EXPONENTIAL STABILITY OF LINEAR SYSTEMS WITH INTERVAL TIME - VARYING DELAYS USING A NEW BOUNDING TECHNIQUE

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An Independent Study Submitted in Partial Fulfillment
of the Requirements for the degree of Bachelor
of Science Program in Mathematics
April 2018
University of Phayao

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ACKNOWLEDGEMENTS

This independent study could not be successfully completed without the kindness of advisor, Dr. Teerapong La-inchua, that idea, guidance and motivation which enable us to carry out our study since start until successfully. Including, He gave appreciate suggestions, insightful comments and encouragement of this independent study. So, we would like to our independent study advisor.

In addition, we would like to thank for all lectures in Department of Mathematics, University of Phayao, who opened the door of knowledge to us.

Finally, we most gratefully acknowledge our parents and my friends for all their support throughout the period of this independent study.

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ชื่อเรื่อง	เสถียรภาพแบบเลขชี้กำลังของระบบเชิงเส้นที่มีตัวหน่วงแปรผันตามเวลาโดยใช้วิธีประมาณค่าขอบเขตใหม่
ผู้ศึกษาค้นคว้า	นางสาวกนกวรรณ หนูโพนทา นายทชา จานคูณ นางสาวพัชรา ทะจีน
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คำสำคัญ	เสถียรภาพแบบชี้กำลัง ตัวหน่วงแปรผันตามเวลา ฟังก์ชัน ไลปูนอฟ อสมการเมทริกซ์เชิงเส้น

บทคัดย่อ

ในการศึกษาอิสระนี้ศึกษาเกี่ยวกับปัญหาเสถียรภาพเลขชี้กำลังของระบบเชิงเส้นที่มีตัวหน่วงแปรผันตามเวลา โดยตัวหน่วงดังกล่าวเป็นฟังก์ชันต่อเนื่องและอยู่ในช่วงที่กำหนด แต่ไม่จำเป็นต้องหาอนุพันธ์ได้ โดยการใช้ฟังก์ชันนอล ไลปูนอฟ คราโซฟสกี้ กับอสมการปริพันธ์เมทริกซ์อิสระ ทำให้เราได้เงื่อนไขใหม่เพียงพอที่ขึ้นอยู่กับตัวหน่วง สำหรับเสถียรภาพเลขชี้กำลัง ซึ่งอยู่ในรูปของอสมการเมทริกซเชิงเส้น (LMI) ในตอนท้ายของการศึกษาอิสระนี้ได้มีการยกตัวอย่างเชิงตัวเลขเพื่อแสดงให้เห็นถึงประสิทธิภาพของผลลัพธ์นั้น

Title	Exponential stability of linear systems with interval time-varying delay using a new bounding technique
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Bachelor of Science	Program in Mathematics
Keywords	Exponential stability, Time-varying delay Lyapunov function, Linear matrix inequalities

ABSTRACT

This independent study, we investigate exponential stability problem for a class of linear and uncertain linear systems with time-varying delays. The time-delays is assumed to be a continuous function belonging to a given interval, but not necessary to be differentiable. By introduce a set of augmented Lyapunov-Krasovskii functionals combined with the Free-matrix-based integral inequality, new delays-dependent sufficient conditions for the exponential stability of the system is first established in terms of linear matrix inequalities (LMIs). Numerical examples are given to show the effectiveness of our obtained results.

LIST OF CONTENTS

Page

LIST OF FIGURES

Figures **Page**

LIST OF TABLE

Table **Page**

CHAPTER 1
Introduction

1.1 Introduction

Time-delay systems are widely used to model concrete systems in engineering sciences, such as biology, chemistry, mechanics and so on. So the stability analysis of time-delay systems strongly requires before experimental stage. As these reason, the stability analysis of time-delay system have been an attractive study research field during the past years.

The derivative of the Lyapunov functional in order to make it easy to handle. Stability analysis of linear systems with time-varying delays $\dot{x}(t) = Ax(t) + Dx(t - h(t))$ is fundamental to many practical problems and has received considerable attention. Most of the known results on this problem are derived assuming only that the time-varying delay $h(t)$ is a continuously differentiable function, satisfying some boundedness condition on its derivative : $h(t) \leq \delta < 1$ In delay-dependent stability criteria, the main concern is to enlarge the feasible region of stability criteria in given time-delay interval. By constructing augmented Lyapunov functionals and utilizing free weight matrices.

Uncertainty is one of the main features of complex and intelligent decision making systems. Various approaches, methods and techniques in this field have been developed for several decades, starting with such concepts and tools as adaptation, stochastic optimization and statistical decision theory. Another category of approaches is based on the functionals with prescribed derivative. The idea is to apply the functional, that is appropriate for a nominal system and does not depend on the uncertainties, for analysis of the uncertain one. Since our approach belongs to this category, we address the issue in more detail below. One of the crucial goals of the theory was to construct a functional that admits a quadratic lower bound what is of paramount importance for robustness analysis in particular. Such functional was derived in was called the functional of complete type. Its derivative depends on the whole state of a system, and this functional particularly was applied in analysis of systems with delay uncertainties, interesting applications of the functional. It is worth mentioning that there exist other definitions of the complete-type functionals, which are also applied in development of the topic. All these functionals came from the functional with a simple derivative $x^T(t)Wx(t)$ for which a quadratic lower bound does not exist, here W is a positive definite matrix. There is a certain problem when we apply this simple functional for analysis of uncertain systems : its time-derivative along the solutions of a perturbed system is not negative definite, thus the Krasovskii theorem does not hold.

In this paper, we present a new approach for stability analysis of linear time-invariant systems with delay uncertainties, either constant or time-varying, that is developed applying Free matrix based integral inequality. Motivated by the above discretion, we shall desired new criteria for the exponential stability of systems with interval time-varying non-differentiable delay. By introduction a set of improved Lyapunov functionals combined with the NewtonLeibniz formula, we propose new criteria for the exponential stability of the system. The delay-dependent stability conditions are formulated in terms of LMIs, being thus solvable by utilizing MATLAB LMI Control Toolbox available in the literature to date. The approach allows us to apply in exponential stability of uncertain linear systems with interval time-varying delays.

The independent study is organized as follows : Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Delay-dependent exponential stability conditions of the system with illustrative numerical examples are show in Section 4. Section 5 gives the conclusions of the paper.

CHAPTER 2
Problem formulation and preliminaries

2.1 Problem formulation and preliminaries

The following notations will be used in this paper. R^+ denotes the set of all real non-negative numbers ; R^n denotes the n-dimensional space with the scalar product $x^T y$ and the vector norm $\| \,.\, \|$; $M^{n\times r}$ denotes the space of all matrices of $(n \times r)$ -dimensions ; A^T denotes the transpose of matrix A ; A is symmetric if A = A^T; I denotes the identity matrix ; $\lambda(A)$ denotes the set of all eigenvalues of A ; $\lambda_{\min/\max}(A)$ =min/max $\{Re\lambda : \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h, 0]\}$, $\| x_t \| = sup_{s\in[-h_2,0]}\{\| x(t+s) \|\}$; $C^1([0,t], R^n)$ denotes the set of all R^n -valued continuously differentiable functions on $[0,t]$; Matrix A is called semi-positive definite $(A \geq 0)$ if $\langle Ax, x\rangle \geq 0$, for all x $\in R^n$; A is positive definite $(A > 0)$ if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; $*$ denotes the symmetric term in a matrix.

Consider a linear system with interval time-varying delay of the form :

$$\dot{x}(t) = Ax(t) + Dx(t - h(t)), \quad t \in R^+,$$
$$x(t) = \phi(t), t \in [-h_2, 0] \tag{1}$$

where $x(t) \in R^n$ is the state ; $A, D \in M^{n\times n}$, and $\phi(t) \in C^1([-h_2, 0], R^n)$ is the initial function with the norm $\| \phi \| = sup_{-h_2\leq t\leq 0}\{\| \phi(t) \|, \| \dot{\phi}(t) \|\}$. The time-varying delay function $h(t)$ satisfies

$$0 \leq h_1 \leq h(t) \leq h_2, \quad t \in R^+.$$

Definition 2.1 Given $\alpha > 0$. The zero solution of system (1) is α -exponentially stable if there exist a positive number $N > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\| x(t, \phi) \| \leq Ne^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Proposition 2.1 (Cauchy inequality). For any symmetric positive definite matrix $N \in M^{n\times n}$ and $a, b \in R^n$ we have

$$\pm a^T b \leq a^T N a + b^T N^{-1} b.$$

Proposition 2.2 For any symmetric positive definite matrix $M \in M^{n\times n}$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds

$$\Big(\int_0^\gamma \omega(s)ds\Big)^T M \Big(\int_0^\gamma \omega(s)ds\Big) \leq \gamma \Big(\int_0^\gamma \omega^T(s) M \omega(s) ds\Big).$$

Proposition 2.3 Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \leq I$. For any $\epsilon > 0$, we have

$$EFH + H^T F^T E^T \leq \epsilon E E^T + \epsilon^{-1} H^T H.$$

Proposition 2.4 (Schur complement lemma). Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$, if and only if

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \text{ or } \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0 .$$

Lemma 2.1 (Free matrix based integral inequality [3]).

Let $x(r) \in R^{n\times n}$ be a continuous function : $\{x(r) | r \in [a, b]\}$. For symmetric matrices $M, N \in R^{3n\times 3n}, R \in R^{n\times n}$, matrices $x \in R^{3n\times 3n}, W, Y \in R^{3n\times n}$ satisfying

$$\theta = \begin{bmatrix} M & X & Y \\ * & N & W \\ * & * & R \end{bmatrix} \geq 0,$$

the following inequality holds :

$$-\int_a^b \dot{X}^T R \dot{X}(r) dr \leq \xi^T \Big((b-a)M + \frac{b-a}{3} N + He\{Y\phi_1 + W\phi_2\}\Big)\xi,$$

where $\phi_1 = \begin{bmatrix} I & -I & 0 \end{bmatrix}$, $\phi_2 = \begin{bmatrix} -I & -I & 2I \end{bmatrix}$,

$$\xi = col\Big\{x(b), x(a), \frac{1}{b-a}\int_a^b x(s)ds\Big\} .$$

CHAPTER 3
Main results

3.1 Main results

The following notations will be used throughout this paper.

$$J_{11} = A^T P + PA + 2Q - h_1 e^{-2\alpha h_1} \Phi_1 - h_2 e^{-2\alpha h_2} \Psi_1 - (S_1 A + A^T S_1^T),$$

$$J_{12} = -h_1 e^{-2\alpha h_1} \Phi_4 - S_2 A,$$

$$J_{13} = -h_2 e^{-2\alpha h_2} \Psi_4 - S_3 A,$$

$$J_{14} = J_{41} = PD - S_1 D - S_4 D,$$

$$J_{15} = -h_1 e^{-2\alpha h_1} \Phi_7 - S_5 A,$$

$$J_{16} = -h_2 e^{-2\alpha h_2} \Psi_7 - S_6 A,$$

$$J_{17} = J_{71} = -S_7 A,$$

$$J_{18} = J_{81} = S_1 - S_8 A,$$

$$J_{21} = -h_1 e^{-2\alpha h_1} \Psi_2 - S_2 A,$$

$$J_{22} = e^{2\alpha h_1} Q - h_1 e^{-2\alpha h_1} \Phi_5 - (h_2 - h_1) e^{-2\alpha h_2} \Omega_1,$$

$$J_{23} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_4 - (0.5)(e^{2\alpha h_1} + e^{2\alpha h_2}) Q,$$

$$J_{24} = J_{42} = -S_2 D,$$

$$J_{25} = -h_1 e^{-2\alpha h_1} \Phi_8,$$

$$J_{26} = J_{62} = 0,$$

$$J_{27} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_7,$$

$$J_{28} = J_{82} = S_2,$$

$$J_{31} = -h_2 e^{-2\alpha h_2} \Psi_2 - S_3 A,$$

$$J_{32} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_2 - (0.5)(e^{2\alpha h_1} + e^{2\alpha h_2}) Q,$$

$$J_{33} = -e^{-2\alpha h_2} Q - h_2 e^{-2\alpha h_2} \Psi_5 - (h_2 - h_1) e^{-2\alpha h_2} \Omega_5,$$

$$J_{34} = J_{43} = -S_3 D,$$

$$J_{35} = J_{53} = 0,$$

$$J_{36} = -h_2 e^{-2\alpha h_2} \Psi_8,$$

$$J_{37} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_8,$$

$$J_{38} = J_{83} = S_3,$$

$$J_{44} = -2 S_4 D = -(S_4 D + D^T S_4^T),$$

$$J_{45} = J_{54} = -S_5 D,$$

$$J_{46} = J_{64} = -S_6 D,$$

$$J_{47} = J_{74} = -S_7 D,$$

$$J_{48} = J_{84} = S_4 - S_8 D,$$

$$J_{51} = -h_1 e^{-2\alpha h_1} \Phi_3 - S_5 A,$$

$$J_{52} = -h_1 e^{-2\alpha h_1} \Phi_5,$$

$$J_{55} = -h_1 e^{-2\alpha h_1} \Phi_9,$$

$$J_{56} = J_{65} = 0,$$

$$J_{57} = J_{75} = 0,$$

$$J_{58} = J_{85} = S_5,$$

$$J_{61} = -h_2 e^{-2\alpha h_2} \Psi_3 - S_6 A,$$

$$J_{63} = -h_2 e^{-2\alpha h_2} \Psi_6,$$

$$J_{66} = -h_2 e^{-2\alpha h_2} \Psi_9,$$

$$J_{67} = J_{76} = 0,$$

$$J_{68} = S_6,$$

$$J_{72} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_3,$$

$$J_{73} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_6,$$

$$J_{77} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_9,$$

$$J_{78} = J_{87} = S_7,$$

$$J_{88} = (h_1^2 + h_2^2) R + (h_2 - h_1)^2 U + S_8 + S_8^T.$$

Theorem 3.1 Given $\alpha > 0$. The zero solution of system (1) is α -exponentially stable if there exist positive matrices P,Q,R,U, positive semi-definite matrices $M_{ii}, N_{ii}(i = 1,2,3)$, any matrices $M_{ij}, N_{ij}(i = 1,2,3, i \neq j)$ and $S_i(i = 1,2,..,8)$ such that following LMIs hold :

$$\theta = \begin{bmatrix} M & X & Y \\ * & N & W \\ * & * & R \end{bmatrix} \geq 0,$$

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} & -S_7A & -S_1 - S_8A \\ * & J_{22} & J_{23} & -S_2D & J_{25} & 0 & J_{27} & S_2 \\ * & * & J_{33} & -S_3D & 0 & J_{36} & J_{37} & S_4 \\ * & * & * & -2S_4D & -S_5D & -S_6D & -S_7D & S_4 - S_8D \\ * & * & * & * & J_{55} & 0 & 0 & S_5 \\ * & * & * & * & * & J_{66} & 0 & S_6 \\ * & * & * & * & * & * & J_{77} & S_7 \\ * & * & * & * & * & * & * & J_{88} \end{bmatrix} < 0, \tag{3.1}$$

where
$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix},$$

$$N = \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{bmatrix}.$$

Proof We introduce the following LyapunovKrasovskii functional for the system (1)

$$V(t, x_t) = \sum_{i=1}^{7} V_i \ ,$$

where

$$V_1 = x^T(t)Px(t), \tag{3.1.1}$$

$$V_2 = \int_{t-h_1}^{t} e^{2\alpha(s-t)} x^T(s)Qx(s)ds, \tag{3.1.2}$$

$$V_3 = \int_{t-h_2}^{t} e^{2\alpha(s-t)} x^T(s)Qx(s)ds, \tag{3.1.3}$$

$$V_4 = \int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)} x^T(s)Qx(s)ds, \tag{3.1.4}$$

$$V_5 = h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{-2\alpha(\tau-t)} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau \ ds, \tag{3.1.5}$$

$$V_6 = h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{-2\alpha(\tau-t)} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau \ ds, \tag{3.1.6}$$

$$V_7 = (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+s}^{t} e^{2\alpha(\tau-t)} \dot{x}^T(\tau)U\dot{x}(\tau)d\tau \ ds. \tag{3.1.7}$$

From (3.1.1), we have

$$\lambda_{\min}(P) \parallel x \parallel^2 \leq x^T(t)Px(t) \leq V(t, x_t) \ .$$

From (3.1.2),we have

$$\begin{aligned}
V_2 &= \int_{t-h_1}^{t} e^{2\alpha s} \cdot e^{-2\alpha t} x^T(s)Qx(s)ds \\
&\leq \int_{t-h_1}^{t} x^T(s)Qx(s)ds \\
&\leq \int_{t-h_1}^{t} \lambda_{\max}(Q) \parallel x(t) \parallel^2 ds \\
&= \lambda_{\max}(Q) \parallel x(t) \parallel^2 \int_{t-h_1}^{t} 1ds \\
&\leq \lambda_{\max}(Q) \parallel x(t) \parallel^2 \ h_1 \\
&\leq \lambda_{\max}(Q) \ h \ \parallel x(t) \parallel^2 \\
&\leq \lambda_{\max}(Q) \parallel x_t \parallel^2 \ h_1 \ .
\end{aligned}$$

