([1 6 6 R1],.5*12,-1,'s');         % LMI #1: -12*R1 (NON SYMMETRIC?)
```

```
lmiterm([1 7 1 -P15],h2*A',1);                  % LMI #1: h2*A'*P15'
lmiterm([1 7 1 P45],h2,1);                      % LMI #1: h2*P45
lmiterm([1 7 1 -P55],h2,1);                     % LMI #1: h2*P55'
lmiterm([1 7 1 R2],6,1);                        % LMI #1: 6*R2
lmiterm([1 7 2 P25],h2,1);                      % LMI #1: h2*P25
lmiterm([1 7 2 P45],h2,-1);                     % LMI #1: -h2*P45
lmiterm([1 7 3 P35],h2,1);                      % LMI #1: h2*P35
lmiterm([1 7 3 -P55],h2,-1);                    % LMI #1: -h2*P55'
lmiterm([1 7 3 R2],6,1);                        % LMI #1: 6*R2
lmiterm([1 7 7 R2],.5*12,-1,'s');               % LMI #1: -12*R2 (NON SYMMETRIC?)
lmiterm([1 8 1 -P11],1,Ad);                     % LMI #1: P11'*Ad
lmiterm([1 8 1 S1],A',Ad);                      % LMI #1: A'*S1*Ad
lmiterm([1 8 1 S2],A',Ad);                      % LMI #1: A'*S2*Ad
lmiterm([1 8 1 R1],h1^(2)*A',Ad);               % LMI #1: h1^(2)*A'*R1*Ad
lmiterm([1 8 1 R2],h2^(2)*A',Ad);               % LMI #1: h2^(2)*A'*R2*Ad
lmiterm([1 8 1 W],A',-1);                       % LMI #1: -A'*W
lmiterm([1 8 1 T],1,a);                         % LMI #1: T*a
lmiterm([1 8 2 P12],1,Ad);                      % LMI #1: P12*Ad
lmiterm([1 8 3 P13],1,Ad);                      % LMI #1: P13*Ad
lmiterm([1 8 6 P14],h1,Ad);                     % LMI #1: h1*P14*Ad
lmiterm([1 8 7 P15],h2,Ad);                     % LMI #1: h2*P15*Ad
lmiterm([1 8 8 S1],Ad',Ad);                     % LMI #1: Ad'*S1*Ad
lmiterm([1 8 8 S2],Ad',Ad);                     % LMI #1: Ad'*S2*Ad
lmiterm([1 8 8 R1],.5*h1^(2)*Ad',Ad,'s');       % LMI #1: h1^(2)*Ad'*R1*Ad (NON SYMMETRIC?)
lmiterm([1 8 8 R2],.5*h2^(2)*Ad',Ad,'s');       % LMI #1: h2^(2)*Ad'*R2*Ad (NON SYMMETRIC?)
lmiterm([1 8 8 W],1,-Ad,'s');                   % LMI #1: -W*Ad-Ad'*W
lmiterm([1 8 8 T],.5*2,-a,'s');                 % LMI #1: -2*T*a (NON SYMMETRIC?)
lmiterm([1 9 1 -P11],1,C);                      % LMI #1: P11'*C
lmiterm([1 9 1 S1],A',C);                       % LMI #1: A'*S1*C
```

```
lmiterm([1 9 1 S2],A',C);                    % LMI #1: A'*S2*C
lmiterm([1 9 1 R1],h1^(2)*A',C);             % LMI #1: h1^(2)*A'*R1*C
lmiterm([1 9 1 R2],h2^(2)*A',C);             % LMI #1: h2^(2)*A'*R2*C
lmiterm([1 9 1 W],A',-1);                    % LMI #1: -A'*W
lmiterm([1 9 2 P12],1,C);                    % LMI #1: P12*C
lmiterm([1 9 3 P13],1,C);                    % LMI #1: P13*C
lmiterm([1 9 6 P14],h1,C);                   % LMI #1: h1*P14*C
lmiterm([1 9 7 P15],h2,C);                   % LMI #1: h2*P15*C
lmiterm([1 9 8 S1],Ad',C);                   % LMI #1: Ad'*S1*C
lmiterm([1 9 8 S2],Ad',C);                   % LMI #1: Ad'*S2*C
lmiterm([1 9 8 R1],h1^(2)*Ad',C);            % LMI #1: h1^(2)*Ad'*R1*C
lmiterm([1 9 8 R2],h2^(2)*Ad',C);            % LMI #1: h2^(2)*Ad'*R2*C
lmiterm([1 9 8 W],1,-C);                     % LMI #1: -W*C
lmiterm([1 9 8 W],Ad',-1);                   % LMI #1: -Ad'*W
lmiterm([1 9 9 S1],C',C);                    % LMI #1: C'*S1*C
lmiterm([1 9 9 S2],C',C);                    % LMI #1: C'*S2*C
lmiterm([1 9 9 R1],.5*h1^(2)*C',C,'s');      % LMI #1: h1^(2)*C'*R1*C (NON SYMMETRIC?)
lmiterm([1 9 9 R2],.5*h2^(2)*C',C,'s');      % LMI #1: h2^(2)*C'*R2*C (NON SYMMETRIC?)
lmiterm([1 9 9 W],1,-C,'s');                 % LMI #1: -W*C-C'*W
lmiterm([1 9 9 N],.5*2,-1,'s');              % LMI #1: -2*N (NON SYMMETRIC?)
lmiterm([1 9 9 0],n*(1-D)^(2)*I);            % LMI #1: n*(1-D)^(2)*I
lmiterm([1 10 1 W],A',1);                    % LMI #1: A'*W
lmiterm([1 10 8 W],Ad',1);                   % LMI #1: Ad'*W
lmiterm([1 10 8 W],1,1);                     % LMI #1: W
lmiterm([1 10 9 W],C',1);                    % LMI #1: C'*W
lmiterm([1 10 9 W],1,1);                     % LMI #1: W
lmiterm([1 10 10 W],.5*2,-1,'s');            % LMI #1: -2*W (NON SYMMETRIC?)
lmiterm([1 11 8 T],1,-1);                    % LMI #1: -T
lmiterm([1 11 11 M],.5*2,-1,'s');            % LMI #1: -2*M (NON SYMMETRIC?)
```