From (3.1.3), we have

$$\begin{aligned}
V_3 &= \int_{t-h_2}^{t} e^{2\alpha t} \cdot e^{-2\alpha t} x^T(s)Qx(s)ds \\
&= \int_{t-h_2}^{t} e^{2\alpha t} x^T(s)Qx(s)ds \\
&\leq \int_{t-h_2}^{t} \lambda_{\max}(Q) \parallel x(t) \parallel^2 ds \\
&= \lambda_{\max}(Q) \parallel x(t) \parallel^2 \int_{t-h_2}^{t} 1ds \\
&= \lambda_{\max}(Q) \parallel x(t) \parallel^2 \ h_2
\end{aligned}$$

$$\leq \lambda_{\max}(Q)\ h_2\ \ \| x(t) \|^2$$

$$\leq \lambda_{\max}(Q) \| x_t \|^2\ \ h_2\ .$$

From (3.1.4), we have

$$V_4 = \int_{t-h_2}^{t-h_1} e^{2\alpha s} \cdot e^{-2\alpha t} x^T(s)Qx(s)ds$$

$$\leq \int_{t-h_2}^{t-h_1} e^{2\alpha(t-h_1)} \cdot e^{-2\alpha t} x^T(s)Qx(s)ds$$

$$= \int_{t-h_2}^{t-h_1} e^{-2\alpha h_1} x^T(s)Qx(s)ds$$

$$= e^{-2\alpha h_1} \int_{t-h_2}^{t-h_1} x^T(s)Qx(s)ds$$

$$\leq e^{-2\alpha h_1} \int_{t-h_2}^{t-h_1} \lambda_{\max}(Q) \| x(t) \|^2\ ds$$

$$= e^{-2\alpha h_1} \lambda_{\max}(Q) \| x(t) \|^2 \int_{t-h_2}^{t-h_1} 1 ds$$

$$= e^{-2\alpha h_1} \lambda_{\max}(Q) \| x(t) \|^2\ \ (h_2 - h_1)$$

$$\leq \lambda_{\max}(Q) \| x_t \|^2\ \ (h_2 - h_1)\ .$$

From (3.1.5), we have

$$V_5 \leq h_1 \int_{-h_1}^{0} \int_{t+h_1}^{t} e^{2\alpha h_1} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds$$

$$= h_1 e^{2\alpha h_1} \int_{t-h_1}^{t} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau \int_{-h_1}^{0} 1\ ds$$

$$= h_1 e^{2\alpha h_1} \int_{t-h_1}^{t} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ (h_1)$$

$$= (h_1)^2 \int_{t-h_1}^{t} \lambda_{\max} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau$$

$$\leq \lambda_{\max}(R) \| \dot{x}(t) \|^2\ \ (h_1)^2$$

$$= \lambda_{\max}(R)(\| Ax(t)\ + Dx(t-h(t)) \|)^2\ (h_1)^2$$

$$= \lambda_{\max}(R)(h_1)^2\ (\| A \|^2 \| x(t) \|^2$$

$$+2 \| A \| \| x(t) \| \| D \| \| x(t-h(t)) \| + \| D \|^2 \| x(t-h(t)) \|^2)$$

$$\leq \lambda_{\max}(R)(h_1)^2 (\| A \|^2 + 2 \| A \| \| D \| + \| D \|^2) \| x(t) \|^2$$

$$\leq \lambda_{\max}(R)(h_1)^2 (\| A \|^2 + 2 \| A \| \| D \| + \| D \|^2) \| x_t \|^2\ .$$

From (3.1.6), we have

$$V_6 \leq h_2 \int_{-h_2}^{0} \int_{t+h_2}^{t} e^{2\alpha h_2} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds$$

$$= h_2 e^{2\alpha h_2} \int_{t-h_2}^{t} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau \int_{-h_1}^{0} 1\ ds$$

$$= h_2 e^{2\alpha h_2} \int_{t-h_2}^{t} \dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ (h_2)$$

$$= (h_2)^2 \int_{t-h_2}^{t} \lambda_{\max} \dot{x}^T(\tau) R \dot{x}(\tau) d\tau$$
$$\leq \lambda_{\max}(R) \parallel \dot{x}(t) \parallel^2 \ (h_2)^2$$
$$= \lambda_{\max}(h_2)^2(R)(\parallel Ax(t) \ + Dx(t-h(t)) \parallel)^2$$
$$= \lambda_{\max}(h_2)^2(R) \ (\parallel A \parallel^2 \parallel x(t) \parallel^2$$
$$+2 \parallel A \parallel \parallel x(t) \parallel \parallel D \parallel \parallel x(t-h(t)) \parallel + \parallel D \parallel^2 \parallel x(t-h(t)) \parallel^2)$$
$$\leq \lambda_{\max}(h_2)^2(R)(\parallel A \parallel^2 + 2 \parallel A \parallel \parallel D \parallel + \parallel D \parallel^2) \parallel x(t) \parallel^2$$
$$\leq \lambda_{\max}(h_2)^2(R)(\parallel A \parallel^2 + 2 \parallel A \parallel \parallel D \parallel + \parallel D \parallel^2) \parallel x_t \parallel^2 .$$

From (3.1.7), we have

$$V_7 \leq (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t-h_2}^{t} e^{2\alpha h_2} \dot{x}^T(\tau) U \dot{x}(\tau) d\tau \ ds$$
$$= (h_2 - h_1) e^{2\alpha h_2} \int_{t-h_2}^{t} \dot{x}^T(\tau) U \dot{x}(\tau) d\tau \int_{-h_2}^{-h_1} 1 \ ds$$
$$= (h_2 - h_1) e^{2\alpha h_2} \int_{t-h_2}^{t} \dot{x}^T(\tau) U \dot{x}(\tau) d\tau \ (h_2 - h_1)$$
$$= (h_2 - h_1)^2 \int_{t-h_2}^{t} \lambda_{\max} \dot{x}^T(\tau) U \dot{x}(\tau) d\tau$$
$$\leq \lambda_{\max}(U) \parallel \dot{x}(t) \parallel^2 (h_2 - h_1)(h_2 - h_1)$$
$$= \lambda_{\max}(U)(h_2 - h_1)^2(\parallel A \parallel^2 \parallel x(t) \parallel^2$$
$$+2 \parallel A \parallel \parallel x(t) \parallel \parallel D \parallel \parallel x(t-h(t) \parallel + \parallel D \parallel^2 \parallel x-(t-h(t)) \parallel^2$$
$$\leq \lambda_{\max}(U)(h_2 - h_1)^2(\parallel A \parallel^2 + 2 \parallel A \parallel \parallel D \parallel + \parallel D \parallel^2) \parallel x(t) \parallel^2$$
$$\leq \lambda_{\max}(U)(h_2 - h_1)^2(\parallel A \parallel^2 + 2 \parallel A \parallel \parallel D \parallel + \parallel D \parallel^2) \parallel x_t \parallel^2 .$$

It easy to verify that

$$\lambda_1 \parallel x(t) \parallel^2 \leq V(t, x_t) \leq \lambda_2 \parallel x_t \parallel^2 , \quad \forall_t \geq 0. \tag{3.2}$$

By taking the derivative of V_1 along the solution of system (1), we have

$$\dot{V}_1 = x^T P x(t)$$
$$= x^T(t) \frac{d}{dx} P x(t) + \frac{d}{dx} x^T(t) P x(t)$$
$$= x^T(t) P \dot{x}(t) + \dot{x}^T(t) P x(t)$$
$$= 2x^T(t) P \dot{x}(t)$$
$$= 2x^T(t) P \big[Ax(t) + Dx(t-h(t)) \big]$$
$$= 2x^T(t) P A x(t) + 2x^T(t) P D x(t-h(t))$$
$$= x^T(t) P A x(t) + x^T(t) P A x(t) + 2x^T P D x(t-h(t))$$
$$= x^T(t) \big[PA + PA \big] x(t) + 2x^T(t) P D x(t-h(t))$$

$$= x^T(t)\big[A^TP + PA\big]x(t) + 2x^T(t)PDx(t-h(t)) \ .$$

$$\begin{aligned}
\dot{V}_2 &= \frac{d}{dt}\int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= \frac{d}{dt}\Big(e^{-2\alpha t}\int_{t-h_1}^{t} e^{2\alpha s}x^T(s)Qx(s)ds\Big) \\
&= e^{-2\alpha t}\frac{d}{dt}\int_{t-h_1}^{t} e^{2\alpha s}x^T(s)Qx(s)ds + \int_{t-h_1}^{t} e^{2\alpha s}x^T(s)Qx(s)ds\frac{d}{dt}e^{-2\alpha t} \\
&= \Big[e^{-2\alpha t}e^{2\alpha t}x^T(t)Qx(t) - e^{-2\alpha t}e^{2\alpha(t-h_1)}x^T(t-h_1)Qx(t-h_1)\Big] \\
&\quad -2\alpha\int_{t-h_2}^{t} e^{2\alpha s}x^T(s)Qx(s)dse^{-2\alpha t} \\
&= \Big[e^{-2\alpha t}e^{2\alpha t}x^T(t)Qx(t) - e^{-2\alpha t}e^{2\alpha(t-h_1)}x^T(t-h_1)Qx(t-h_1)\Big] \\
&\quad -2\alpha\int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= x^T(t)Qx(t) - e^{-2\alpha h_1}x^T(t-h_1)Qx(t-h_1) \ -2\alpha V_2 \ .
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3 &= \frac{d}{dt}\int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= \frac{d}{dt}\Big(e^{-2\alpha t}\int_{t-h_2}^{t} e^{2\alpha s}x^T(s)Qx(s)ds\Big) \\
&= e^{-2\alpha t}\frac{d}{dt}\int_{t-h_2}^{t} e^{2\alpha s}x^T(s)Qx(s)ds + \int_{t-h_2}^{t} e^{2\alpha s}x^T(s)Qx(s)ds\frac{d}{dt}e^{-2\alpha t} \\
&= \Big[e^{-2\alpha t}e^{2\alpha t}x^T(t)Qx(t) - e^{-2\alpha t}e^{2\alpha(t-h_2)}x^T(t-h_2)Qx(t-h_2)\Big] \\
&\quad -2\alpha\int_{t-h_2}^{t} e^{2\alpha s}x^T(s)Qx(s)dse^{-2\alpha t} \\
&= \Big[e^{-2\alpha t}e^{2\alpha t}x^T(t)Qx(t) - e^{-2\alpha t}e^{2\alpha(t-h_2)}x^T(t-h_2)Qx(t-h_2)\Big] \\
&\quad -2\alpha\int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= x^T(t)Qx(t) - e^{-2\alpha h_2}x^T(t-h_2)Qx(t-h_2) \ -2\alpha V_3 \ .
\end{aligned}$$

$$\begin{aligned}
\dot{V}_4 &= \frac{d}{dt}\int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= \frac{d}{dt}\Big(e^{-2\alpha t}\int_{t-h_2}^{t-h_1} e^{2\alpha s}x^T(s)Qx(s)ds\Big) \\
&= e^{-2\alpha t}\frac{d}{dt}\int_{t-h_2}^{t-h_1} e^{2\alpha s}x^T(s)Qx(s)ds + \int_{t-h_2}^{t-h_1} e^{2\alpha s}x^T(s)Qx(s)ds\frac{d}{dt}e^{-2\alpha t} \\
&= e^{-2\alpha h_1}\Big[x^T(t-h_1)Qx(t-h_1) - e^{2\alpha h_2}x^T(t-h_2)Qx(t-h_2)\Big] \\
&\quad -2e^{-2\alpha}V_4.
\end{aligned}$$

$$
\begin{aligned}
\dot{V}_5 &= \frac{d}{dt}\Big(h_1 e^{-2\alpha t}\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds\Big)\\
&= h_1 e^{-2\alpha t}\frac{d}{dt}\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds\\
&\quad + \textstyle\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds \frac{d}{dt}h_1 e^{-2\alpha t}\\
&= h_1 e^{-2\alpha t}\frac{d}{dt}\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds\\
&\quad -2\alpha h_1 \textstyle\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds\\
&= h_1 e^{-2\alpha t}\frac{d}{dt}\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds - 2\alpha V_5\\
&= h_1 e^{-2\alpha t}\int_{-h_1}^{0} e^{2\alpha t}\dot{x}^T(t)R\dot{x}(t) - e^{2\alpha(t+s)}\dot{x}^T(t+s)R\dot{x}(t+s)ds - 2\alpha V_5\\
&= h_1 e^{-2\alpha t}e^{2\alpha t}\int_{-h_1}^{0}\Big(\dot{x}^T(t)R\dot{x}(t) - e^{2\alpha s}\dot{x}^T(t+s)R\dot{x}(t+s)\Big)ds - 2\alpha V_5\\
&= h_1\int_{-h_1}^{0}\Big(\dot{x}^T(t)R\dot{x}(t) - e^{2\alpha s}\dot{x}^T(t+s)R\dot{x}(t+s)\Big)ds - 2\alpha V_5\\
&= h_1\int_{-h_1}^{0}\dot{x}^T(t)R\dot{x}(t)ds - h_1\int_{-h_1}^{0} e^{2\alpha s}\dot{x}^T(t+s)R\dot{x}(t+s)ds - 2\alpha V_5\\
&= h_1\dot{x}^T(t)R\dot{x}(t)\int_{-h_1}^{0} ds - h_1\int_{-h_1}^{0} e^{2\alpha s}\dot{x}^T(t+s)R\dot{x}(t+s)ds - 2\alpha V_5\\
&= h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1\int_{-h_1}^{0} e^{2\alpha s}\dot{x}^T(t+s)R\dot{x}(t+s)ds - 2\alpha V_5\\
&= h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1\int_{t-h_1}^{t} e^{2\alpha(\theta-t)}\dot{x}^T(\theta)R\dot{x}(\theta)d\theta - 2\alpha V_5\\
&= h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1\int_{t-h_1}^{t} e^{2\alpha(s-t)}\dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_5\\
&= h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1\int_{t-h_1}^{t} e^{2\alpha s}e^{-2\alpha t}\dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_5\\
&= h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1\int_{t-h_1}^{t} e^{2\alpha(t-h_1)}e^{-2\alpha t}\dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_5\\
&\le h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1 e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_5\ .
\end{aligned}
$$

By applying Proposition 2.2, Newton-Leibniz formula and Lemma 2.1, we obtain

$$
\begin{aligned}
\dot{V}_5 \le\ & h_1^2\dot{x}^T(t)R\dot{x}(t) + x(t)\Phi_1 h_1 e^{-2\alpha h_1}x^T(t) + x(t-h_1)\Phi_4 h_2 e^{-2\alpha h_1}x^T(t)\\
&+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_7 h_1 e^{-2\alpha h_1}x^T(t) + x(t)\Phi_2 h_1 e^{-2\alpha h_1}x^T(t-h_1)\\
&+x(t-h_1)\Phi_5 h_1 e^{-2\alpha h_1}x^T(t-h_1)+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_8 h_1 e^{-2\alpha h_1}x^T(t-h_1)
\end{aligned}
$$

$$+x(t)\Phi_3 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T + x(t-h_1)\Phi_6 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T$$
$$+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_9 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T - 2\alpha V_5\ ,$$

$$\begin{aligned}
\dot{V}_6 &= h_2 e^{-2\alpha t}\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds \\
&= h_2 e^{-2\alpha t}\frac{d}{dt}\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds \\
&\quad + \textstyle\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds\frac{d}{dt}h_2 e^{-2\alpha t} \\
&= h_2 e^{-2\alpha t}\frac{d}{dt}\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds \\
&\quad -2\alpha h_2 \textstyle\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds \\
&= h_2 e^{-2\alpha t}\frac{d}{dt}\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds - 2\alpha V_6 \\
&= h_2 e^{-2\alpha t}\int_{-h_2}^{0} e^{2\alpha t}\Big(\dot{x}^\tau(t)R\dot{x}(t) - e^{2\alpha s}\dot{x}^\tau(t+s)R\dot{x}(t+s)\Big)ds - 2\alpha V_6 \\
&= h_2\int_{-h_2}^{0}\dot{x}^\tau(t)R\dot{x}(t) - e^{2\alpha s}\dot{x}^\tau(t+s)R\dot{x}(t+s)ds - 2\alpha V_6 \\
&= h_2\int_{-h_2}^{0}\dot{x}^\tau(t)R\dot{x}(t)ds - h_2\int_{-h_2}^{0} e^{2\alpha s}\dot{x}^\tau(t+s)R\dot{x}(t+s)ds - 2\alpha V_6 \\
&= h_2\dot{x}^\tau(t)R\dot{x}(t)\int_{-h_2}^{0} ds - h_2\int_{-h_2}^{0} e^{2\alpha s}\dot{x}^\tau(t+s)R\dot{x}(t+s)ds - 2\alpha V_6 \\
&= h_2\dot{x}^\tau(t)R\dot{x}(t)\Big[0-(-h_2)\Big] - h_2\int_{-h_2}^{0} e^{2\alpha s}\dot{x}^\tau(t+s)R\dot{x}(t+s)ds - 2\alpha V_6 \\
&= h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2\int_{-h_2}^{0} e^{2\alpha s}\dot{x}^\tau(t+s)R\dot{x}(t+s)ds - 2\alpha V_6 \\
&= h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2\int_{t-h_2}^{t} e^{2\alpha(\theta-t)}\dot{x}^\tau(\theta)R\dot{x}(\theta)d\theta - 2\alpha V_6 \\
&= h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2\int_{t-h_2}^{t} e^{2\alpha(s-t)}\dot{x}^\tau(s)R\dot{x}(s)ds - 2\alpha V_6 \\
&\le h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2\int_{t-h_2}^{t} e^{2\alpha s}e^{-2\alpha t}\dot{x}^\tau(s)R\dot{x}(s)ds - 2\alpha V_6 \\
&\le h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2\int_{t-h_2}^{t} e^{2\alpha(t-h_2)}e^{-2\alpha t}\dot{x}^\tau(s)R\dot{x}(s)ds - 2\alpha V_6 \\
&\le h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2 e^{-2\alpha h_2}\int_{t-h_2}^{t}\dot{x}^\tau(s)R\dot{x}(s)ds - 2\alpha V_6\ .
\end{aligned}$$

From Proposition 2.2, Newton-Leibniz formula and Lemma 2.1 , we have

$$\begin{aligned}
\dot{V}_6 &\le h_2^2\dot{x}^\tau(t)R\dot{x}(t) + x(t)\Psi_1 h_2 e^{-2\alpha h_2}x^T(t) + x(t-h_2)\Psi_4 h_2 e^{-2\alpha h_2}x^T(t) \\
&\quad +\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_7 h_2 e^{-2\alpha h_2}x^T(t)\ + x(t)\Psi_2 h_2 e^{-2\alpha h_2}x^T(t-h_2)
\end{aligned}$$

$$+x(t-h_2)\Psi_5 h_2 e^{-2\alpha h_2} x^T(t-h_2) + \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds \Psi_8 h_2 e^{-2\alpha h_2} x^T(t-h_2)$$
$$+x(t)\Psi_3 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T + x(t-h_2)\Psi_6 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds \Psi_9 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T - 2\alpha V_6 \ ,$$

$$\dot{V}_7 = (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}e^{-2\alpha t}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds$$
$$= (h_2-h_1)e^{-2\alpha t}\int_{-h_2}^{-h_1}\left[\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau\right]ds$$
$$= (h_2-h_1)e^{-2\alpha t}\frac{d}{dt}\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds$$
$$+\frac{d}{dt}(h_2-h_1)e^{-2\alpha t}\left[\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds\right]$$
$$= (h_2-h_1)e^{-2\alpha t}\frac{d}{dt}\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds$$
$$-2\alpha(h_2-h_1)e^{-2\alpha t}\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds$$
$$= (h_2-h_1)e^{-2\alpha t}\frac{d}{dt}\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds$$
$$-2\alpha\left[(h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds\right]$$
$$= (h_2-h_1)e^{-2\alpha t}\frac{d}{dt}\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds - 2\alpha V_7$$
$$= (h_2-h_1)e^{-2\alpha t}\int_{-h_2}^{-h_1}\Big(e^{2\alpha t}\dot{x}^T(t)U\dot{x}(t) - e^{2\alpha(t+s)}\dot{x}^T(t+s)U\dot{x}(t+s)\Big)ds$$
$$-2\alpha V_7$$
$$= (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) - (h_2-h_1)\int_{-h_2}^{-h_1} e^{2\alpha s}\dot{x}^T(t+s)U\dot{x}(t+s)ds - 2\alpha V_7$$
$$= (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) - (h_2-h_1)\int_{t-h_2}^{t-h_1} e^{2\alpha(t-h_2-t)}\dot{x}^T(s)U\dot{x}(s)ds$$
$$= (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) - (h_2-h_1)e^{2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds - 2\alpha V_7 \ .$$

By using Proposition 2.2, Newton-Leibniz formula and Lemma 2.1, we obtain

$$\dot{V}_7 \le (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1)$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1)$$
$$+\tfrac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1)$$

$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2)$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2)$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2)$$
$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$-2\alpha V_7\ .$$

Therefore , we have

$$\dot{V}(.)+2\alpha V(.)$$
$$\leq x^T(t)\Big[A^TP+PA\Big]x(t)+2x^T(t)PDx(t-h(t))+x^TQx(t)$$
$$-e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_2)\Big)$$
$$+e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_1)\Big)-e^{2\alpha h_2}x^T\Big(t-h_2Qx(t-h_2)\Big)+h_1^2\dot{x}^T(t)R\dot{x}(t)$$
$$-h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds+h_2^2\dot{x}^T(t)R\dot{x}(t)-h_2e^{-2\alpha h_2}\int_{t-h_2}^{t}\dot{x}^T(s)R\dot{x}(s)ds$$
$$+(h_2-h_1)^2\dot{x}^T(t)R\dot{x}(t)-(h_2-h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R\dot{x}(s)ds$$
$$\leq x^T(t)\Big[A^TP+PA\Big]x(t)+2x^T(t)PDx(t-h(t))+x^TQx(t)$$
$$-e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_2)\Big)+e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_1)\Big)$$
$$-e^{2\alpha h_2}x^T\Big(t-h_2Qx(t-h_2)\Big)$$
$$+h_1^2\dot{x}^T(t)R\dot{x}(t)+x(t)\Phi_1h_1e^{-2\alpha h_1}x^T(t)+x(t-h_1)\Phi_4h_2e^{-2\alpha h_1}x^T(t)$$
$$+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_7h_1e^{-2\alpha h_1}x^T(t)+x(t)\Phi_2h_1e^{-2\alpha h_1}x^T(t-h_1)$$
$$+x(t-h_1)\Phi_5h_1e^{-2\alpha h_1}x^T(t-h_1)+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_8h_1e^{-2\alpha h_1}x^T(t-h_1)$$
$$+x(t)\Phi_3h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T+x(t-h_1)\Phi_6h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T$$
$$+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_9h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_7h_2e^{-2\alpha h_2}x^T(t)\ +x(t)\Psi_2h_2e^{-2\alpha h_2}x^T(t-h_2)$$
$$+x(t-h_2)\Psi_5h_2e^{-2\alpha h_2}x^T(t-h_2)+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_8h_2e^{-2\alpha h_2}x^T(t-h_2)$$
$$+x(t)\Psi_3h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T+x(t-h_2)\Psi_6h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T$$

$$
\begin{aligned}
&+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_9 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T \\
&+(h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1) \\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1) \\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1) \\
&+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2) \\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2) \\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2) \\
&+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\tfrac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T \\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T \\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T. \qquad (3.3)
\end{aligned}
$$

By using the following identity relation

$$\dot{x}(t) - Ax(t) - Dx(t-h(t)) = 0,$$

we have

$$
\begin{aligned}
&2x^T(t)S_1\dot{x}(t) - 2x^T(t)S_1Ax(t) - 2x^T(t)S_1Dx(t-h(t)) = 0 \\
&2x^T(t-h_1)S_2\dot{x}(t) - 2x^T(t-h_1)S_2Ax(t) - 2x^T(t-h_1)S_2Dx(t-h(t)) = 0 \\
&2x^T(t-h_2)S_3\dot{x}(t) - 2x^T(t-h_2)S_3Ax(t) - 2x^T(t-h_2)S_3Dx(t-h(t)) = 0 \\
&2x^T(t-h(t))S_4\dot{x}(t) - 2x^T(t-h(t))S_4Ax(t) - 2x^T(t-h(t))S_4Dx(t-h(t)) = 0 \\
&2\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T S_5\dot{x}(t) - 2\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T S_5Ax(t) \\
&-2\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T S_5Dx(t-h(t)) = 0 \\
&2\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T S_6\dot{x}(t) - 2\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T S_6Ax(t) \\
&-2\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T S_6Dx(t-h(t)) = 0 \\
&2\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T S_7\dot{x}(t) - 2\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_2} x(s)ds\Big)^T S_7Ax(t) \\
&-2\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T S_7Dx(t-h(t)) = 0. \\
&2\dot{x}^T(t)S_8\dot{x}(t) - 2\dot{x}^T(t)S_8Ax(t) - 2\dot{x}^T(t)S_8Dx(t-h(t)) = 0. \qquad (3.4)
\end{aligned}
$$

By adding all the zero items of (3.4) into (3.3) , we obtain

$\dot{V}(.) + 2\alpha V(.)$

$$
\begin{aligned}
&\leq x^T\Big[A^TP+PA+2Q-h_1e^{-2\alpha h_1}\Phi_1-h_2e^{-2\alpha h_2}\Psi_1-(S_1A+A^TS_1^T)\Big]x(t)\\
&+x^T(t)\Big[-h_1e^{-2\alpha h_1}\Phi_4-S_2A\Big]x(t-h_1)+x^T(t)\Big[-h_2e^{-2\alpha h_1}\Psi_4-S_3A\Big]x(t-h_2)\\
&+x^T\Big[PD-S_1D-S_4D\Big]x(t-h(t))+x^T(t)\Big[-h_1e^{-2\alpha h_1}\Phi_7-S_5A\Big]\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\\
&+x^T(t)\Big[-h_2e^{-2\alpha h_2}\Psi_7-S_6A\Big]\frac{1}{h_2}\int_{t-h_2}^t x(s)ds+x^T(t)[-S_7A]\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\\
&+x^T[S_1-S_8A]\dot{x}(t)+x^T(t-h_1)\Big[-h_1e^{-2\alpha h_1}\Phi_2-S_2A\Big]x(t)\\
&+x^T(t-h_1)\Big[2e^{-2\alpha h_1}Q-h_1e^{-2\alpha h_1}\Phi_5-(h_2-h_1)e^{-2\alpha h_2}\Omega_1\Big]x(t-h_1)\\
&+x^T(t-h_1)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_4-(0.5)(e^{2\alpha h_1}+e^{2\alpha h_2}Q)\Big]x(t-h_2)\\
&+x^T(t-h_1)[-S_2D]x(t-h(t))+x^T(t-h_1)\Big[-h_1e^{-2\alpha h_1}\Phi_8\Big]\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\\
&+x^T(t-h_1)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_7\Big]\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\\
&+x^T(t-h_1)S_2\dot{x}(t)+x^T(t-h_2)\Big[-h_2e^{-2\alpha h_2}\Psi_2-S_3A\Big]x(t)\\
&+x^T(t-h_2)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_2-(0.5)(e^{2\alpha h_1}+e^{2\alpha h_2})Q\Big]x(t-h_1)\\
&+x^T(t-h_2)\Big[-e^{-2\alpha h_2}Q-h_2e^{-2\alpha h_2}\Psi_5-(h_2-h_1)e^{-2\alpha h_2}\Omega_5\Big]x(t-h_2)\\
&+x^T(t-h_2)[-S_3D]x(t-h(t))+x^T(t-h_2)\Big[-h_2e^{-2\alpha h_2}\Psi_8\Big]\frac{1}{h_2}\int_{t-h_2}^t x(s)ds\\
&+x^T(t-h_2)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_8\Big]\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\\
&+x^T(t-h_2)[S_3]\dot{x}(t)+x^T(t-h(t))\Big[PD-S_4A-S_1D\Big]x(t)\\
&+x^T(t-h(t))[-S_2D]x(t-h_1)+x^T(t-h(t))[-S_3D]x(t-h_2)\\
&+x^T(t-h(t))[-2S_4D]x(t-h(t))+x^T(t-h(t))[-S_5D]\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\\
&+x^T(t-h(t))[-S_6D]\frac{1}{h_2}\int_{t-h_2}^t x(s)ds+x^T(t-h(t))[-S_7D]\frac{1}{h_2-h_1}\int_{t-h}^{t-h_1} x(s)ds\\
&+x^T(t-h(t))[S_4-S_8D]\dot{x}(t)+(\frac{1}{h_1}\int_{t-h_1}^t x(s)ds)^T\Big[-h_1e^{-2\alpha h_1}\Phi_3-S_5A\Big]x(t)\\
&+(\frac{1}{h_1}\int_{t-h_1}^t x(s)ds)^T\Big[-h_1e^{-2\alpha h_1}\Phi_6\Big]x(t-h_1)\\
&+\Big(\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\Big)^T[-S_5D]x(t-h(t))\\
&+\Big(\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\Big)^T\Big[-h_1e^{-2\alpha h_1}\Phi_9\Big]\Big(\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\Big)\\
&+\Big(\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\Big)^T S_5\Big(\frac{1}{h_1}\int_{t-h_1}^t x(s)ds\Big)\\
&+\Big(\frac{1}{h_2}\int_{t-h_2}^t x(s)ds\Big)^T\Big[-h_2e^{-2\alpha h_2}\Psi_3-S_6A\Big]x(t)
\end{aligned}
$$

$$
\begin{aligned}
&+\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T\Big[-h_2e^{-2\alpha h_2}\Psi_6\Big]x(t-h_2)\\
&+\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T[-S_6D]x(t-h(t))\\
&+\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T\Big[-h_2e^{-2\alpha h_2}\Psi_9\Big]\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)\\
&+\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T[S_6]\dot{x}(t)+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T[-S_7]x(t)\\
&+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T[-\Big(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big]x(t-h_1)\\
&+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big]x(t-h_2)\\
&+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T[-S_7D]x(t-h(t))\\
&+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big]\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)\\
&+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T[S_7]\dot{x}(t)\\
&+\dot{x}^T(t)[-S_8A+S_1]x(t)+\dot{x}^T(t)[S_2]x(t-h_1)+\dot{x}^T(t)[S_3]x(t-h_2)\\
&+\dot{x}^T(t)[-S_8D+S_4]x(t-h(t))+\dot{x}^T(t)[S_2]x(t-h_1)+\dot{x}^T(t)S_5\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)\\
&+\dot{x}^T(t)S_6\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)+\dot{x}^T(t)S_7\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)\\
&+\dot{x}^T(t)\Big[(h_1^2+h_2^2)R+(h_2-h_1)^2U+S_8+S_8^T\Big]\dot{x}(t)\\
&=\xi^T(t)J\xi(t).
\end{aligned}
$$

where

$$
\xi(t)=\Bigg[x(t),x(t-h_2),x(t-h_2),x(t-h(t)),\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds,\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds,
\frac{1}{h_2-h_2}\int_{t-h_2}^{t-h_1} x(s)ds,\dot{x}(t)\Bigg].
$$

According to condition (3.1), we obtain

$$
\begin{aligned}
&\dot{V}(t,x_t)+2\alpha V(t,x_t)\le 0\\
&\dot{V}(t,x_t)\le -2\alpha V(t,x_t),\quad \forall t\in R^+. \qquad (3.5)
\end{aligned}
$$

From (3.5), we have

$$
\begin{aligned}
&\dot{V}(t,x_t)=-2\alpha V(t,x_t)\\
&\frac{dV}{dt}(t,x_t)=-2\alpha V(t,x_t) \qquad (3.6)
\end{aligned}
$$

By interesting both system of (3.6) , we obtain

$$
\int_0^t(\frac{1}{V})\frac{dV}{dt}dt=\int_0^t(-2\alpha)dt
$$

$$\ln(V(t))|_0^t = (-2\alpha(t))|_0^t$$
$$\ln(V(t)) - \ln(V(0)) = -2\alpha(t) - 2\alpha(0)$$
$$\ln\frac{V(t)}{V(0)} = -2\alpha t$$
$$\frac{V(t)}{V(0)} = e^{-2\alpha t}$$
$$V(t) = V(o)e^{-2\alpha t}$$

$\therefore V(t, x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \in R^+$

From $\lambda_{\min}\|x(t)\|^2 \leq V(t, x_t) \leq \lambda_{\max}\|x(t)\|^2$

$$\lambda_1\|x(t,\phi)\|^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t}\|\phi\|^2$$
$$\|x(t,\phi)\|^2 \leq \frac{\lambda_2 e^{-2\alpha t}\|\phi\|^2}{\lambda_1} \leq \frac{\lambda_2}{\lambda_1}e^{-2\alpha t}\|\phi\|^2 \ .$$

Then,we have

$$\|x(t,\phi)\| \leq \sqrt{(\frac{\lambda_2}{\lambda_1})e^{-2\alpha t}\|\phi\|^2} \leq \sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\alpha t}\|\phi\|, \quad t \in R^+.$$

From definition 2.1 , we concludes that the zero equation of system(1) is α - exponentially stable. □

Next , we consider the following uncertain linear systems with interval time-varying delay:

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [D + \Delta D(t)]x(t - h(t)), t \in R^+,$$
$$x(t) = \phi(t), t \in [h_2, 0] \tag{2}$$

where the time-varying uncertain matrices$\Delta A(t), \Delta D(t)$are given by :

$$\Delta A(t) = E_a F_a(t) H_a, \Delta D(t) = E_d F_d(t) H_d$$

and E_a, E_d, H_a, H_d are known constant matrices with appropriate dimensions, F_a, F_d are unknown uncertain matrices satisfying

$$F_a^T(t)F_a(t) \leq I, F_d^T(t)F_a(t) \leq I, t \in R^+.$$

The following notations will be used throughout this paper.

$$M_{11} = A^T P + PA + 2Q - h_1 e^{-2\alpha h_1}\Phi_1 - h_2 e^{-2\alpha h_2}\Psi_1 + H_a^T H_a + H_a^T H_a,$$
$$M_{12} = -h_1 e^{-2\alpha h_1}\Phi_4,$$
$$M_{13} = -h_2 e^{-2\alpha h_2}\Psi_4,$$
$$M_{14} = J_{41} = PD + 0.5 + H_d^T H_d - \bar{S}_1 A,$$

$$M_{15} = -h_1 e^{-2\alpha h_1} \Phi_7,$$

$$M_{16} = -h_2 e^{-2\alpha h_2} \Psi_7,$$

$$M_{17} = J_{71} = 0,$$

$$M_{18} = J_{81} = -\bar{S}_2 A,$$

$$M_{21} = -h_1 e^{-2\alpha h_1} \Psi_2,$$

$$M_{22} = e^{2\alpha h_1} Q - h_1 e^{-2\alpha h_1} \Phi_5 - (h_2 - h_1) e^{-2\alpha h_2} \Omega_1,$$

$$M_{23} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_4,$$

$$M_{24} = J_{42} = 0,$$

$$M_{25} = -h_1 e^{-2\alpha h_1} \Phi_8,$$

$$M_{26} = J_{62} = 0,$$

$$M_{27} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_7,$$

$$M_{28} = J_{82} = 0,$$

$$M_{31} = -h_2 e^{-2\alpha h_2} \Psi_2,$$

$$M_{32} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_2,$$

$$M_{33} = -e^{-2\alpha h_2} Q - h_2 e^{-2\alpha h_2} \Psi_5 - (h_2 - h_1) e^{-2\alpha h_2} \Omega_5,$$

$$M_{34} = J_{43} = 0,$$

$$M_{35} = J_{53} = 0,$$

$$M_{36} = -h_2 e^{-2\alpha h_2} \Psi_8,$$

$$M_{37} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_8,$$

$$M_{38} = J_{83} = 0,$$

$$M_{44} = -2S_4 D = H_d^T H_d + H_d^T H_d + H_d^T H_d,$$

$$M_{45} = J_{54} = 0,$$

$$M_{46} = J_{64} = 0,$$

$$M_{47} = J_{74} = 0,$$

$$M_{48} = J_{84} = \bar{S}_1,$$

$$M_{51} = -h_1 e^{-2\alpha h_1} \Phi_3,$$

$$M_{52} = -h_1 e^{-2\alpha h_1} \Phi_5,$$

$$M_{55} = -h_1 e^{-2\alpha h_1} \Phi_9$$

$$M_{56} = J_{65} = 0,$$

$$M_{57} = J_{75} = 0,$$

$$M_{58} = J_{85} = 0,$$

$$M_{61} = -h_2 e^{-2\alpha h_2} \Psi_3,$$

$$M_{63} = -h_2 e^{-2\alpha h_2} \Psi_6,$$

$$M_{66} = -h_2 e^{-2\alpha h_2} \Psi_9,$$

$$M_{67} = J_{76} = 0,$$

$$M_{68} = 0,$$

$$M_{72} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_3,$$

$$M_{73} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_6,$$

$$M_{77} = -(h_2 - h_1) e^{-2\alpha h_2} \Omega_9,$$

$$M_{78} = J_{87} = 0,$$

$$M_{88} = (h_1^2 + h_2^2) R + (h_2 - h_1)^2 U + \bar{S}_2 + \bar{S}_2^T + H_d^T H_d + H_d^T H_d,$$