```
lmiterm([1 11 11 0],m*a^(2)*I);          %LMI #1: m*a^(2)*I
lmiterm([1 12 9 -N],1,1);                % LMI #1: N'
lmiterm([1 12 12 0],-n*I);               % LMI #1: -n*I
lmiterm([1 13 11 -M],1,1);               % LMI #1: M'
lmiterm([1 13 13 0],-m*I);               % LMI #1: -m*I

lmiterm([-2 1 1 S1],1,1);                % LMI #2: S1

lmiterm([-3 1 1 S2],1,1);                % LMI #3: S2

lmiterm([-4 1 1 Q1],1,1);                % LMI #4: Q1

lmiterm([-5 1 1 Q2],1,1);                % LMI #5: Q2

lmiterm([-6 1 1 R1],1,1);                % LMI #6: R1

lmiterm([-7 1 1 R2],1,1);                % LMI #7: R2

lmiterm([-8 1 1 W],1,1);                 % LMI #8: W

lmiterm([-9 1 1 P11],1,1);               % LMI #9: P11

lmiterm([-10 1 1 P22],1,1);              % LMI #10: P22

lmiterm([-11 1 1 P33],1,1);              % LMI #11: P33

lmiterm([-12 1 1 P44],1,1);              % LMI #12: P44

lmiterm([-13 1 1 P55],1,1);              % LMI #13: P55
```

```
lmiterm([-14 1 1 N],1,1);                    % LMI #14: N

lmiterm([-15 1 1 M],1,1);                    % LMI #15: M

lmiterm([-16 1 1 T],1,1);                    % LMI #16: T

IS281=getlmis;
[tmin,xfeas]=feasp(IS281)
Q1=dec2mat(IS281,xfeas,Q1)
Q2=dec2mat(IS281,xfeas,Q2)
R1=dec2mat(IS281,xfeas,R1)
R2=dec2mat(IS281,xfeas,R2)
S1=dec2mat(IS281,xfeas,S1)
S2=dec2mat(IS281,xfeas,S2)
W=dec2mat(IS281,xfeas,W)
N=dec2mat(IS281,xfeas,N)
M=dec2mat(IS281,xfeas,M)
 T=dec2mat(IS281,xfeas,T)
P11=dec2mat(IS281,xfeas,P11)
P12=dec2mat(IS281,xfeas,P12)
P13=dec2mat(IS281,xfeas,P13)
P14=dec2mat(IS281,xfeas,P14)
P15=dec2mat(IS281,xfeas,P15)
P22=dec2mat(IS281,xfeas,P22)
P23=dec2mat(IS281,xfeas,P23)
P24=dec2mat(IS281,xfeas,P24)
P25=dec2mat(IS281,xfeas,P25)
P33=dec2mat(IS281,xfeas,P33)
```

```
P34=dec2mat(IS281,xfeas,P34)
P35=dec2mat(IS281,xfeas,P35)
P44=dec2mat(IS281,xfeas,P44)
P45=dec2mat(IS281,xfeas,P45)
P55=dec2mat(IS281,xfeas,P55)
tmin
```

2. MATLAB CODE for finding solution of examples 2

```
A=[-0.2,0;0,1]
Ad=[-0.1,0;0,0.1]
C=[0.1,0;0,-0.1]
I=[1,0;0,1]
h1=0
h2=1
D=0.5
n=0.9
m=0.5
a=0.2
    setlmis([]);
S1=lmivar(1,[2,1]);
S2=lmivar(1,[2,1]);
Q1=lmivar(1,[2,1]);
Q2=lmivar(1,[2,1]);
R1=lmivar(1,[2,1]);
R2=lmivar(1,[2,1]);
W=lmivar(1,[2,1]);
P11=lmivar(1,[2,1]);
P12=lmivar(2,[2,2]);
```

```
P13=lmivar(2,[2,2]);
P14=lmivar(2,[2,2]);
P15=lmivar(2,[2,2]);
P22=lmivar(1,[2,1]);
P23=lmivar(2,[2,2]);
P24=lmivar(2,[2,2]);
P25=lmivar(2,[2,2]);
P33=lmivar(1,[2,1]);
P34=lmivar(2,[2,2]);
P35=lmivar(2,[2,2]);
P44=lmivar(1,[2,1]);
P45=lmivar(2,[2,2]);
P55=lmivar(1,[2,1]);
N=lmivar(1,[2,1]);
M=lmivar(1,[2,1]);
T=lmivar(1,[2,1]);
lmiterm([1 1 1 P11],1,A,'s');            % LMI #1: P11*A+A'*P11
lmiterm([1 1 1 P14],1,1,'s');            % LMI #1: P14+P14'
lmiterm([1 1 1 P15],1,1,'s');            % LMI #1: P15+P15'
lmiterm([1 1 1 Q1],1,1);                 % LMI #1: Q1
lmiterm([1 1 1 Q2],1,1);                 % LMI #1: Q2
lmiterm([1 1 1 S1],A',A);                % LMI #1: A'*S1*A
lmiterm([1 1 1 S2],A',A);                % LMI #1: A'*S2*A
lmiterm([1 1 1 R1],.5*h1^(2)*A',A,'s');  % LMI #1: h1^(2)*A'*R1*A (NON SYMMETRIC?)
lmiterm([1 1 1 R1],.5*4,-1,'s');         % LMI #1: -4*R1 (NON SYMMETRIC?)
lmiterm([1 1 1 R2],.5*h2^(2)*A',A,'s');  % LMI #1: h2^(2)*A'*R2*A (NON SYMMETRIC?)
lmiterm([1 1 1 R2],.5*4,-1,'s');         % LMI #1: -4*R2 (NON SYMMETRIC?)
lmiterm([1 2 1 -P14],1,-1);              % LMI #1: -P14'
lmiterm([1 2 1 -P12],A',1);              % LMI #1: A'*P12'
```

```
lmiterm([1 2 1 -P24],1,1);          % LMI #1: P24'
lmiterm([1 2 1 -P25],1,1);          % LMI #1: P25'
lmiterm([1 2 1 R1],2,-1);           % LMI #1: -2*R1
lmiterm([1 2 2 P24],1,-1,'s');      % LMI #1: -P24-P24'
lmiterm([1 2 2 Q1],1,-1);           % LMI #1: -Q1
lmiterm([1 2 2 R1],.5*4,-1,'s');    % LMI #1: -4*R1 (NON SYMMETRIC?)
lmiterm([1 3 1 -P15],1,-1);         % LMI #1: -P15'
lmiterm([1 3 1 -P13],A',1);         % LMI #1: A'*P13'
lmiterm([1 3 1 -P34],1,1);          % LMI #1: P34'
lmiterm([1 3 1 -P35],1,1);          % LMI #1: P35'
lmiterm([1 3 1 R2],2,-1);           % LMI #1: -2*R2
lmiterm([1 3 2 -P25],1,-1);         % LMI #1: -P25'
lmiterm([1 3 2 -P34],1,-1);         % LMI #1: -P34'
lmiterm([1 3 3 P35],1,-1,'s');      % LMI #1: -P35-P35'
lmiterm([1 3 3 Q2],1,-1);           % LMI #1: -Q2
lmiterm([1 3 3 R2],.5*4,-1,'s');    % LMI #1: -4*R2 (NON SYMMETRIC?)
lmiterm([1 4 1 -P12],1,1);          % LMI #1: P12'
lmiterm([1 4 2 -P22],1,1);          % LMI #1: P22'
lmiterm([1 4 3 P23],1,1);           % LMI #1: P23
lmiterm([1 4 4 S1],1,-1);           % LMI #1: -S1
lmiterm([1 5 1 -P13],1,1);          % LMI #1: P13'
lmiterm([1 5 2 -P23],1,1);          % LMI #1: P23'
lmiterm([1 5 3 -P33],1,1);          % LMI #1: P33'
lmiterm([1 5 5 S2],1,-1);           % LMI #1: -S2
lmiterm([1 6 1 -P14],h1*A',1);      % LMI #1: h1*A'*P14'
lmiterm([1 6 1 -P44],h1,1);         % LMI #1: h1*P44'
lmiterm([1 6 1 -P45],h1,1);         % LMI #1: h1*P45'
lmiterm([1 6 1 R1],6,1);            % LMI #1: 6*R1
lmiterm([1 6 2 -P44],h1,-1);        % LMI #1: -h1*P44'
```