Theorem 3.2 Given $\alpha > 0$ The zero solution of the system (2) is α-exponentially stable if there exist symmetric positive definite matrices P, Q, R, U, and any matrices $\overline{S}_i, i = 1, 2$ such that the following LMI hold

$$M_1 = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} & M_{18} \\ * & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} & M_{27} & M_{28} \\ * & * & M_{33} & M_{34} & M_{35} & M_{36} & M_{37} & M_{38} \\ * & * & * & M_{44} & M_{45} & M_{46} & M_{47} & M_{48} \\ * & * & * & * & M_{55} & M_{56} & M_{57} & M_{58} \\ * & * & * & * & * & M_{66} & M_{67} & M_{68} \\ * & * & * & * & * & * & M_{77} & M_{78} \\ * & * & * & * & * & * & * & M_{88} \end{bmatrix} < 0, \quad (3.7)$$

$$\mathcal{M}_2 = \begin{bmatrix} 2PA & PE_a & PE_d & \bar{S}_1 E_a & \bar{S}_2 E_a \\ * & -I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (3.8)$$

$$\mathcal{M}_3 = \begin{bmatrix} -2\bar{S}_1 D & \bar{S}_1 E_d & \bar{S}_2 E_d \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (3.9)$$

Proof We consider the following Lyapunov - Krasovskii functional for the system (2)

$$V(t, x_t) = \sum_{i=1}^{7} V_i \ ,$$

By taking the derivative of V_1 along the solution of system (2) we have

$$\begin{aligned}
\dot{V}_1 &= x^T Px(t) \\
&= x^T(t)\tfrac{d}{dx}Px(t) + \tfrac{d}{dx}x^T(t)Px(t) \\
&= x^T(t)P\dot{x}(t) + \dot{x}^T(t)Px(t) \\
&= 2x^T(t)P\dot{x}(t) \\
&= 2x^T(t)P[(A + \Delta A(t))x(t) + (D + \Delta D(t)x(t-h(t))] \\
&= 2x^T(t)P[Ax(t) + \Delta A(t)x(t) + Dx(t-h(t) + \Delta D(t)x(t-h(t)] \\
&= 2x^T(t)PAx(t) + 2x^T(t)P\Delta A(t)x(t) + 2x^T(t)PDx(t-h(t)) \\
&\quad +2x^T(t)P\Delta D(t)x(t-h(t)) \\
&= x^T(t)PAx(t) + x^T(t)PAx(t) + x^T(t)P\Delta A(t)x(t) + x^T(t)P\Delta A(t)x(t) \\
&\quad +x^T(t)PDx(t-h(t)) + x^T(t)PDx(t-h(t) + x^T(t)P\Delta D(t)x(t-h(t)) \\
&\quad +x^T(t)P\Delta D(t)x(t-h(t)) \\
&= x^T(t)[PA + A^TP]x(t) + x^T(t)[P\Delta A(t) \\
&\quad +\Delta A^TP]x(t) + x^T(t)[PD + D^TP]x(t-h(t)) + x^T(t)[P\Delta D \\
&\quad +\Delta D^TP](t)x(t-h(t)) \\
&= x^T(t)[PA + A^TP]x(t) + x^T(t)[PE_aF_a(t)H_a + PH_a^TF_a^T(t)E_a^T]x(t) \\
&\quad +x^T(t)[PD + D^TP]x(t-h(t)) + x^T(t)[PE_dF_d(t)H_d \\
&\quad +PH_d^TF_d^T(t)E_d^T](t)x(t-h(t)) \\
&= x^T(t)[PA + A^TP]x(t) + x^T(t)[(PE_a)(PE_a)^T + H_a^TH_a]x(t) \\
&\quad +x^T(t)[PD + D^TP]x(t-h(t)) \\
&\quad +x^T(t)[(PE_d)(PE_d)^T + H_d^TH_d](t)x(t-h(t))
\end{aligned} \tag{3.10}$$

$$\begin{aligned}
\dot{V}_2 &= \frac{d}{dt}\int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= x^T(t)Qx(t) - e^{-2\alpha h_1}x^T(t-h_1)Qx(t-h_1) - 2\alpha V_2 \ ,
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\dot{V}_3 &= \frac{d}{dt}\int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= x^T(t)Qx(t) - e^{-2\alpha h_2}x^T(t-h_2)Qx(t-h_2) - 2\alpha V_3 \ ,
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
\dot{V}_4 &= \frac{d}{dt}\int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)}x^T(s)Qx(s)ds \\
&= e^{-2\alpha h_1}\Big[x^T(t-h_1)Qx(t-h_1) - e^{2\alpha h_2}x^T(t-h_2)Qx(t-h_2)\Big] - 2e^{-2\alpha}V_4,
\end{aligned}$$

$$
\begin{aligned}
\dot{V}_5 &= \frac{d}{dt}\Big(h_1 e^{-2\alpha t}\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds\Big) \\
&\le h_1^2\dot{x}^T(t)R\dot{x}(t) + x(t)\Phi_1 h_1 e^{-2\alpha h_1}x^T(t) + x(t-h_1)\Phi_4 h_2 e^{-2\alpha h_1}x^T(t) \\
&\quad +\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_7 h_1 e^{-2\alpha h_1}x^T(t) + x(t)\Phi_2 h_1 e^{-2\alpha h_1}x^T(t-h_1) \\
&\quad +x(t-h_1)\Phi_5 h_1 e^{-2\alpha h_1}x^T(t-h_1)+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_8 h_1 e^{-2\alpha h_1}x^T(t-h_1) \\
&\quad +x(t)\Phi_3 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T+x(t-h_1)\Phi_6 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T \\
&\quad +\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_9 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T-2\alpha V_5\,,
\end{aligned}
\tag{3.13, 3.14}
$$

$$
\begin{aligned}
\dot{V}_6 &= h_2 e^{-2\alpha t}\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds \\
&\le h_2^2\dot{x}^\tau(t)R\dot{x}(t) + x(t)\Psi_1 h_2 e^{-2\alpha h_2}x^T(t) + x(t-h_2)\Psi_4 h_2 e^{-2\alpha h_2}x^T(t) \\
&\quad +\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_7 h_2 e^{-2\alpha h_2}x^T(t)\ + x(t)\Psi_2 h_2 e^{-2\alpha h_2}x^T(t-h_2) \\
&\quad +x(t-h_2)\Psi_5 h_2 e^{-2\alpha h_2}x^T(t-h_2)+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_8 h_2 e^{-2\alpha h_2}x^T(t-h_2) \\
&\quad +x(t)\Psi_3 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T+x(t-h_2)\Psi_6 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T \\
&\quad +\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_9 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T-2\alpha V_6\,,
\end{aligned}
\tag{3.15}
$$

$$
\begin{aligned}
\dot{V}_7 &= (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}e^{-2\alpha t}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds \\
&\le (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1) \\
&\quad +x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1) \\
&\quad +\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1) \\
&\quad +x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2) \\
&\quad +x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2) \\
&\quad +\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2) \\
&\quad +x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T \\
&\quad +x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T \\
&\quad +\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T \\
&\quad -2\alpha V_7\,.
\end{aligned}
\tag{3.16}
$$

Hence , we that

$$\dot{V}(.) + 2\alpha V(.)$$

$$\begin{aligned}
&\leq x^T(t)[PA + A^TP]x(t) + x^T(t)[(PE_a)(PE_a)^T + H_a^TH_a]x(t) \\
&\quad + x^T(t)[PD + D^TP]x(t - h(t)) + x^T(t)[(PE_d)(PE_d)^T \\
&\quad + H_d^TH_d]x(t - h(t)) + x^TQx(t) - e^{2\alpha h_1}x^T\Big(t - h_1Qx(t - h_2)\Big) \\
&\quad + e^{2\alpha h_1}x^T\Big(t - h_1Qx(t - h_1)\Big) - e^{2\alpha h_2}x^T\Big(t - h_2Qx(t - h_2)\Big) + h_1^2\dot{x}^T(t)R\dot{x}(t) \\
&\quad - h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds + h_2^2\dot{x}^T(t)R\dot{x}(t) - h_2e^{-2\alpha h_2}\int_{t-h_2}^{t}\dot{x}^T(s)R\dot{x}(s)ds \\
&\quad + (h_2 - h_1)^2\dot{x}^T(t)R\dot{x}(t) - (h_2 - h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R\dot{x}(s)ds \\
&\leq x^T(t)[PA + A^TP]x(t) + x^T(t)[(PE_a)(PE_a)^T + H_a^TH_a]x(t) \\
&\quad + x^T(t)[PD + D^TP]x(t - h(t)) + x^T(t)[(PE_d)(PE_d)^T \\
&\quad + H_d^TH_d]x(t - h(t)) + x^TQx(t) \\
&\quad - e^{2\alpha h_1}x^T\Big(t - h_1Qx(t - h_2)\Big) + e^{2\alpha h_1}x^T\Big(t - h_1Qx(t - h_1)\Big) \\
&\quad - e^{2\alpha h_2}x^T\Big(t - h_2Qx(t - h_2)\Big) \\
&\quad + h_1^2\dot{x}^T(t)R\dot{x}(t) + x(t)\Phi_1h_1e^{-2\alpha h_1}x^T(t) + x(t - h_1)\Phi_4h_2e^{-2\alpha h_1}x^T(t) \\
&\quad + \frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_7h_1e^{-2\alpha h_1}x^T(t) + x(t)\Phi_2h_1e^{-2\alpha h_1}x^T(t - h_1) \\
&\quad + x(t - h_1)\Phi_5h_1e^{-2\alpha h_1}x^T(t - h_1) + \frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_8h_1e^{-2\alpha h_1}x^T(t - h_1) \\
&\quad + x(t)\Phi_3h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T + x(t - h_1)\Phi_6h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T \\
&\quad + \frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_9h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T \\
&\quad + \frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_7h_2e^{-2\alpha h_2}x^T(t) + x(t)\Psi_2h_2e^{-2\alpha h_2}x^T(t - h_2) \\
&\quad + x(t - h_2)\Psi_5h_2e^{-2\alpha h_2}x^T(t - h_2) + \frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_8h_2e^{-2\alpha h_2}x^T(t - h_2) \\
&\quad + x(t)\Psi_3h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T + x(t - h_2)\Psi_6h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T \\
&\quad + \frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_9h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T \\
&\quad + (h_2 - h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t - h_1)(h_2 - h_1)e^{-2\alpha h_2}\Omega_1x^T(t - h_1) \\
&\quad + x(t - h_2)(h_2 - h_1)e^{-2\alpha h_2}\Omega_4x^T(t - h_1) \\
&\quad + \frac{1}{h_2 - h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2 - h_1)e^{-2\alpha h_2}\Omega_7x^T(t - h_1) \\
&\quad + x(t - h_1)(h_2 - h_1)e^{-2\alpha h_2}\Omega_2x^T(t - h_2) \\
&\quad + x(t - h_2)(h_2 - h_1)e^{-2\alpha h_2}\Omega_5x^T(t - h_2)
\end{aligned}$$

$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2)$$
$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\tfrac{1}{h_2-h_1}\textstyle\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T \qquad (3.17)$$

By using the following identity relation

$$\dot{x}(t)-[A-\Delta A(t)]x(t)+[D-\Delta D(t)]x(t-h(t))=0\ ,$$

we have

$$-2x^T(t)S_1\dot{x}(t)+2x^T(t)S_1[A+\Delta A(t)]x(t)$$
$$+2x^TS_1[D+\Delta D(t)]x(t-h(t))=0 \qquad (3.18)$$
$$2x^T(t-h(t))S_4\dot{x}(t)-2x^T(t-h(t))S_4[A+\Delta A(t)]x(t)$$
$$-2x^T(t-h(t))S_4[D+\Delta D(t)]x(t-h(t))=0 \qquad (3.19)$$
$$2\dot{x}^T(t)S_8\dot{x}(t)-2\dot{x}^T(t)S_8[A+\Delta A(t)]x(t)$$
$$-2\dot{x}^T(t)S_8[D+\Delta D(t)]x(t-h(t))=0 \qquad (3.20)$$

From (3.17),(3.18),(3.19) and (3.20) , we have

$$\dot{V}+2\alpha V\le x^T(t)[PA+A^TP]x(t)+x^T(t)[(PE_a)(PE_a)^T+H_a^TH_a]x(t)$$
$$+x^T(t)[PD+D^TP]x(t-h(t))+x^T(t)[(PE_d)(PE_d)^T$$
$$+H_d^TH_d]x(t-h(t))+x^TQx(t)$$
$$-e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_2)\Big)+e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_1)\Big)$$
$$-e^{2\alpha h_2}x^T\Big(t-h_2Qx(t-h_2)\Big)$$
$$+h_1^2\dot{x}^T(t)R\dot{x}(t)+x(t)\Phi_1h_1e^{-2\alpha h_1}x^T(t)+x(t-h_1)\Phi_4h_2e^{-2\alpha h_1}x^T(t)$$
$$+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_7h_1e^{-2\alpha h_1}x^T(t)+x(t)\Phi_2h_1e^{-2\alpha h_1}x^T(t-h_1)$$
$$+x(t-h_1)\Phi_5h_1e^{-2\alpha h_1}x^T(t-h_1)+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_8h_1e^{-2\alpha h_1}x^T(t-h_1)$$
$$+x(t)\Phi_3h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T+x(t-h_1)\Phi_6h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T$$
$$+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_9h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_7h_2e^{-2\alpha h_2}x^T(t)\ +x(t)\Psi_2h_2e^{-2\alpha h_2}x^T(t-h_2)$$
$$+x(t-h_2)\Psi_5h_2e^{-2\alpha h_2}x^T(t-h_2)+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_8h_2e^{-2\alpha h_2}x^T(t-h_2)$$

$$+x(t)\Psi_3 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T + x(t-h_2)\Psi_6 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_9 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T$$
$$+(h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1)$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1)$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1)$$
$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2)$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2)$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2)$$
$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\tfrac{1}{h_2-h_1}\textstyle\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$
$$-2x^T(t)S_1\dot{x}(t) + 2x^T(t)S_1[A+\Delta A(t)]x(t)$$
$$+2x^T S_1[D+\Delta D(t)]x(t-h(t))$$
$$+2x^T(t-h(t))S_4\dot{x}(t) - 2x^T(t-h(t))S_4[A+\Delta A(t)]x(t)$$
$$-2x^T(t-h(t))S_4[D+\Delta D(t)]x(t-h(t))$$
$$+2\dot{x}^T(t)S_8\dot{x}(t) - 2\dot{x}^T(t)S_8[A+\Delta A(t)]x(t)$$
$$-2\dot{x}^T(t)S_8[D+\Delta D(t)]x(t-h(t))\ .$$
$$= \xi^T(t)M_1\xi(t)$$

Hence , we have

$$\dot{V}(t,x_t) - 2\alpha V(t,x_t) \le \xi^T(t)M_1\xi(t) + x^T(t)M_2x(t) + x^T(t-h(t))M_3x(t-h(t)) \tag{3.21}$$

By using the similar approach as in Theorem 3.1 with taking $S_1 = P, S_2 = S_3 = S_5 = S_6 = S_7 = 0, S_4 = \bar{S}1, S_8 = \bar{S}2$, we obtain

$$2PA + PE_aE_a^TP + PE_aE_a^TP + PE_dE_d^TP + \bar{S}_1E_aE_a^T\bar{S}_1 + \bar{S}_2E_aE_a^T\bar{S}_2 < 0$$

By equivalent , we have

$$\mathcal{M}_2 = \begin{bmatrix} 2PA & PE_a & PE_d & \bar{S}_1E_a & \bar{S}_2E_a \\ * & -I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (3.22)$$

$$-2\bar{S}_1D + \bar{S}_1E_dE_d^T\bar{S}_1 + \bar{S}_2E_dE_d^T\bar{S}_2 < 0$$

By equivalent , we have

$$\mathcal{M}_3 = \begin{bmatrix} -2\bar{S}_1D & \bar{S}_1E_d & \bar{S}_2E_d \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (3.23)$$

$$M_2 = 2PA + PE_aE_a^TP + PE_dE_d^TP + \bar{S}_1E_aE_a^T\bar{S}_1 + \bar{S}_2E_aE_a^T\bar{S}_2 < 0$$

$$M_3 = -2\bar{S}_1D + \bar{S}_1E_dE_d^T\bar{S}_1 + \bar{S}_2E_dE_d^T\bar{S}_2 < 0$$

From (3.21) , we have

$$\begin{aligned} \dot{V}(t,x_t) + 2\alpha V(t,x_t) &\leq \xi^T(t)M_1\xi(t) + x^T(t)M_2x(t) \\ &\quad + x^T(t-h(t))M_3x(t-h(t)) \\ \dot{V}(t,x_t) + 2\alpha V(.) &\leq \xi^T(t)M_1\xi(t) \\ &\leq 0 \\ \dot{V}(t,x_t) &\leq -2\alpha V(t,x_t), \quad \forall t \in R^+. \end{aligned} \quad (3.24)$$

From (3.24) , we have

$$\begin{aligned} \dot{V}(t,x_t) &\leq -2\alpha V(t,x_t) \\ \frac{dV}{dt}(t,x_t) &= -2\alpha V(t,x_t) \end{aligned} \quad (3.25)$$

By interesting both systems of (3.25) , we obtain

$$\begin{aligned} \int_0^t (\frac{1}{V})\frac{dV}{dt}dt &= \int_0^t (-2\alpha)dt \\ \ln(V(t))|_0^t &= (-2\alpha(t))|_0^t \\ \ln(V(t)) - \ln(V(0)) &= -2\alpha(t) - 2\alpha(0) \\ \ln\frac{V(t)}{V(0)} &= -2\alpha t \\ \frac{V(t)}{V(0)} &= e^{-2\alpha t} \\ V(t) &= V(o)e^{-2\alpha t} \end{aligned}$$

$$\therefore V(t,x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \in R^+$$

From $\quad \lambda_{min}\|x(t)\|^2 \leq V(t,x_t) \leq \lambda_{max}\|x(t)\|^2$

$$
\begin{aligned}
\lambda_1\|x(t,\phi)\|^2 &\le V(x_t) \le V(\phi)e^{-2\alpha t} \le \lambda_2 e^{-2\alpha t}\|\phi\|^2 \\
\|x(t,\phi)\|^2 &\le \frac{\lambda_2 e^{-2\alpha t}\|\phi\|^2}{\lambda_1} \\
&\le \frac{\lambda_2}{\lambda_1} e^{-2\alpha t}\|\phi\|^2 \ .
\end{aligned}
$$

Then,we have

$$
\begin{aligned}
\|x(t,\phi)\| &\le \sqrt{(\frac{\lambda_2}{\lambda_1})e^{-2\alpha t}\|\phi\|^2} \\
&\le \sqrt{\frac{\lambda_2}{\lambda_1}} e^{-\alpha t}\|\phi\|, \quad t \in R^+.
\end{aligned}
$$

From definition 2.1 , we concludes that the zero equation of system(2) is α - exponentially stable. □

CHAPTER 4
Numerical Examples

4.1 Numerical Examples

In the sequel, we illustrate the effectiveness of the proposed method which yields a computationally solution to the exponential stability and robust stability in the context of LMIs.