```
lmiterm([1 6 2 R1],6,1);                        % LMI #1: 6*R1
lmiterm([1 6 3 -P45],h1,-1);                    % LMI #1: -h1*P45'
lmiterm([1 6 4 P24],h1,1);                      % LMI #1: h1*P24
lmiterm([1 6 5 P34],h1,1);                      % LMI #1: h1*P34
lmiterm([1 6 6 R1],.5*12,-1,'s');               % LMI #1: -12*R1 (NON SYMMETRIC?)
lmiterm([1 7 1 -P15],h2*A',1);                  % LMI #1: h2*A'*P15'
lmiterm([1 7 1 P45],h2,1);                      % LMI #1: h2*P45
lmiterm([1 7 1 -P55],h2,1);                     % LMI #1: h2*P55'
lmiterm([1 7 1 R2],6,1);                        % LMI #1: 6*R2
lmiterm([1 7 2 P25],h2,1);                      % LMI #1: h2*P25
lmiterm([1 7 2 P45],h2,-1);                     % LMI #1: -h2*P45
lmiterm([1 7 3 P35],h2,1);                      % LMI #1: h2*P35
lmiterm([1 7 3 -P55],h2,-1);                    % LMI #1: -h2*P55'
lmiterm([1 7 3 R2],6,1);                        % LMI #1: 6*R2
lmiterm([1 7 7 R2],.5*12,-1,'s');               % LMI #1: -12*R2 (NON SYMMETRIC?)
lmiterm([1 8 1 -P11],1,Ad);                     % LMI #1: P11'*Ad
lmiterm([1 8 1 S1],A',Ad);                      % LMI #1: A'*S1*Ad
lmiterm([1 8 1 S2],A',Ad);                      % LMI #1: A'*S2*Ad
lmiterm([1 8 1 R1],h1^(2)*A',Ad);               % LMI #1: h1^(2)*A'*R1*Ad
lmiterm([1 8 1 R2],h2^(2)*A',Ad);               % LMI #1: h2^(2)*A'*R2*Ad
lmiterm([1 8 1 W],A',-1);                       % LMI #1: -A'*W
lmiterm([1 8 1 T],1,a);                         % LMI #1: T*a
lmiterm([1 8 2 P12],1,Ad);                      % LMI #1: P12*Ad
lmiterm([1 8 3 P13],1,Ad);                      % LMI #1: P13*Ad
lmiterm([1 8 6 P14],h1,Ad);                     % LMI #1: h1*P14*Ad
lmiterm([1 8 7 P15],h2,Ad);                     % LMI #1: h2*P15*Ad
lmiterm([1 8 8 S1],Ad',Ad);                     % LMI #1: Ad'*S1*Ad
lmiterm([1 8 8 S2],Ad',Ad);                     % LMI #1: Ad'*S2*Ad
lmiterm([1 8 8 R1],.5*h1^(2)*Ad',Ad,'s');       % LMI #1: h1^(2)*Ad'*R1*Ad (NON SYMMETRIC?)
```

```
lmiterm([1 8 8 R2],.5*h2^(2)*Ad',Ad,'s');     % LMI #1: h2^(2)*Ad'*R2*Ad (NON SYMMETRIC?)
lmiterm([1 8 8 W],1,-Ad,'s');                 % LMI #1: -W*Ad-Ad'*W
lmiterm([1 8 8 T],.5*2,-a,'s');               % LMI #1: -2*T*a (NON SYMMETRIC?)
lmiterm([1 9 1 -P11],1,C);                    % LMI #1: P11'*C
lmiterm([1 9 1 S1],A',C);                     % LMI #1: A'*S1*C
lmiterm([1 9 1 S2],A',C);                     % LMI #1: A'*S2*C
lmiterm([1 9 1 R1],h1^(2)*A',C);              % LMI #1: h1^(2)*A'*R1*C
lmiterm([1 9 1 R2],h2^(2)*A',C);              % LMI #1: h2^(2)*A'*R2*C
lmiterm([1 9 1 W],A',-1);                     % LMI #1: -A'*W
lmiterm([1 9 2 P12],1,C);                     % LMI #1: P12*C
lmiterm([1 9 3 P13],1,C);                     % LMI #1: P13*C
lmiterm([1 9 6 P14],h1,C);                    % LMI #1: h1*P14*C
lmiterm([1 9 7 P15],h2,C);                    % LMI #1: h2*P15*C
lmiterm([1 9 8 S1],Ad',C);                    % LMI #1: Ad'*S1*C
lmiterm([1 9 8 S2],Ad',C);                    % LMI #1: Ad'*S2*C
lmiterm([1 9 8 R1],h1^(2)*Ad',C);             % LMI #1: h1^(2)*Ad'*R1*C
lmiterm([1 9 8 R2],h2^(2)*Ad',C);             % LMI #1: h2^(2)*Ad'*R2*C
lmiterm([1 9 8 W],1,-C);                      % LMI #1: -W*C
lmiterm([1 9 8 W],Ad',-1);                    % LMI #1: -Ad'*W
lmiterm([1 9 9 S1],C',C);                     % LMI #1: C'*S1*C
lmiterm([1 9 9 S2],C',C);                     % LMI #1: C'*S2*C
lmiterm([1 9 9 R1],.5*h1^(2)*C',C,'s');       % LMI #1: h1^(2)*C'*R1*C (NON SYMMETRIC?)
lmiterm([1 9 9 R2],.5*h2^(2)*C',C,'s');       % LMI #1: h2^(2)*C'*R2*C (NON SYMMETRIC?)
lmiterm([1 9 9 W],1,-C,'s');                  % LMI #1: -W*C-C'*W
lmiterm([1 9 9 N],.5*2,-1,'s');               % LMI #1: -2*N (NON SYMMETRIC?)
lmiterm([1 9 9 0],n*(1-D)^(2)*I);             % LMI #1: n*(1-D)^(2)*I
lmiterm([1 10 1 W],A',1);                     % LMI #1: A'*W
lmiterm([1 10 8 W],Ad',1);                    % LMI #1: Ad'*W
lmiterm([1 10 8 W],1,1);                      % LMI #1: W
```

```
lmiterm([1 10 9 W],C',1);                 % LMI #1: C'*W
lmiterm([1 10 9 W],1,1);                  % LMI #1: W
lmiterm([1 10 10 W],.5*2,-1,'s');         % LMI #1: -2*W (NON SYMMETRIC?)
lmiterm([1 11 8 T],1,-1);                 % LMI #1: -T
lmiterm([1 11 11 M],.5*2,-1,'s');         % LMI #1: -2*M (NON SYMMETRIC?)
lmiterm([1 11 11 0],m*a^(2)*I);           %LMI #1: m*a^(2)*I
lmiterm([1 12 9 -N],1,1);                 % LMI #1: N'
lmiterm([1 12 12 0],-n*I);                % LMI #1: -n*I
lmiterm([1 13 11 -M],1,1);                % LMI #1: M'
lmiterm([1 13 13 0],-m*I);                % LMI #1: -m*I

lmiterm([-2 1 1 S1],1,1);                 % LMI #2: S1

lmiterm([-3 1 1 S2],1,1);                 % LMI #3: S2

lmiterm([-4 1 1 Q1],1,1);                 % LMI #4: Q1

lmiterm([-5 1 1 Q2],1,1);                 % LMI #5: Q2

lmiterm([-6 1 1 R1],1,1);                 % LMI #6: R1

lmiterm([-7 1 1 R2],1,1);                 % LMI #7: R2

lmiterm([-8 1 1 W],1,1);                  % LMI #8: W

lmiterm([-9 1 1 P11],1,1);                % LMI #9: P11

lmiterm([-10 1 1 P22],1,1);               % LMI #10: P22
```

```
lmiterm([-11 1 1 P33],1,1);                     % LMI #11: P33

lmiterm([-12 1 1 P44],1,1);                     % LMI #12: P44

lmiterm([-13 1 1 P55],1,1);                     % LMI #13: P55

lmiterm([-14 1 1 N],1,1);                       % LMI #14: N

lmiterm([-15 1 1 M],1,1);                       % LMI #15: M

lmiterm([-16 1 1 T],1,1);                       % LMI #16: T

IS281=getlmis;
[tmin,xfeas]=feasp(IS281)
Q1=dec2mat(IS281,xfeas,Q1)
Q2=dec2mat(IS281,xfeas,Q2)
R1=dec2mat(IS281,xfeas,R1)
R2=dec2mat(IS281,xfeas,R2)
S1=dec2mat(IS281,xfeas,S1)
S2=dec2mat(IS281,xfeas,S2)
W=dec2mat(IS281,xfeas,W)
N=dec2mat(IS281,xfeas,N)
M=dec2mat(IS281,xfeas,M)
 T=dec2mat(IS281,xfeas,T)
P11=dec2mat(IS281,xfeas,P11)
P12=dec2mat(IS281,xfeas,P12)
P13=dec2mat(IS281,xfeas,P13)
P14=dec2mat(IS281,xfeas,P14)
P15=dec2mat(IS281,xfeas,P15)
```

```
P22=dec2mat(IS281,xfeas,P22)
P23=dec2mat(IS281,xfeas,P23)
P24=dec2mat(IS281,xfeas,P24)
P25=dec2mat(IS281,xfeas,P25)
P33=dec2mat(IS281,xfeas,P33)
P34=dec2mat(IS281,xfeas,P34)
P35=dec2mat(IS281,xfeas,P35)
P44=dec2mat(IS281,xfeas,P44)
P45=dec2mat(IS281,xfeas,P45)
P55=dec2mat(IS281,xfeas,P55)
tmin
```

3. MATLAB CODE for finding solution of examples 3

```
A=[-4.5,0;0,6]
Ad=[-1,0;0,2]
C=[0.1,0;0,-0.1]
I=[1,0;0,1]
h1=0
h2=1
D=0.5
n=0.9
m=0.5
a=0.2
    setlmis([]);
S1=lmivar(1,[2,1]);
S2=lmivar(1,[2,1]);
Q1=lmivar(1,[2,1]);
Q2=lmivar(1,[2,1]);
R1=lmivar(1,[2,1]);
```