Example 4.1 Consider the linear system with interval time-varying delay(2.1), where

$$A = \begin{bmatrix} -12.0000 & 0.0000 \\ 0.0000 & -19.0000 \end{bmatrix}, D = \begin{bmatrix} -0.0002 & 0.004 \\ 0.003 & -0.0005 \end{bmatrix},$$

$$h(t) = 0.1 + 0.4 \mid \sin^2 t \mid .$$

It is worth noting that, the delay function $h(t)$ is non - differntiable. By using LMI Toolbox in MATLAB, the LMI (3.1) is feasible with $h_1 = 0.1000, h_2 = 0.5000, \alpha = 3.0000$ and

$$P = \begin{bmatrix} 2.2710 & 0.0097 \\ 0.0097 & 3.0316 \end{bmatrix} \cdot 10^7, \quad Q = \begin{bmatrix} 0.8546 & -0.0011 \\ -0.0011 & 0.7895 \end{bmatrix} \cdot 10^{-13},$$

$$R = \begin{bmatrix} 2.2638 & 0.0001 \\ 0.0001 & 2.2525 \end{bmatrix} \cdot 10^8, \quad U = \begin{bmatrix} 1.1778 & 0.0001 \\ 0.0001 & 1.1853 \end{bmatrix} \cdot 10^8,$$

$$S_1 = \begin{bmatrix} -6.6308 & 0.0158 \\ 0.0035 & -2.9118 \end{bmatrix} \cdot 10^7, S_2 = \begin{bmatrix} -1.0084 & -0.0005 \\ -0.0036 & 0.0188 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} -1.2931 & 0.0020 \\ 0.0041 & -0.6819 \end{bmatrix}, \quad S_4 = \begin{bmatrix} -0.1460 & 0.9255 \\ 2.9367 & -0.1563 \end{bmatrix} \cdot 10^4,$$

$$S_5 = \begin{bmatrix} -2.2921 & -0.0018 \\ -0.0066 & -1.5879 \end{bmatrix} \cdot 10^4, S_6 = \begin{bmatrix} 24.0437 & -0.0031 \\ -0.0031 & 15.6813 \end{bmatrix},$$

$$S_7 = \begin{bmatrix} 19.7610 & -0.0017 \\ -0.0017 & 12.8688 \end{bmatrix}, \quad S_8 = \begin{bmatrix} 6.4856 & -0.0009 \\ -0.0009 & 4.2495 \end{bmatrix} \cdot 10^7,$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 7.2093 \cdot 10^{-13} \|\phi\|, \forall t \in R^+ \ .$$

The trajectory of the solution of system (1) in has been show in Figure 1.

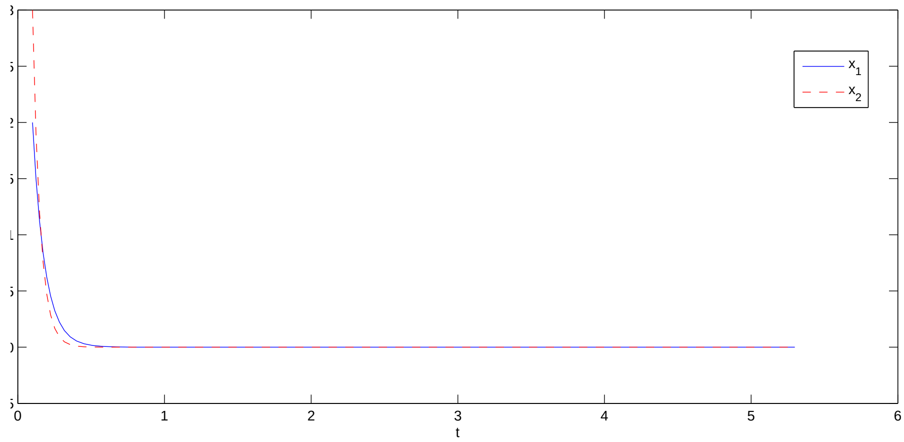

Figure 4.1: The trajectory of the solution of system (1) in Example 4.1 .

Table 4.1: Maximum allowable upper bounds h_2 of the time-varying delay for different values of the lower bounds h_1.

Method	0.1	1.0
Zhang et al. (2016) [9]	4.7000	2.2000
Alexandre Seuret [1]	4.7100	2.2400
Hao-Tian Xu et al. [8]	4.6421	2.1630
Liu et al. [5]	4.4700	2.3820
Park et al. [6]	4.7800	2.4140
Lee et al. [4]	3.6400	2.4980
Theorem 3.1	4.8215	2.5546

Example 4.2 Consider the uncertain linear system with interval time-varying delay(2)with time delay function h(t) with $h_1 = 0.1000, h_2 = 3.1495$ and

$$A = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & 0.3 \\ 0.2 & -0.5 \end{bmatrix},$$

$$Ha = \begin{bmatrix} 0.04 & 1 \\ -1 & -1 \end{bmatrix}, Hd = \begin{bmatrix} 0.04 & 1 \\ -1 & -1 \end{bmatrix},$$

$$Ea = \begin{bmatrix} -0.07 & 0.004 \\ 0.005 & 0.075 \end{bmatrix}, Ed = \begin{bmatrix} -0.045 & 0.002 \\ 0.001 & 0.04 \end{bmatrix}.$$

$$h(t) = 0.1 + 0.4 \mid \sin^2 t \mid .$$

By using LMI Toolbox in MATLAB, the LMI (3.2) of theorem 3.2 are feasible with $\alpha = 5$ and

$$P = \begin{bmatrix} 3.4056 & -1.8341 \\ -1.8341 & 7.1534 \end{bmatrix} \cdot 10^7, \qquad Q = \begin{bmatrix} 0.1700 & -0.0058 \\ -0.0058 & 0.1703 \end{bmatrix} \cdot 10^{-12},$$

$$R = \begin{bmatrix} 2.2195 & -0.0545 \\ -0.0545 & 2.4783 \end{bmatrix} \cdot 10^8, \qquad U = \begin{bmatrix} 1.0957 & 0.0195 \\ 0.0195 & 0.9966 \end{bmatrix} \cdot 10^8,$$

$$\bar{S}_1 = \begin{bmatrix} 0.0903 & -1.6926 \\ 0.6436 & -0.4598 \end{bmatrix} \cdot 10^8, \qquad \bar{S}_2 = \begin{bmatrix} 0.1525 & 1.3081 \\ 0.1319 & 0.6377 \end{bmatrix} \cdot 10^6.$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 1.8041 \cdot 10^{-12} \|\phi\|, \forall t \in R^+ \ .$$

The trajectory of the solution of system (2) in has been show in Figure 2.

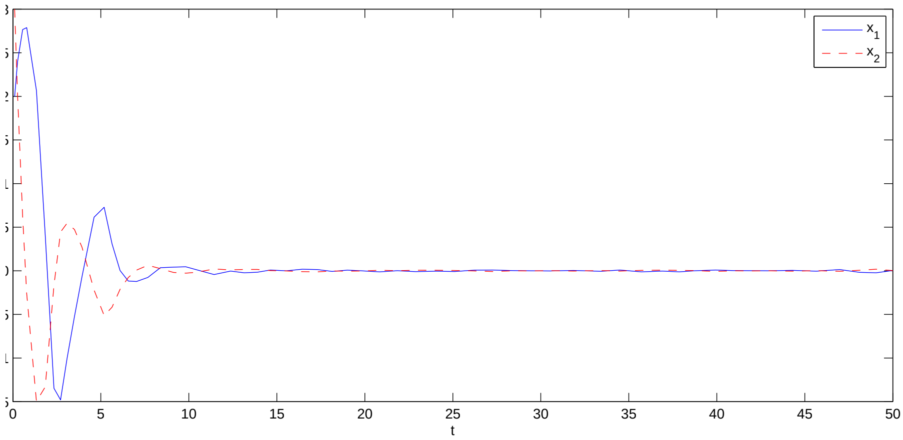

Figure 4.2: The trajectory of the solution of system (2) in Example 4.2 .

CHAPTER 5
Conclusions

5.1 Conclusions

In this independent study, new delay-dependent conditions for the exponential stability of linear systems with non-differentiable interval time-varying delay have been derived in terms of solutions of LMIs. By introducing a set of improved Lyapunov-Krasovskii functional and using Free matrix based integral inequality, the conditions for the exponential stability of the systems have been established. In the future work,the Free matrix-based integral inequality may be applied to stability analysis of other systems such as neural network system,fuzzy system and switched system.

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BIBLIOGRAPHY

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APPENDIX

Exponential stability of linear systems with interval time-varying delays using a new bounding technique

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Abstract

This paper study, we investigate exponential stability problem for a class of linear and uncertain linear systems with time-varying delay. The time-delay is assumed to be a continuous function belonging to a given interval, but not necessary to be differentiable. By introduce a set of augmented Lyapunov-Krasovskii functionals combined with the Free-matrix-based integral inequality, new delay-dependent sufficient conditions for the exponential stability of the system is first established in terms of linear matrix inequalities (LMIs). Numerical examples are given to show the effectiveness of our obtained results.

1 Introduction

Time-delay systems are widely used to model concrete systems in engineering sciences, such as biology, chemistry, mechanics and so on. So the stability analysis of time-delay systems strongly requires before experimental stage. As these reason, the stability analysis of time-delay system have been an attractive study research field during the past years.

The derivative of the Lyapunov functional in order to make it easy to handle. Stability analysis of linear systems with time-varying delays $\dot{x}(t) = Ax(t) + Dx(t - h(t))$ is fundamental to many practical problems and has received considerable attention. Most of the known results on this problem are derived assuming only that the time-varying delay $h(t)$ is a continuously differentiable function, satisfying some boundedness condition on its derivative : $h(t) \leq \delta < 1$ In delay-dependent stability criteria, the main concern is to enlarge the feasible region of stability criteria in given time-delay interval. By constructing augmented Lyapunov functionals and utilizing free weight matrices.

Uncertainty is one of the main features of complex and intelligent decision making systems. Various approaches, methods and techniques in this field have been developed for several decades, starting with such concepts and tools as adaptation, stochastic optimization and statistical decision theory. Another category of approaches is based on the functionals with prescribed derivative. The idea is to apply the functional, that is appropriate for a nominal system and does not depend on the uncertainties, for analysis of the uncertain one. Since our approach belongs to this category, we address the issue in more detail below. One of the crucial goals of the theory was to construct a functional that admits a quadratic lower bound what is of paramount importance for robustness analysis in particular. Such functional was derived in was called the functional of complete type. Its derivative depends on the whole state of a system, and this functional particularly was applied in analysis of systems with delay uncertainties, interesting applications of the functional. It is worth mentioning that there exist other definitions of the complete-type functionals, which are also applied in development of the topic. All these functionals came from the functional with a simple derivative $x^T(t)Wx(t)$ for which a quadratic lower bound does not exist , here W is a positive definite matrix. There is a certain problem when we apply this simple functional for analysis of uncertain systems: its time-derivative along the solutions of a perturbed system is not negative definite, thus the Krasovskii theorem does not hold.

In this paper, we present a new approach for stability analysis of linear time-invariant systems with delay uncertainties, either constant or time-varying, that is developed applying Free matrix based integral inequality. Motivated by the above discretion, we shall desired new criteria for the exponential stability of systems with interval time-varying non-differentiable delay. By introduction a set of improved Lyapunov functionals combined with the NewtonLeibniz formula, we propose new criteria for the exponential stability of the system. The delay-dependent stability conditions are formulated in terms of LMIs, being thus solvable by utilizing MATLAB LMI Control Toolbox available in the literature to date. The approach allows us to apply in exponential stability of uncertain linear systems with interval time-varying delays.

The independent study is organized as follows: Section 2 presents definitions and some well-known technical propositions needed for the proof of the main results. Delay-dependent exponential stability conditions of the system with illustrative numerical examples are show in Section 4. Section 5 gives the conclusions of the paper.

2 Problem formulation and preliminaries

The following notations will be used in this paper. R^+ denotes the set of all real non-negative numbers ; R^n denotes the n-dimensional space with the scalar product x^Ty and the vector norm $\| . \|$; $M^{n\times r}$ denotes the space of all matrices of $(n\times r)$ -dimensions ; A^T denotes the transpose of matrix A ; A is symmetric if A $=$ A^T; I denotes the identity matrix ; $\lambda(A)$ denotes the set of all eigenvalues of A ; $\lambda_{\min/\max}(A)$ =min/max $\{Re\lambda : \lambda \in \lambda(A)\}$; $x_t := \{x(t+s) : s \in [-h, 0]\}$,
$\| x_t \| = sup_{s\in[-h_2,0]}\{\| x(t+s) \|\}$; $C^1([0,t], R^n)$ denotes the set of all R^n -valued continuously differentiable functions on $[0,t]$; Matrix A is called semi-positive definite $(A \geq 0)$ if $\langle Ax, x\rangle \geq 0$, for all x $\in R^n$; A is positive definite $(A > 0)$ if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A > B$ means $A - B > 0$; $*$ denotes the symmetric term in a matrix.

Consider a linear system with interval time-varying delay of the form :

$$\dot{x}(t) = Ax(t) + Dx(t - h(t)), \quad t \in R^+$$
$$x(t) = \phi(t), t \in [-h_2, 0], \tag{1}$$

where $x(t) \in R^n$ is the state ; $A, D \in M^{n\times n}$, and $\phi(t) \in C^1([-h_2, 0], R^n)$ is the initial function with the norm $\| \phi \| = sup_{-h_2 \le t \le 0}\{\| \phi(t) \|, \| \dot{\phi}(t) \|\}$. The time-varying delay function $h(t)$ satisfies

$$0 \le h_1 \le h(t) \le h_2, \quad t \in R^+.$$

Definition 2.1 Given $\alpha > 0$. The zero solution of system (1) is α -exponentially stable if there exist a positive number $N > 0$ such that every solution $x(t, \phi)$ satisfies the following condition:

$$\| x(t, \phi) \| \le Ne^{-\alpha t} \| \phi \|, \quad \forall t \in R^+.$$

We end this section with the following technical well-known propositions, which will be used in the proof of the main results.

Proposition 2.1 (Cauchy inequality). For any symmetric positive definite matrix $N \in M^{n\times n}$ and $a, b \in R^n$ we have

$$\pm a^T b \le a^T N a + b^T N^{-1} b.$$

Proposition 2.2 For any symmetric positive definite matrix $M \in M^{n\times n}$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \to R^n$ such that the integrations concerned are well defined, the following inequality holds

$$\Big(\int_0^\gamma \omega(s)ds\Big)^T M \Big(\int_0^\gamma \omega(s)ds\Big) \le \gamma \Big(\int_0^\gamma \omega^T(s) M \omega(s) ds\Big).$$

Proposition 2.3 Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \le I$. For any $\epsilon > 0$, we have

$$EFH + H^T F^T E^T \le \epsilon E E^T + \epsilon^{-1} H^T H.$$

Proposition 2.4 (Schur complement lemma). Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T, Y = Y^T > 0$. Then $X + Z^T Y^{-1} Z < 0$, if and only if

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0 \text{ or } \begin{pmatrix} -Y & Z \\ Z^T & X \end{pmatrix} < 0 \ .$$

Lemma 2.1 (Free matrix based integral inequality [3]).

Let $x(r) \in R^{n\times n}$ be a continuous function : $\{x(r)|r \in [a,b]\}$. For symmetric matrices $M, N \in R^{3n\times 3n}, R \in R^{n\times n}$, matrices $x \in R^{3n\times 3n}, W, Y \in R^{3n\times n}$ satisfying

$$\theta = \begin{bmatrix} M & X & Y \\ * & N & W \\ * & * & R \end{bmatrix} \geq 0,$$

the following inequality holds :

$$- \int_a^b \dot{X}^T R\dot{X}(r)dr \leq \xi^T\Big((b-a)M + \frac{b-a}{3}N + He\{Y\phi_1 + W\phi_2\}\Big)\xi,$$

where $\phi_1 = \begin{bmatrix} I & -I & 0 \end{bmatrix}$, $\phi_2 = \begin{bmatrix} -I & -I & 2I \end{bmatrix}$,

$$\xi = col\Big\{x(b), x(a), \frac{1}{b-a}\int_a^b x(s)ds\Big\} .$$

3 Main results

The following notations will be used throughout this paper.

$J_{11} = A^TP+PA+2Q-h_1e^{-2\alpha h_1}\Phi_1-h_2e^{-2\alpha h_2}\Psi_1-(S_1A+A^TS_1^T)$, $J_{12} = -h_1e^{-2\alpha h_1}\Phi_4-S_2A$,
$J_{13} = -h_2e^{-2\alpha h_2}\Psi_4 - S_3A$, $J_{14} = J_{41} = PD - S_1D - S_4D$, $J_{15} = -h_1e^{-2\alpha h_1}\Phi_7 - S_5A$,
$J_{16} = -h_2e^{-2\alpha h_2}\Psi_7 - S_6A$, $J_{17} = J_{71} = -S_7A$, $J_{18} = J_{81} = S_1 - S_8A$,
$J_{21} = -h_1e^{-2\alpha h_1}\Psi_2 - S_2A$, $J_{22} = e^{2\alpha h_1}Q - h_1e^{-2\alpha h_1}\Phi_5 - (h_2-h_1)e^{-2\alpha h_2}\Omega_1$,
$J_{23} = -(h_2-h_1)e^{-2\alpha h_2}\Omega_4 - (0.5)(e^{2\alpha h_1} + e^{2\alpha h_2})Q$, $J_{24} = J_{42} = -S_2D$, $J_{25} = -h_1e^{-2\alpha h_1}\Phi_8$,
$J_{26} = J_{62} = 0$, $J_{27} = -(h_2-h_1)e^{-2\alpha h_2}\Omega_7$, $J_{28} = J_{82} = S_2$, $J_{31} = -h_2e^{-2\alpha h_2}\Psi_2 - S_3A$,
$J_{32} = -(h_2-h_1)e^{-2\alpha h_2}\Omega_2 - (0.5)(e^{2\alpha h_1} + e^{2\alpha h_2})Q$, $J_{34} = J_{43} = -S_3D$, $J_{35} = J_{53} = 0$,
$J_{33} = -e^{-2\alpha h_2}Q - h_2e^{-2\alpha h_2}\Psi_5 - (h_2-h_1)e^{-2\alpha h_2}\Omega_5$, $J_{36} = -h_2e^{-2\alpha h_2}\Psi_8$,
$J_{37} = -(h_2-h_1)e^{-2\alpha h_2}\Omega_8$, $J_{38} = J_{83} = S_3$, $J_{44} = -2S_4D = -(S_4D + D^TS_4^T)$,
$J_{45} = J_{54} = -S_5D$, $J_{46} = J_{64} = -S_6D$, $J_{47} = J_{74} = -S_7D$, $J_{48} = J_{84} = S_4 - S_8D$,
$J_{51} = -h_1e^{-2\alpha h_1}\Phi_3 - S_5A$, $J_{52} = -h_1e^{-2\alpha h_1}\Phi_5$, $J_{55} = -h_1e^{-2\alpha h_1}\Phi_9$, $J_{56} = J_{65} = 0$,
$J_{57} = J_{75} = 0$, $J_{58} = J_{85} = S_5$, $J_{61} = -h_2e^{-2\alpha h_2}\Psi_3 - S_6A$, $J_{63} = -h_2e^{-2\alpha h_2}\Psi_6$,
$J_{66} = -h_2e^{-2\alpha h_2}\Psi_9$, $J_{67} = J_{76} = 0$,$J_{68} = S_6$, $J_{72} = -(h_2-h_1)e^{-2\alpha h_2}\Omega_3$,
$J_{73} = -(h_2-h_1)e^{-2\alpha h_2}\Omega_6$, $J_{77} = -(h_2-h_1)e^{-2\alpha h_2}\Omega_9$, $J_{78} = J_{87} = S_7$,
$J_{88} = (h_1^2 + h_2^2)R + (h_2-h_1)^2U + S_8 + S_8^T$.

Theorem 3.1 Given $\alpha > 0$. The zero solution of system (1) is α -exponentially stable if there exist positive matrices P,Q,R,U, positive semi-definite matrices $M_{ii}, N_{ii} (i = 1, 2, 3)$, any matrices $M_{ij}, N_{ij} (i = 1, 2, 3, i \neq j)$ and $S_i (i = 1, 2, .., 8)$ such that following LMIs hold :

$$\theta = \begin{bmatrix} M & X & Y \\ * & N & W \\ * & * & R \end{bmatrix} \geq 0,$$

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} & -S_7A & -S_1 - S_8A \\ * & J_{22} & J_{23} & -S_2D & J_{25} & 0 & J_{27} & S_2 \\ * & * & J_{33} & -S_3D & 0 & J_{36} & J_{37} & S_4 \\ * & * & * & -2S_4D & -S_5D & -S_6D & -S_7D & S_4 - S_8D \\ * & * & * & * & J_{55} & 0 & 0 & S_5 \\ * & * & * & * & * & J_{66} & 0 & S_6 \\ * & * & * & * & * & * & J_{77} & S_7 \\ * & * & * & * & * & * & * & J_{88} \end{bmatrix} < 0, \quad (3)$$

where $$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}, \quad N = \begin{bmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{bmatrix}.$$

Proof We introduce the following LyapunovKrasovskii functional for the system (1)

$$V(t, x_t) = \sum_{i=1}^{7} V_i \ ,$$

where

$$V_1 = x^T(t)Px(t), \quad (3.1.1)$$

$$V_2 = \int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds, \quad (3.1.2)$$

$$V_3 = \int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds, \quad (3.1.3)$$

$$V_4 = \int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)}x^T(s)Qx(s)ds, \quad (3.1.4)$$

$$V_5 = h_1 \int_{-h_1}^{0} \int_{t+s}^{t} e^{-2\alpha(\tau-t)}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau \ ds, \quad (3.1.5)$$

$$V_6 = h_2 \int_{-h_2}^{0} \int_{t+s}^{t} e^{-2\alpha(\tau-t)}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau \ ds, \quad (3.1.6)$$

$$V_7 = (h_2 - h_1) \int_{-h_2}^{-h_1} \int_{t+s}^{t} e^{2\alpha(\tau-t)}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau \ ds. \quad (3.1.7)$$

It easy to verify that

$$\lambda_1 \parallel x(t) \parallel^2 \leq V(t, x_t) \leq \lambda_2 \parallel x_t \parallel^2 , \quad \forall_t \geq 0. \quad (3.2)$$

By taking the derivative of V_1 along the solution of system (1), we have

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t)$$
$$= x^T(t)\big[A^TP + PA\big]x(t) + 2x^T(t)PDx(t-h(t))\ .$$
$$\dot{V}_2 = x^T(t)Qx(t) - e^{-2\alpha h_1}x^T(t-h_1)Qx(t-h_1)\ -2\alpha V_2\ .$$
$$\dot{V}_3 = x^T(t)Qx(t) - e^{-2\alpha h_2}x^T(t-h_2)Qx(t-h_2)\ -2\alpha V_3\ .$$
$$\dot{V}_4 = e^{-2\alpha h_1}\Big[x^T(t-h_1)Qx(t-h_1) - e^{2\alpha h_2}x^T(t-h_2)Qx(t-h_2)\Big] - 2e^{-2\alpha}V_4.$$
$$\dot{V}_5 = h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1\int_{t-h_1}^{t} e^{2\alpha(t-h_1)}e^{-2\alpha t}\dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_5$$
$$\leq h_1^2\dot{x}^T(t)R\dot{x}(t) - h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds - 2\alpha V_5\ .$$

By applying Proposition 2.2, Newton-Leibniz formula and Lemma 2.1, we obtain

$$\dot{V}_5 \leq h_1^2\dot{x}^T(t)R\dot{x}(t) + x(t)\Phi_1h_1e^{-2\alpha h_1}x^T(t) + x(t-h_1)\Phi_4h_2e^{-2\alpha h_1}x^T(t)$$
$$+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_7h_1e^{-2\alpha h_1}x^T(t) + x(t)\Phi_2h_1e^{-2\alpha h_1}x^T(t-h_1)$$
$$+x(t-h_1)\Phi_5h_1e^{-2\alpha h_1}x^T(t-h_1) + \frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_8h_1e^{-2\alpha h_1}x^T(t-h_1)$$
$$+x(t)\Phi_3h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T + x(t-h_1)\Phi_6h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T$$
$$+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_9h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T - 2\alpha V_5\ ,$$

$$\dot{V}_6 = h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2\int_{t-h_2}^{t} e^{2\alpha(s-t)}\dot{x}^\tau(s)R\dot{x}(s)ds - 2\alpha V_6$$

$\leq h_2^2\dot{x}^\tau(t)R\dot{x}(t) - h_2e^{-2\alpha h_2}\int_{t-h_2}^{t}\dot{x}^\tau(s)R\dot{x}(s)ds - 2\alpha V_6$. From Proposition 2.2, Newton-Leibniz formula and Lemma 2.1, we have

$$\dot{V}_6 \leq h_2^2\dot{x}^\tau(t)R\dot{x}(t) + x(t)\Psi_1h_2e^{-2\alpha h_2}x^T(t) + x(t-h_2)\Psi_4h_2e^{-2\alpha h_2}x^T(t)$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_7h_2e^{-2\alpha h_2}x^T(t)\ + x(t)\Psi_2h_2e^{-2\alpha h_2}x^T(t-h_2)$$
$$+x(t-h_2)\Psi_5h_2e^{-2\alpha h_2}x^T(t-h_2) + \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_8h_2e^{-2\alpha h_2}x^T(t-h_2)$$
$$+x(t)\Psi_3h_2e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T + x(t-h_2)\Psi_6h_2e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_9h_2e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T - 2\alpha V_6\ ,$$

$$\dot{V}_7 = (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) - (h_2-h_1)\int_{-h_2}^{-h_1} e^{2\alpha s}\dot{x}^T(t+s)U\dot{x}(t+s)ds - 2\alpha V_7$$

$$\leq (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) - (h_2-h_1)e^{2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)U\dot{x}(s)ds - 2\alpha V_7\ .$$

By using Proposition 2.2, Newton-Leibniz formula and Lemma 2.1, we obtain

$$\begin{aligned}
\dot{V}_7 &\leq (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1)\\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1) + \frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1)\\
&+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2) + x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2)\\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2)\\
&+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T\\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T\\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T - 2\alpha V_7\ .
\end{aligned}$$

Therefore , we have

$$\begin{aligned}
&\dot{V}(.) + 2\alpha V(.)\\
&\leq x^T(t)\Big[A^TP+PA\Big]x(t) + 2x^T(t)PDx(t-h(t)) + x^TQx(t) - e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_2)\Big)\\
&+e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_1)\Big) - e^{2\alpha h_2}x^T\Big(t-h_2Qx(t-h_2)\Big) + h_1^2\dot{x}^T(t)R\dot{x}(t)\\
&-h_1e^{-2\alpha h_1}\int_{t-h_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds + h_2^2\dot{x}^T(t)R\dot{x}(t) - h_2e^{-2\alpha h_2}\int_{t-h_2}^{t}\dot{x}^T(s)R\dot{x}(s)ds\\
&+(h_2-h_1)^2\dot{x}^T(t)R\dot{x}(t) - (h_2-h_1)e^{-2\alpha h_2}\int_{t-h_2}^{t-h_1}\dot{x}^T(s)R\dot{x}(s)ds\\
&\leq x^T(t)\Big[A^TP+PA\Big]x(t) + 2x^T(t)PDx(t-h(t)) + x^TQx(t)\\
&-e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_2)\Big) + e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_1)\Big) - e^{2\alpha h_2}x^T\Big(t-h_2Qx(t-h_2)\Big)\\
&+h_1^2\dot{x}^T(t)R\dot{x}(t) + x(t)\Phi_1h_1e^{-2\alpha h_1}x^T(t) + x(t-h_1)\Phi_4h_2e^{-2\alpha h_1}x^T(t)\\
&+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_7h_1e^{-2\alpha h_1}x^T(t) + x(t)\Phi_2h_1e^{-2\alpha h_1}x^T(t-h_1)\\
&+x(t-h_1)\Phi_5h_1e^{-2\alpha h_1}x^T(t-h_1) + \frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_8h_1e^{-2\alpha h_1}x^T(t-h_1)
\end{aligned}$$

$$+x(t)\Phi_3 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T + x(t-h_1)\Phi_6 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T$$

$$+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_9 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T$$

$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_7 h_2 e^{-2\alpha h_2} x^T(t) \;+ x(t)\Psi_2 h_2 e^{-2\alpha h_2} x^T(t-h_2)$$

$$+x(t-h_2)\Psi_5 h_2 e^{-2\alpha h_2} x^T(t-h_2) + \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_8 h_2 e^{-2\alpha h_2} x^T(t-h_2)$$

$$+x(t)\Psi_3 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T + x(t-h_2)\Psi_6 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T$$

$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_9 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T + x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1)$$

$$+(h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1)$$

$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2)$$

$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2) + \frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2)$$

$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\tfrac{1}{h_2-h_1}\textstyle\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$

$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$

$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T \qquad (3.3)$$

By using the following identity relation

$$\dot{x}(t) - Ax(t) - Dx(t-h(t)) = 0,$$

we have

$$2x^T(t)S_1\dot{x}(t) - 2x^T(t)S_1Ax(t) - 2x^T(t)S_1Dx(t-h(t)) = 0$$

$$2x^T(t-h_1)S_2\dot{x}(t) - 2x^T(t-h_1)S_2Ax(t) - 2x^T(t-h_1)S_2Dx(t-h(t)) = 0$$

$$2x^T(t-h_2)S_3\dot{x}(t) - 2x^T(t-h_2)S_3Ax(t) - 2x^T(t-h_2)S_3Dx(t-h(t)) = 0$$

$$2x^T(t-h(t))S_4\dot{x}(t) - 2x^T(t-h(t))S_4Ax(t) - 2x^T(t-h(t))S_4Dx(t-h(t)) = 0$$

$$2\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T S_5\dot{x}(t) - 2\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T S_5Ax(t)$$

$$-2\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T S_5Dx(t-h(t)) = 0$$

$$2\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T S_6\dot{x}(t) - 2\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T S_6 Ax(t)$$
$$-2\Big(\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Big)^T S_6 Dx(t-h(t)) = 0$$
$$2\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T S_7\dot{x}(t) - 2\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_2} x(s)ds\Big)^T S_7 Ax(t)$$
$$-2\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T S_7 Dx(t-h(t)) = 0.$$
$$2\dot{x}^T(t)S_8\dot{x}(t) - 2\dot{x}^T(t)S_8Ax(t) - 2\dot{x}^T(t)S_8Dx(t-h(t)) = 0. \tag{3.4}$$

By adding all the zero items of (3.4) into (3.3) , we obtain

$$\begin{aligned}
&\dot{V}(.) + 2\alpha V(.) \\
&\le x^T\Big[A^TP + PA + 2Q - h_1e^{-2\alpha h_1}\Phi_1 - h_2e^{-2\alpha h_2}\Psi_1 - (S_1A + A^TS_1^T)\Big]x(t) \\
&\quad + x^T(t)\Big[-h_1e^{-2\alpha h_1}\Phi_4 - S_2A\Big]x(t-h_1) + x^T(t)\Big[-h_2e^{-2\alpha h_1}\Psi_4 - S_3A\Big]x(t-h_2) \\
&\quad + x^T\Big[PD - S_1D - S_4D\Big]x(t-h(t)) + x^T(t)\Big[-h_1e^{-2\alpha h_1}\Phi_7 - S_5A\Big]\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds \\
&\quad + x^T(t)\Big[-h_2e^{-2\alpha h_2}\Psi_7 - S_6A\Big]\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds + x^T(t)[-S_7A]\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds \\
&\quad + x^T[S_1 - S_8A]\dot{x}(t) + x^T(t-h_1)\Big[-h_1e^{-2\alpha h_1}\Phi_2 - S_2A\Big]x(t) + x^T(t-h_1)[-S_2D]x(t-h(t)) \\
&\quad + x^T(t-h_1)\Big[2e^{-2\alpha h_1}Q - h_1e^{-2\alpha h_1}\Phi_5 - (h_2-h_1)e^{-2\alpha h_2}\Omega_1\Big]x(t-h_1) \\
&\quad + x^T(t-h_1)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_4 - (0.5)(e^{2\alpha h_1} + e^{2\alpha h_2}Q)\Big]x(t-h_2) \\
&\quad + x^T(t-h_1)\Big[-h_1e^{-2\alpha h_1}\Phi_8\Big]\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds + x^T(t-h_1)S_2\dot{x}(t) \\
&\quad + x^T(t-h_1)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_7\Big]\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds \\
&\quad + x^T(t-h_2)\Big[-h_2e^{-2\alpha h_2}\Psi_2 - S_3A\Big]x(t) + x^T(t-h_2)[-S_3D]x(t-h(t)) \\
&\quad + x^T(t-h_2)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_2 - (0.5)(e^{2\alpha h_1} + e^{2\alpha h_2})Q\Big]x(t-h_1) \\
&\quad + x^T(t-h_2)\Big[-e^{-2\alpha h_2}Q - h_2e^{-2\alpha h_2}\Psi_5 - (h_2-h_1)e^{-2\alpha h_2}\Omega_5\Big]x(t-h_2) \\
&\quad + x^T(t-h_2)\Big[-h_2e^{-2\alpha h_2}\Psi_8\Big]\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds + x^T(t-h(t))\Big[PD - S_4A - S_1D\Big]x(t)
\end{aligned}$$

$$+x^T(t-h_2)\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_8\Big]\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds+x^T(t-h_2)[S_3]\dot{x}(t)$$

$$+x^T(t-h(t))[-S_2D]x(t-h_1)+x^T(t-h(t))[-S_3D]x(t-h_2)$$

$$+x^T(t-h(t))[-2S_4D]x(t-h(t))+x^T(t-h(t))[-S_5D]\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds$$

$$+x^T(t-h(t))[-S_6D]\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds+x^T(t-h(t))[-S_7D]\frac{1}{h_2-h_1}\int_{t-h}^{t-h_1}x(s)ds$$

$$+x^T(t-h(t))[S_4-S_8D]\dot{x}(t)+(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds)^T\Big[-h_1e^{-2\alpha h_1}\Phi_3-S_5A\Big]x(t)$$

$$+(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds)^T\Big[-h_1e^{-2\alpha h_1}\Phi_6\Big]x(t-h_1)+\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T[-S_5D]x(t-h(t))$$

$$+\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T\Big[-h_1e^{-2\alpha h_1}\Phi_9\Big]\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)$$

$$+\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T S_5\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)+\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T\Big[-h_2e^{-2\alpha h_2}\Psi_3-S_6A\Big]x(t)$$

$$+\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T\Big[-h_2e^{-2\alpha h_2}\Psi_6\Big]x(t-h_2)+\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T[-S_6D]x(t-h(t))$$

$$+\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T\Big[-h_2e^{-2\alpha h_2}\Psi_9\Big]\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)+\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T[S_6]\dot{x}(t)$$

$$+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T[-S_7]x(t)+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T[-S_7D]x(t-h(t))$$

$$+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T[-\Big(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big]x(t-h_1)$$

$$+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big]x(t-h_2)$$

$$+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T\Big[-(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big]\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)$$

$$+\dot{x}^T(t)[-S_8A+S_1]x(t)+\dot{x}^T(t)[S_2]x(t-h_1)+\dot{x}^T(t)[S_3]x(t-h_2)$$

$$+\dot{x}^T(t)[-S_8D+S_4]x(t-h(t))+\dot{x}^T(t)[S_2]x(t-h_1)+\dot{x}^T(t)S_5\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)$$

$$+\dot{x}^T(t)S_6\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)+\dot{x}^T(t)S_7\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)$$

$$+\dot{x}^T(t)\Big[(h_1^2+h_2^2)R+(h_2-h_1)^2U+S_8+S_8^T\Big]\dot{x}(t)+\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T[S_7]\dot{x}(t)$$