```
R2=lmivar(1,[2,1]);
W=lmivar(1,[2,1]);
P11=lmivar(1,[2,1]);
P12=lmivar(2,[2,2]);
P13=lmivar(2,[2,2]);
P14=lmivar(2,[2,2]);
P15=lmivar(2,[2,2]);
P22=lmivar(1,[2,1]);
P23=lmivar(2,[2,2]);
P24=lmivar(2,[2,2]);
P25=lmivar(2,[2,2]);
P33=lmivar(1,[2,1]);
P34=lmivar(2,[2,2]);
P35=lmivar(2,[2,2]);
P44=lmivar(1,[2,1]);
P45=lmivar(2,[2,2]);
P55=lmivar(1,[2,1]);
N=lmivar(1,[2,1]);
M=lmivar(1,[2,1]);
T=lmivar(1,[2,1]);
lmiterm([1 1 1 P11],1,A,'s');                % LMI #1: P11*A+A'*P11
lmiterm([1 1 1 P14],1,1,'s');                % LMI #1: P14+P14'
lmiterm([1 1 1 P15],1,1,'s');                % LMI #1: P15+P15'
lmiterm([1 1 1 Q1],1,1);                     % LMI #1: Q1
lmiterm([1 1 1 Q2],1,1);                     % LMI #1: Q2
lmiterm([1 1 1 S1],A',A);                    % LMI #1: A'*S1*A
lmiterm([1 1 1 S2],A',A);                    % LMI #1: A'*S2*A
lmiterm([1 1 1 R1],.5*h1^(2)*A',A,'s');      % LMI #1: h1^(2)*A'*R1*A (NON SYMMETRIC?)
lmiterm([1 1 1 R1],.5*4,-1,'s');             % LMI #1: -4*R1 (NON SYMMETRIC?)
```

```
lmiterm([1 1 1 R2],.5*h2^(2)*A',A,'s');     % LMI #1: h2^(2)*A'*R2*A (NON SYMMETRIC?)
lmiterm([1 1 1 R2],.5*4,-1,'s');            % LMI #1: -4*R2 (NON SYMMETRIC?)
lmiterm([1 2 1 -P14],1,-1);                 % LMI #1: -P14'
lmiterm([1 2 1 -P12],A',1);                 % LMI #1: A'*P12'
lmiterm([1 2 1 -P24],1,1);                  % LMI #1: P24'
lmiterm([1 2 1 -P25],1,1);                  % LMI #1: P25'
lmiterm([1 2 1 R1],2,-1);                   % LMI #1: -2*R1
lmiterm([1 2 2 P24],1,-1,'s');              % LMI #1: -P24-P24'
lmiterm([1 2 2 Q1],1,-1);                   % LMI #1: -Q1
lmiterm([1 2 2 R1],.5*4,-1,'s');            % LMI #1: -4*R1 (NON SYMMETRIC?)
lmiterm([1 3 1 -P15],1,-1);                 % LMI #1: -P15'
lmiterm([1 3 1 -P13],A',1);                 % LMI #1: A'*P13'
lmiterm([1 3 1 -P34],1,1);                  % LMI #1: P34'
lmiterm([1 3 1 -P35],1,1);                  % LMI #1: P35'
lmiterm([1 3 1 R2],2,-1);                   % LMI #1: -2*R2
lmiterm([1 3 2 -P25],1,-1);                 % LMI #1: -P25'
lmiterm([1 3 2 -P34],1,-1);                 % LMI #1: -P34'
lmiterm([1 3 3 P35],1,-1,'s');              % LMI #1: -P35-P35'
lmiterm([1 3 3 Q2],1,-1);                   % LMI #1: -Q2
lmiterm([1 3 3 R2],.5*4,-1,'s');            % LMI #1: -4*R2 (NON SYMMETRIC?)
lmiterm([1 4 1 -P12],1,1);                  % LMI #1: P12'
lmiterm([1 4 2 -P22],1,1);                  % LMI #1: P22'
lmiterm([1 4 3 P23],1,1);                   % LMI #1: P23
lmiterm([1 4 4 S1],1,-1);                   % LMI #1: -S1
lmiterm([1 5 1 -P13],1,1);                  % LMI #1: P13'
lmiterm([1 5 2 -P23],1,1);                  % LMI #1: P23'
lmiterm([1 5 3 -P33],1,1);                  % LMI #1: P33'
lmiterm([1 5 5 S2],1,-1);                   % LMI #1: -S2
lmiterm([1 6 1 -P14],h1*A',1);              % LMI #1: h1*A'*P14'
```