$= \xi^T(t)J\xi(t),$

where

$$\xi(t) = \left[x(t), x(t-h_2), x(t-h_2), x(t-h(t)), \frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds, \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds,\right.$$
$$\left.\frac{1}{h_2-h_2}\int_{t-h_2}^{t-h_1} x(s)ds, \dot{x}(t)\right].$$

According to condition (3.1), we obtain

$$\dot{V}(t,x_t) + 2\alpha V(t,x_t) \leq 0$$
$$\dot{V}(t,x_t) \leq -2\alpha V(t,x_t), \quad \forall t \in R^+. \tag{3.5}$$

From (3.5), we have

$\therefore V(t,x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \in R^+$

From $\lambda_{\min}\|x(t)\|^2 \leq V(t,x_t) \leq \lambda_{\max}\|x(t)\|^2$

$$\lambda_1\|x(t,\phi)\|^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t}\|\phi\|^2$$
$$\|x(t,\phi)\|^2 \leq \frac{\lambda_2 e^{-2\alpha t}\|\phi\|^2}{\lambda_1}$$
$$\leq \frac{\lambda_2}{\lambda_1}e^{-2\alpha t}\|\phi\|^2 \ .$$

Then,we have

$$\|x(t,\phi)\| \leq \sqrt{(\frac{\lambda_2}{\lambda_1})e^{-2\alpha t}\|\phi\|^2}$$
$$\leq \sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\alpha t}\|\phi\|, \quad t \in R^+.$$

From definition 2.1, we concludes that the zero equation of system(1) is α - exponentially stable. □

Next , we consider the following uncertain linear systems with interval time-varying delay:

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [D + \Delta D(t)]x(t-h(t)), \quad t \in R^+,$$
$$x(t) = \phi(t), t \in [h_2, 0] \tag{3.2}$$

where the time-varying uncertain matrices$\Delta A(t), \Delta D(t)$are given by :

$$\Delta A(t) = E_a F_a(t) H_a, \Delta D(t) = E_d F_d(t) H_d$$

and E_a, E_d, H_a, H_d are known constant matrices with appropriate dimensions, F_a, F_d are unknown uncertain matrices satisfying

$$F_a^T(t)F_a(t) \leq I, F_d^T(t)F_a(t) \leq I, t \in R^+.$$

The following notations will be used throughout this paper.

$M_{11} = A^TP + PA + 2Q - h_1e^{-2\alpha h_1}\Phi_1 - h_2e^{-2\alpha h_2}\Psi_1 + H_a^T H_a + H_a^T H_a,$

$M_{12} = -h_1e^{-2\alpha h_1}\Phi_4, M_{13} = -h_2e^{-2\alpha h_2}\Psi_4, M_{14} = J_{41} = PD + 0.5 + H_d^T H_d - \bar{S}_1A,$

$M_{15} = -h_1e^{-2\alpha h_1}\Phi_7, M_{16} = -h_2e^{-2\alpha h_2}\Psi_7, M_{17} = J_{71} = 0, M_{18} = J_{81} = -\bar{S}_2A,$

$M_{21} = -h_1e^{-2\alpha h_1}\Psi_2, M_{22} = e^{2\alpha h_1}Q - h_1e^{-2\alpha h_1}\Phi_5 - (h_2 - h_1)e^{-2\alpha h_2}\Omega_1,$

$M_{23} = -(h_2 - h_1)e^{-2\alpha h_2}\Omega_4, M_{24} = J_{42} = 0, M_{25} = -h_1e^{-2\alpha h_1}\Phi_8, M_{26} = J_{62} = 0,$

$M_{27} = -(h_2 - h_1)e^{-2\alpha h_2}\Omega_7, M_{28} = J_{82} = 0, M_{31} = -h_2e^{-2\alpha h_2}\Psi_2, M_{32} = -(h_2 - h_1)e^{-2\alpha h_2}\Omega_2,$

$M_{33} = -e^{-2\alpha h_2}Q - h_2e^{-2\alpha h_2}\Psi_5 - (h_2 - h_1)e^{-2\alpha h_2}\Omega_5, M_{34} = J_{43} = 0, M_{35} = J_{53} = 0,$

$M_{36} = -h_2e^{-2\alpha h_2}\Psi_8, M_{37} = -(h_2 - h_1)e^{-2\alpha h_2}\Omega_8, M_{38} = J_{83} = 0, M_{45} = J_{54} = 0,$

$M_{44} = -2S_4D = H_d^T H_d + H_d^T H_d + H_d^T H_d, M_{46} = J_{64} = 0, M_{47} = J_{74} = 0, M_{48} = J_{84} = \bar{S}_1,$

$M_{51} = -h_1e^{-2\alpha h_1}\Phi_3, M_{52} = -h_1e^{-2\alpha h_1}\Phi_5, M_{55} = -h_1e^{-2\alpha h_1}\Phi_9 M_{56} = J_{65} = 0, M_{78} = J_{87} = 0,$

$M_{57} = J_{75} = 0, M_{58} = J_{85} = 0, M_{61} = -h_2e^{-2\alpha h_2}\Psi_3, M_{63} = -h_2e^{-2\alpha h_2}\Psi_6, M_{66} = -h_2e^{-2\alpha h_2}\Psi_9,$

$M_{67} = J_{76} = 0, M_{68} = 0, M_{72} = -(h_2 - h_1)e^{-2\alpha h_2}\Omega_3, M_{73} = -(h_2 - h_1)e^{-2\alpha h_2}\Omega_6,$

$M_{77} = -(h_2 - h_1)e^{-2\alpha h_2}\Omega_9, M_{88} = (h_1^2 + h_2^2)R + (h_2 - h_1)^2U + \bar{S}_2 + \bar{S}_2^T + H_d^T H_d + H_d^T H_d.$

Theorem 3.2 Given $\alpha > 0$. The zero solution of the system (2) is α-exponentially stable if there exist symmetric positive definite matrices P, Q, R, U, and any matrices $\overline{S}_i, i = 1, 2$ such that the following LMI hold

$$M_1 = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & M_{16} & M_{17} & M_{18} \\ * & M_{22} & M_{23} & M_{24} & M_{25} & M_{26} & M_{27} & M_{28} \\ * & * & M_{33} & M_{34} & M_{35} & M_{36} & M_{37} & M_{38} \\ * & * & * & M_{44} & M_{45} & M_{46} & M_{47} & M_{48} \\ * & * & * & * & M_{55} & M_{56} & M_{57} & M_{58} \\ * & * & * & * & * & M_{66} & M_{67} & M_{68} \\ * & * & * & * & * & * & M_{77} & M_{78} \\ * & * & * & * & * & * & * & M_{88} \end{bmatrix} < 0, \tag{3.7}$$

$$\mathcal{M}_2 = \begin{bmatrix} 2PA & PE_a & PE_d & \bar{S}_1E_a & \bar{S}_2E_a \\ * & -I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{3.8}$$

$$\mathcal{M}_3 = \begin{bmatrix} -2\bar{S}_1D & \bar{S}_1E_d & \bar{S}_2E_d \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{3.9}$$

Proof We consider the following Lyapunov - Krasovskii functional for the system (2)

$$V(t,x_t)=\sum_{i=1}^{7}V_i\ .$$

By taking the derivative of V_1 along the solution of system (2) we have

$$\begin{aligned}\dot{V}_1 &= 2x^T(t)P\dot{x}(t)\\ &= x^T(t)[PA+A^TP]x(t)+x^T(t)[(PE_a)(PE_a)^T+H_a^TH_a]x(t)+x^T(t)[PD+D^TP]x(t-h(t))\\ &\quad +x^T(t)[(PE_d)(PE_d)^T+H_d^TH_d](t)x(t-h(t)) \end{aligned}\tag{3.10}$$

$$\begin{aligned}\dot{V}_2 &= \frac{d}{dt}\int_{t-h_1}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds\\ &= x^T(t)Qx(t)-e^{-2\alpha h_1}x^T(t-h_1)Qx(t-h_1)-2\alpha V_2\ , \end{aligned}\tag{3.11}$$

$$\begin{aligned}\dot{V}_3 &= \frac{d}{dt}\int_{t-h_2}^{t} e^{2\alpha(s-t)}x^T(s)Qx(s)ds\\ &= x^T(t)Qx(t)-e^{-2\alpha h_2}x^T(t-h_2)Qx(t-h_2)-2\alpha V_3\ , \end{aligned}\tag{3.12}$$

$$\begin{aligned}\dot{V}_4 &= \frac{d}{dt}\int_{t-h_2}^{t-h_1} e^{2\alpha(s-t)}x^T(s)Qx(s)ds\\ &= e^{-2\alpha h_1}\Big[x^T(t-h_1)Qx(t-h_1)-e^{2\alpha h_2}x^T(t-h_2)Qx(t-h_2)\Big]-2e^{-2\alpha}V_4, \end{aligned}\tag{3.13}$$

$$\begin{aligned}\dot{V}_5 &= \frac{d}{dt}\Big(h_1e^{-2\alpha t}\int_{-h_1}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau ds\Big)\\ &\le h_1^2\dot{x}^T(t)R\dot{x}(t)+x(t)\Phi_1h_1e^{-2\alpha h_1}x^T(t)+x(t-h_1)\Phi_4h_2e^{-2\alpha h_1}x^T(t)\\ &\quad +\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_7h_1e^{-2\alpha h_1}x^T(t)+x(t)\Phi_2h_1e^{-2\alpha h_1}x^T(t-h_1)\\ &\quad +x(t-h_1)\Phi_5h_1e^{-2\alpha h_1}x^T(t-h_1)+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_8h_1e^{-2\alpha h_1}x^T(t-h_1)\\ &\quad +x(t)\Phi_3h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T+x(t-h_1)\Phi_6h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T\\ &\quad +\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_9h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T-2\alpha V_5\ , \end{aligned}\tag{3.14}$$

$$\begin{aligned}\dot{V}_6 &= h_2e^{-2\alpha t}\int_{-h_2}^{0}\int_{t+s}^{t} e^{2\alpha\tau}\dot{x}^T(\tau)R\dot{x}(\tau)d\tau\ ds\\ &\le h_2^2\dot{x}^\tau(t)R\dot{x}(t)+x(t)\Psi_1h_2e^{-2\alpha h_2}x^T(t)+x(t-h_2)\Psi_4h_2e^{-2\alpha h_2}x^T(t)\\ &\quad +\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_7h_2e^{-2\alpha h_2}x^T(t)\ +x(t)\Psi_2h_2e^{-2\alpha h_2}x^T(t-h_2) \end{aligned}$$

$$+x(t-h_2)\Psi_5 h_2 e^{-2\alpha h_2} x^T(t-h_2) + \frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_8 h_2 e^{-2\alpha h_2} x^T(t-h_2)$$

$$+x(t)\Psi_3 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T + x(t-h_2)\Psi_6 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T$$

$$+\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds\Psi_9 h_2 e^{-2\alpha h_2}\frac{1}{h_2}\int_{t-h_2}^{t} x(s)ds^T - 2\alpha V_6\,, \tag{3.15}$$

$$\dot{V}_7 = (h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+s}^{t} e^{2\alpha\tau}e^{-2\alpha t}\dot{x}^T(\tau)U\dot{x}(\tau)d\tau ds$$

$$\le (h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t) + x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1)$$

$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1)$$

$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1)$$

$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2) + x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2)$$

$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2)$$

$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$

$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$

$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1} x(s)ds\Big)^T$$

$$-2\alpha V_7\,. \tag{3.16}$$

Hence , we that

$$\dot{V}(.) + 2\alpha V(.)$$

$$\le x^T(t)[PA + A^TP]x(t) + x^T(t)[(PE_a)(PE_a)^T + H_a^T H_a]x(t)$$

$$+x^T(t)[PD + D^TP]x(t-h(t)) + x^T(t)[(PE_d)(PE_d)^T + H_d^T H_d]x(t-h(t)) + x^TQx(t)$$

$$-e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_2)\Big) + e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_1)\Big) - e^{2\alpha h_2}x^T\Big(t-h_2Qx(t-h_2)\Big)$$

$$+h_1^2\dot{x}^T(t)R\dot{x}(t) + x(t)\Phi_1 h_1 e^{-2\alpha h_1}x^T(t) + x(t-h_1)\Phi_4 h_2 e^{-2\alpha h_1}x^T(t)$$

$$+\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_7 h_1 e^{-2\alpha h_1}x^T(t) + x(t)\Phi_2 h_1 e^{-2\alpha h_1}x^T(t-h_1)$$

$$+x(t-h_1)\Phi_5 h_1 e^{-2\alpha h_1}x^T(t-h_1) + \frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Phi_8 h_1 e^{-2\alpha h_1}x^T(t-h_1)$$

$$+x(t)\Phi_3 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T + x(t-h_1)\Phi_6 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t} x(s)ds\Big)^T$$

$$+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_9 h_1 e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_7 h_2 e^{-2\alpha h_2}x^T(t) \ +x(t)\Psi_2 h_2 e^{-2\alpha h_2}x^T(t-h_2)$$
$$+x(t-h_2)\Psi_5 h_2 e^{-2\alpha h_2}x^T(t-h_2)+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_8 h_2 e^{-2\alpha h_2}x^T(t-h_2)$$
$$+x(t)\Psi_3 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T+x(t-h_2)\Psi_6 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T$$
$$+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_9 h_2 e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4 x^T(t-h_1)$$
$$+(h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t)+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1 x^T(t-h_1)$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7 x^T(t-h_1)$$
$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2 x^T(t-h_2)+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5 x^T(t-h_2)$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8 x^T(t-h_2)$$
$$+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\tfrac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T$$
$$+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T$$
$$+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T \tag{3.17}$$

By using the following identity relation

$$\dot{x}(t)-[A-\Delta A(t)]x(t)+[D-\Delta D(t)]x(t-h(t))=0 \ ,$$

we have

$$-2x^T(t)S_1\dot{x}(t)+2x^T(t)S_1[A+\Delta A(t)]x(t)+2x^TS_1[D+\Delta D(t)]x(t-h(t))=0 \tag{3.18}$$

$$2x^T(t-h(t))S_4\dot{x}(t)-2x^T(t-h(t))S_4[A+\Delta A(t)]x(t)-2x^T(t-h(t))S_4[D+\Delta D(t)]x(t-h(t))=0 \tag{3.19}$$

$$2\dot{x}^T(t)S_8\dot{x}(t)-2\dot{x}^T(t)S_8[A+\Delta A(t)]x(t)-2\dot{x}^T(t)S_8[D+\Delta D(t)]x(t-h(t))=0 \tag{3.20}$$

From (3.17),(3.18),(3.19) and (3.20) , we have

$$\dot{V}+2\alpha V\leq x^T(t)[PA+A^TP]x(t)+x^T(t)[(PE_a)(PE_a)^T+H_a^TH_a]x(t)$$
$$+x^T(t)[PD+D^TP]x(t-h(t))+x^T(t)[(PE_d)(PE_d)^T+H_d^TH_d]x(t-h(t))+x^TQx(t)$$
$$-e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_2)\Big)+e^{2\alpha h_1}x^T\Big(t-h_1Qx(t-h_1)\Big)$$

$$
\begin{aligned}
&-e^{2\alpha h_2}x^T\Big(t-h_2Qx(t-h_2)\Big)+h_1^2\dot{x}^T(t)R\dot{x}(t)+x(t)\Phi_1h_1e^{-2\alpha h_1}x^T(t)\\
&+x(t-h_1)\Phi_4h_2e^{-2\alpha h_1}x^T(t)+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_7h_1e^{-2\alpha h_1}x^T(t)\\
&+x(t)\Phi_2h_1e^{-2\alpha h_1}x^T(t-h_1)\tfrac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_8h_1e^{-2\alpha h_1}x^T(t-h_1)\\
&+x(t-h_1)\Phi_5h_1e^{-2\alpha h_1}x^T(t-h_1)+x(t)\Phi_3h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T\\
&+x(t-h_1)\Phi_6h_1e^{-2\alpha h_1}\Big(\tfrac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T+\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Phi_9h_1e^{-2\alpha h_1}\Big(\frac{1}{h_1}\int_{t-h_1}^{t}x(s)ds\Big)^T\\
&+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_7h_2e^{-2\alpha h_2}x^T(t)\ +x(t)\Psi_2h_2e^{-2\alpha h_2}x^T(t-h_2)\\
&+x(t-h_2)\Psi_5h_2e^{-2\alpha h_2}x^T(t-h_2)+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_8h_2e^{-2\alpha h_2}x^T(t-h_2)\\
&+x(t)\Psi_3h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T+x(t-h_2)\Psi_6h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T\\
&+\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Psi_9h_2e^{-2\alpha h_2}\Big(\frac{1}{h_2}\int_{t-h_2}^{t}x(s)ds\Big)^T\\
&+(h_2-h_1)^2\dot{x}^T(t)U\dot{x}(t)+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_1x^T(t-h_1)\\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_4x^T(t-h_1)+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_2x^T(t-h_2)\\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_7x^T(t-h_1)\\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_5x^T(t-h_2)\\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_8x^T(t-h_2)\\
&+x(t-h_1)(h_2-h_1)e^{-2\alpha h_2}\Omega_3\Big(\tfrac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T\\
&+x(t-h_2)(h_2-h_1)e^{-2\alpha h_2}\Omega_6\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T\\
&+\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds(h_2-h_1)e^{-2\alpha h_2}\Omega_9\Big(\frac{1}{h_2-h_1}\int_{t-h_2}^{t-h_1}x(s)ds\Big)^T\\
&-2x^T(t)S_1\dot{x}(t)+2x^T(t)S_1[A+\Delta A(t)]x(t)+2x^TS_1[D+\Delta D(t)]x(t-h(t))\\
&+2x^T(t-h(t))S_4\dot{x}(t)-2x^T(t-h(t))S_4[A+\Delta A(t)]x(t)\\
&-2x^T(t-h(t))S_4[D+\Delta D(t)]x(t-h(t))+2\dot{x}^T(t)S_8\dot{x}(t)-2\dot{x}^T(t)S_8[A+\Delta A(t)]x(t)\\
&-2\dot{x}^T(t)S_8[D+\Delta D(t)]x(t-h(t))\ .\\
&=\xi^T(t)M_1\xi(t)
\end{aligned}
$$