```
lmiterm([1 6 1 -P44],h1,1);                 % LMI #1: h1*P44'
lmiterm([1 6 1 -P45],h1,1);                 % LMI #1: h1*P45'
lmiterm([1 6 1 R1],6,1);                    % LMI #1: 6*R1
lmiterm([1 6 2 -P44],h1,-1);                % LMI #1: -h1*P44'
lmiterm([1 6 2 R1],6,1);                    % LMI #1: 6*R1
lmiterm([1 6 3 -P45],h1,-1);                % LMI #1: -h1*P45'
lmiterm([1 6 4 P24],h1,1);                  % LMI #1: h1*P24
lmiterm([1 6 5 P34],h1,1);                  % LMI #1: h1*P34
lmiterm([1 6 6 R1],.5*12,-1,'s');           % LMI #1: -12*R1 (NON SYMMETRIC?)
lmiterm([1 7 1 -P15],h2*A',1);              % LMI #1: h2*A'*P15'
lmiterm([1 7 1 P45],h2,1);                  % LMI #1: h2*P45
lmiterm([1 7 1 -P55],h2,1);                 % LMI #1: h2*P55'
lmiterm([1 7 1 R2],6,1);                    % LMI #1: 6*R2
lmiterm([1 7 2 P25],h2,1);                  % LMI #1: h2*P25
lmiterm([1 7 2 P45],h2,-1);                 % LMI #1: -h2*P45
lmiterm([1 7 3 P35],h2,1);                  % LMI #1: h2*P35
lmiterm([1 7 3 -P55],h2,-1);                % LMI #1: -h2*P55'
lmiterm([1 7 3 R2],6,1);                    % LMI #1: 6*R2
lmiterm([1 7 7 R2],.5*12,-1,'s');           % LMI #1: -12*R2 (NON SYMMETRIC?)
lmiterm([1 8 1 -P11],1,Ad);                 % LMI #1: P11'*Ad
lmiterm([1 8 1 S1],A',Ad);                  % LMI #1: A'*S1*Ad
lmiterm([1 8 1 S2],A',Ad);                  % LMI #1: A'*S2*Ad
lmiterm([1 8 1 R1],h1^(2)*A',Ad);           % LMI #1: h1^(2)*A'*R1*Ad
lmiterm([1 8 1 R2],h2^(2)*A',Ad);           % LMI #1: h2^(2)*A'*R2*Ad
lmiterm([1 8 1 W],A',-1);                   % LMI #1: -A'*W
lmiterm([1 8 1 T],1,a);                     % LMI #1: T*a
lmiterm([1 8 2 P12],1,Ad);                  % LMI #1: P12*Ad
lmiterm([1 8 3 P13],1,Ad);                  % LMI #1: P13*Ad
lmiterm([1 8 6 P14],h1,Ad);                 % LMI #1: h1*P14*Ad
```

```
lmiterm([1 8 7 P15],h2,Ad);                    % LMI #1: h2*P15*Ad
lmiterm([1 8 8 S1],Ad',Ad);                    % LMI #1: Ad'*S1*Ad
lmiterm([1 8 8 S2],Ad',Ad);                    % LMI #1: Ad'*S2*Ad
lmiterm([1 8 8 R1],.5*h1^(2)*Ad',Ad,'s');      % LMI #1: h1^(2)*Ad'*R1*Ad (NON SYMMETRIC?)
lmiterm([1 8 8 R2],.5*h2^(2)*Ad',Ad,'s');      % LMI #1: h2^(2)*Ad'*R2*Ad (NON SYMMETRIC?)
lmiterm([1 8 8 W],1,-Ad,'s');                  % LMI #1: -W*Ad-Ad'*W
lmiterm([1 8 8 T],.5*2,-a,'s');                % LMI #1: -2*T*a (NON SYMMETRIC?)
lmiterm([1 9 1 -P11],1,C);                     % LMI #1: P11'*C
lmiterm([1 9 1 S1],A',C);                      % LMI #1: A'*S1*C
lmiterm([1 9 1 S2],A',C);                      % LMI #1: A'*S2*C
lmiterm([1 9 1 R1],h1^(2)*A',C);               % LMI #1: h1^(2)*A'*R1*C
lmiterm([1 9 1 R2],h2^(2)*A',C);               % LMI #1: h2^(2)*A'*R2*C
lmiterm([1 9 1 W],A',-1);                      % LMI #1: -A'*W
lmiterm([1 9 2 P12],1,C);                      % LMI #1: P12*C
lmiterm([1 9 3 P13],1,C);                      % LMI #1: P13*C
lmiterm([1 9 6 P14],h1,C);                     % LMI #1: h1*P14*C
lmiterm([1 9 7 P15],h2,C);                     % LMI #1: h2*P15*C
lmiterm([1 9 8 S1],Ad',C);                     % LMI #1: Ad'*S1*C
lmiterm([1 9 8 S2],Ad',C);                     % LMI #1: Ad'*S2*C
lmiterm([1 9 8 R1],h1^(2)*Ad',C);              % LMI #1: h1^(2)*Ad'*R1*C
lmiterm([1 9 8 R2],h2^(2)*Ad',C);              % LMI #1: h2^(2)*Ad'*R2*C
lmiterm([1 9 8 W],1,-C);                       % LMI #1: -W*C
lmiterm([1 9 8 W],Ad',-1);                     % LMI #1: -Ad'*W
lmiterm([1 9 9 S1],C',C);                      % LMI #1: C'*S1*C
lmiterm([1 9 9 S2],C',C);                      % LMI #1: C'*S2*C
lmiterm([1 9 9 R1],.5*h1^(2)*C',C,'s');        % LMI #1: h1^(2)*C'*R1*C (NON SYMMETRIC?)
lmiterm([1 9 9 R2],.5*h2^(2)*C',C,'s');        % LMI #1: h2^(2)*C'*R2*C (NON SYMMETRIC?)
lmiterm([1 9 9 W],1,-C,'s');                   % LMI #1: -W*C-C'*W
lmiterm([1 9 9 N],.5*2,-1,'s');                % LMI #1: -2*N (NON SYMMETRIC?)
```

```
lmiterm([1 9 9 0],n*(1-D)^(2)*I);            % LMI #1: n*(1-D)^(2)*I
lmiterm([1 10 1 W],A',1);                    % LMI #1: A'*W
lmiterm([1 10 8 W],Ad',1);                   % LMI #1: Ad'*W
lmiterm([1 10 8 W],1,1);                     % LMI #1: W
lmiterm([1 10 9 W],C',1);                    % LMI #1: C'*W
lmiterm([1 10 9 W],1,1);                     % LMI #1: W
lmiterm([1 10 10 W],.5*2,-1,'s');            % LMI #1: -2*W (NON SYMMETRIC?)
lmiterm([1 11 8 T],1,-1);                    % LMI #1: -T
lmiterm([1 11 11 M],.5*2,-1,'s');            % LMI #1: -2*M (NON SYMMETRIC?)
lmiterm([1 11 11 0],m*a^(2)*I);              %LMI #1: m*a^(2)*I
lmiterm([1 12 9 -N],1,1);                    % LMI #1: N'
lmiterm([1 12 12 0],-n*I);                   % LMI #1: -n*I
lmiterm([1 13 11 -M],1,1);                   % LMI #1: M'
lmiterm([1 13 13 0],-m*I);                   % LMI #1: -m*I

lmiterm([-2 1 1 S1],1,1);                    % LMI #2: S1

lmiterm([-3 1 1 S2],1,1);                    % LMI #3: S2

lmiterm([-4 1 1 Q1],1,1);                    % LMI #4: Q1

lmiterm([-5 1 1 Q2],1,1);                    % LMI #5: Q2

lmiterm([-6 1 1 R1],1,1);                    % LMI #6: R1

lmiterm([-7 1 1 R2],1,1);                    % LMI #7: R2

lmiterm([-8 1 1 W],1,1);                     % LMI #8: W
```

```
lmiterm([-9 1 1 P11],1,1);                    % LMI #9: P11

lmiterm([-10 1 1 P22],1,1);                   % LMI #10: P22

lmiterm([-11 1 1 P33],1,1);                   % LMI #11: P33

lmiterm([-12 1 1 P44],1,1);                   % LMI #12: P44

lmiterm([-13 1 1 P55],1,1);                   % LMI #13: P55

lmiterm([-14 1 1 N],1,1);                     % LMI #14: N

lmiterm([-15 1 1 M],1,1);                     % LMI #15: M

lmiterm([-16 1 1 T],1,1);                     % LMI #16: T

IS281=getlmis;
[tmin,xfeas]=feasp(IS281)
Q1=dec2mat(IS281,xfeas,Q1)
Q2=dec2mat(IS281,xfeas,Q2)
R1=dec2mat(IS281,xfeas,R1)
R2=dec2mat(IS281,xfeas,R2)
S1=dec2mat(IS281,xfeas,S1)
S2=dec2mat(IS281,xfeas,S2)
W=dec2mat(IS281,xfeas,W)
N=dec2mat(IS281,xfeas,N)
M=dec2mat(IS281,xfeas,M)
 T=dec2mat(IS281,xfeas,T)
P11=dec2mat(IS281,xfeas,P11)
```

```
P12=dec2mat(IS281,xfeas,P12)
P13=dec2mat(IS281,xfeas,P13)
P14=dec2mat(IS281,xfeas,P14)
P15=dec2mat(IS281,xfeas,P15)
P22=dec2mat(IS281,xfeas,P22)
P23=dec2mat(IS281,xfeas,P23)
P24=dec2mat(IS281,xfeas,P24)
P25=dec2mat(IS281,xfeas,P25)
P33=dec2mat(IS281,xfeas,P33)
P34=dec2mat(IS281,xfeas,P34)
P35=dec2mat(IS281,xfeas,P35)
P44=dec2mat(IS281,xfeas,P44)
P45=dec2mat(IS281,xfeas,P45)
P55=dec2mat(IS281,xfeas,P55)
tmin
```

BIOGRAPHY

BIOGRAPHY

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A NEUTRAL SYSTEM APPROACH TO STABILITY OF SINGULAR TIME-VARYING DELAY SYSTEM 2018