Hence , we have

$$\dot{V}(t,x_t) - 2\alpha V(t,x_t) \leq \xi^T(t)M_1\xi(t) + x^T(t)M_2x(t) + x^T(t-h(t))M_3x(t-h(t)) \tag{3.21}$$

By using the similar approach as in Theorem 3.1 with taking $S_1 = P, S_2 = S_3 = S_5 = S_6 = S_7 = 0, S_4 = \bar{S}1, S_8 = \bar{S}2$, we obtain

$$2PA + PE_aE_a^TP + PE_aE_a^TP + PE_dE_d^TP + \bar{S}_1E_aE_a^T\bar{S}_1 + \bar{S}_2E_aE_a^T\bar{S}_2 < 0$$

By equivalent , we have

$$\mathcal{M}_2 = \begin{bmatrix} 2PA & PE_a & PE_d & \bar{S}_1E_a & \bar{S}_2E_a \\ * & -I & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{3.22}$$

$$-2\bar{S}_1D + \bar{S}_1E_dE_d^T\bar{S}_1 + \bar{S}_2E_dE_d^T\bar{S}_2 < 0$$

By equivalent , we have

$$\mathcal{M}_3 = \begin{bmatrix} -2\bar{S}_1D & \bar{S}_1E_d & \bar{S}_2E_d \\ * & -I & 0 \\ * & * & -I \end{bmatrix} < 0, \tag{3.23}$$

$$M_2 = 2PA + PE_aE_a^TP + PE_dE_d^TP + \bar{S}_1E_aE_a^T\bar{S}_1 + \bar{S}_2E_aE_a^T\bar{S}_2 < 0$$

$$M_3 = -2\bar{S}_1D + \bar{S}_1E_dE_d^T\bar{S}_1 + \bar{S}_2E_dE_d^T\bar{S}_2 < 0$$

From (3.21) , we have

$$\dot{V}(t,x_t) \leq -2\alpha V(t,x_t)$$

$$\frac{dV}{dt}(t,x_t) = -2\alpha V(t,x_t) \tag{3.25}$$

$$\therefore V(t,x_t) \leq V(\phi)e^{-2\alpha t}, \quad \forall t \in R^+$$

From $\quad \lambda_{\min}\|x(t)\|^2 \leq V(t,x_t) \leq \lambda_{\max}\|x(t)\|^2$

$$\lambda_1\|x(t,\phi)\|^2 \leq V(x_t) \leq V(\phi)e^{-2\alpha t} \leq \lambda_2 e^{-2\alpha t}\|\phi\|^2$$

$$\begin{aligned} \|x(t,\phi)\|^2 &\leq \frac{\lambda_2 e^{-2\alpha t}\|\phi\|^2}{\lambda_1} \\ &\leq \frac{\lambda_2}{\lambda_1}e^{-2\alpha t}\|\phi\|^2 \ . \end{aligned}$$

Then,we have

$$\begin{aligned} \|x(t,\phi)\| &\leq \sqrt{(\frac{\lambda_2}{\lambda_1})e^{-2\alpha t}\|\phi\|^2} \\ &\leq \sqrt{\frac{\lambda_2}{\lambda_1}}e^{-\alpha t}\|\phi\|, \quad t \in R^+. \end{aligned}$$

From definition 2.1 , we concludes that the zero equation of system(2) is α - exponentially stable. □

4 Numerical Examples

In the sequel, we illustrate the effectiveness of the proposed method which yields a computationally solution to the exponential stability and robust stability in the context of LMIs.
Example 4.1 Consider the linear system with interval time-varying delay(3.1), where

$$A = \begin{bmatrix} -12.0000 & 0.0000 \\ 0.0000 & -19.0000 \end{bmatrix}, D = \begin{bmatrix} -0.0002 & 0.004 \\ 0.003 & -0.0005 \end{bmatrix},$$

$$h(t) = 0.1 + 0.4 \mid \sin^2 t \mid .$$

It is worth noting that, the delay function $h(t)$ is non - differntiable. By using LMI Toolbox in MATLAB, the LMI (3.1) is feasible with $h_1 = 0.1000, h_2 = 0.5000, \alpha = 3.0000$ and

$$P = \begin{bmatrix} 2.2710 & 0.0097 \\ 0.0097 & 3.0316 \end{bmatrix} \cdot 10^7, \qquad Q = \begin{bmatrix} 0.8546 & -0.0011 \\ -0.0011 & 0.7895 \end{bmatrix} \cdot 10^{-13},$$

$$R = \begin{bmatrix} 2.2638 & 0.0001 \\ 0.0001 & 2.2525 \end{bmatrix} \cdot 10^8, \qquad U = \begin{bmatrix} 1.1778 & 0.0001 \\ 0.0001 & 1.1853 \end{bmatrix} \cdot 10^8,$$

$$S_1 = \begin{bmatrix} -6.6308 & 0.0158 \\ 0.0035 & -2.9118 \end{bmatrix} \cdot 10^7, S_2 = \begin{bmatrix} -1.0084 & -0.0005 \\ -0.0036 & 0.0188 \end{bmatrix},$$

$$S_3 = \begin{bmatrix} -1.2931 & 0.0020 \\ 0.0041 & -0.6819 \end{bmatrix}, \qquad S_4 = \begin{bmatrix} -0.1460 & 0.9255 \\ 2.9367 & -0.1563 \end{bmatrix} \cdot 10^4,$$

$$S_5 = \begin{bmatrix} -2.2921 & -0.0018 \\ -0.0066 & -1.5879 \end{bmatrix} \cdot 10^4, S_6 = \begin{bmatrix} 24.0437 & -0.0031 \\ -0.0031 & 15.6813 \end{bmatrix},$$

$$S_7 = \begin{bmatrix} 19.7610 & -0.0017 \\ -0.0017 & 12.8688 \end{bmatrix}, \qquad S_8 = \begin{bmatrix} 6.4856 & -0.0009 \\ -0.0009 & 4.2495 \end{bmatrix} \cdot 10^7,$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 7.2093 \cdot 10^{-13} \|\phi\|, \forall t \in R^+ .$$

The trajectory of the solution of system (1) in has been show in Figure 1.

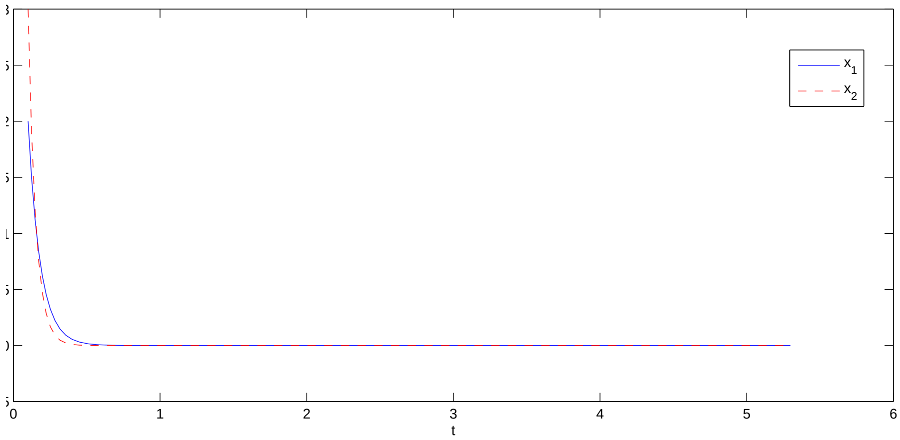

Figure 1: The trajectory of the solution of system (1) in Example 4.1 .

Table 1: Maximum allowable upper bounds h_2 of the time-varying delay for different values of the lower bounds h_1 .

Method	0.1	1.0
Zhang et al. (2016) [9]	4.7000	2.2000
Alexandre Seuret [1]	4.7100	2.2400
Hao-Tian Xu et al. [8]	4.6421	2.1630
Liu et al. [5]	4.4700	2.3820
Park et al. [6]	4.7800	2.4140
Lee et al. [4]	3.6400	2.4980
Theorem 3.1	4.8215	2.5546

Example 4.2 Consider the uncertain linear system with interval time-varying delay(2)with time delay function h(t) with $h_1 = 0.1000, h_2 = 3.1495$ and

$$A = \begin{bmatrix} 0.5 & 1 \\ -1 & -1 \end{bmatrix}, D = \begin{bmatrix} -1 & 0.3 \\ 0.2 & -0.5 \end{bmatrix},$$

$$Ha = \begin{bmatrix} 0.04 & 1 \\ -1 & -1 \end{bmatrix}, Hd = \begin{bmatrix} 0.04 & 1 \\ -1 & -1 \end{bmatrix},$$

$$Ea = \begin{bmatrix} -0.07 & 0.004 \\ 0.005 & 0.075 \end{bmatrix}, Ed = \begin{bmatrix} -0.045 & 0.002 \\ 0.001 & 0.04 \end{bmatrix}.$$

$$h(t) = 0.1 + 0.4 \mid \sin^2 t \mid .$$

By using LMI Toolbox in MATLAB, the LMI (3.2) of theorem 3.2 are feasible with $\alpha = 5$ and

$$P = \begin{bmatrix} 3.4056 & -1.8341 \\ -1.8341 & 7.1534 \end{bmatrix} \cdot 10^7, \qquad Q = \begin{bmatrix} 0.1700 & -0.0058 \\ -0.0058 & 0.1703 \end{bmatrix} \cdot 10^{-12},$$

$$R = \begin{bmatrix} 2.2195 & -0.0545 \\ -0.0545 & 2.4783 \end{bmatrix} \cdot 10^8, \qquad U = \begin{bmatrix} 1.0957 & 0.0195 \\ 0.0195 & 0.9966 \end{bmatrix} \cdot 10^8,$$

$$\bar{S}_1 = \begin{bmatrix} 0.0903 & -1.6926 \\ 0.6436 & -0.4598 \end{bmatrix} \cdot 10^8, \qquad \bar{S}_2 = \begin{bmatrix} 0.1525 & 1.3081 \\ 0.1319 & 0.6377 \end{bmatrix} \cdot 10^6.$$

Moreover, the solution $x(t, \phi)$ of the system satisfies

$$\|x(t, \phi)\| \leq 1.8041 \cdot 10^{-12} \|\phi\|, \forall t \in R^+ \ .$$

The trajectory of the solution of system (2) in has been show in Figure 2.

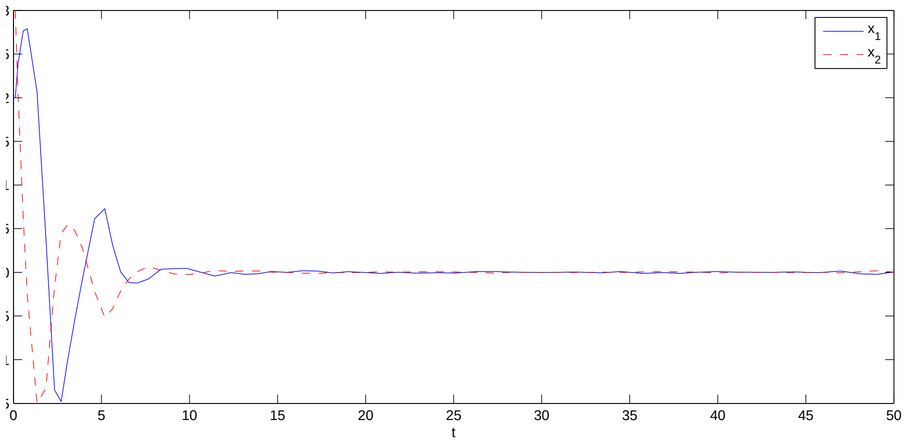

Figure 2: The trajectory of the solution of system (2) in Example 4.2 .

5 Conclusion

In this paper, new delay-dependent conditions for the exponential stability of linear systems with non-differentiable interval time-varying delay have been derived in terms of solutions of LMIs. By introducing a set of improved Lyapunov-Krasovskii functional and using Free matrix based integral inequality, the conditions for the exponential stability of the systems have been established . In the future work,the Free matrix-based integral inequality may be applied to stability analysis of other systems such as neural network system,fuzzy system and switched system.

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MATLAB CODE

MATLAB CODE

1. MATLAB CODE for finding solution of examples 1

```
A=[-12,0;0,-19]; D=[-0.0002,0.004;0.003,-0.0005];h1=0.1;h2=0.5;
e=2.71828;b=1;

setlmis([]);
  P=lmivar(1,[2,1]);
  Q=lmivar(1,[2,1]);
  R=lmivar(1,[2,1]);
  U=lmivar(1,[2,1]);
  S1=lmivar(2,[2,2]);
  S2=lmivar(2,[2,2]);
  S3=lmivar(2,[2,2]);
  S4=lmivar(2,[2,2]);
  S5=lmivar(2,[2,2]);
  S6=lmivar(1,[2,1]);
  S7=lmivar(1,[2,1]);
  S8=lmivar(1,[2,1]);
  M11=lmivar(1,[2,1]);
  M21=lmivar(2,[2,2]);
  M31=lmivar(2,[2,2]);
  M22=lmivar(1,[2,1]);
  M32=lmivar(2,[2,2]);
  M12=lmivar(2,[2,2]);
  M13=lmivar(2,[2,2]);
  M23=lmivar(2,[2,2]);
  M33=lmivar(1,[2,1]);
  N11=lmivar(1,[2,1]);
  N12=lmivar(2,[2,2]);
  N13=lmivar(2,[2,2]);
  N21=lmivar(2,[2,2]);
  N22=lmivar(1,[2,1]);
  N23=lmivar(2,[2,2]);
  N31=lmivar(2,[2,2]);
  N32=lmivar(2,[2,2]);
  N33=lmivar(1,[2,1]);
  y11=lmivar(2,[2,2]);
  y12=lmivar(2,[2,2]);
  y13=lmivar(2,[2,2]);
  y21=lmivar(2,[2,2]);
  y22=lmivar(2,[2,2]);
  y23=lmivar(2,[2,2]);
  y31=lmivar(2,[2,2]);
  y32=lmivar(2,[2,2]);
  y33=lmivar(2,[2,2]);
  w11=lmivar(2,[2,2]);
  w12=lmivar(2,[2,2]);
  w13=lmivar(2,[2,2]);
  w21=lmivar(2,[2,2]);
  w22=lmivar(2,[2,2]);
  w23=lmivar(2,[2,2]);
  w31=lmivar(2,[2,2]);
```

```
w32=lmivar(2,[2,2]);
w33=lmivar(2,[2,2]);

lmiterm([1 1 1 P],A',1,'s');      % LMI #1: A'*P+P*A
lmiterm([1 1 1 Q],.5*2,1,'s');      % LMI #1: 2*Q (NON SYMMETRIC?)
lmiterm([1 1 1 M11],.5*h1*e^(-2*b*h1)*h1,-1,'s'); % LMI #1: -h1*e^(-2*b*h1)*h1*M11 (NON SYMMETRIC?)
lmiterm([1 1 1 N11],.5*h1*e^(-2*b*h1)*(h1/3),-1,'s'); % LMI #1: -h1*e^(-2*b*h1)*(h1/3)*N11 (NON SYMMETRIC?)
lmiterm([1 1 1 y11],.5*h1*e^(-2*b*h1),-1,'s'); % LMI #1: -h1*e^(-2*b*h1)*y11 (NON SYMMETRIC?)
lmiterm([1 1 1 w11],.5*h1*e^(-2*b*h1),1,'s'); % LMI #1: h1*e^(-2*b*h1)*w11 (NON SYMMETRIC?)
lmiterm([1 1 1 M11],.5*h2*e^(-2*b*h2)*h2,-1,'s'); % LMI #1: -h2*e^(-2*b*h2)*h2*M11 (NON SYMMETRIC?)
lmiterm([1 1 1 N11],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s'); % LMI #1: -h2*e^(-2*b*h2)*(h2/3)*N11 (NON SYMMETRIC?)
lmiterm([1 1 1 y11],.5*h2*e^(-2*b*h2),-1,'s'); % LMI #1: -h2*e^(-2*b*h2)*y11 (NON SYMMETRIC?)
lmiterm([1 1 1 w11],.5*h2*e^(-2*b*h2),1,'s'); % LMI #1: h2*e^(-2*b*h2)*w11 (NON SYMMETRIC?)
lmiterm([1 1 1 S1],1,-A,'s'); % LMI #1: -S1*A-A'*S1'
lmiterm([1 2 1 M12],h1*e^(-2*b*h1)*h1,-1); % LMI #1: -h1*e^(-2*b*h1)*h1*M12
lmiterm([1 2 1 N12],h1*e^(-2*b*h1)*(h1/3),-1); % LMI #1: -h1*e^(-2*b*h1)*(h1/3)*N12
lmiterm([1 2 1 y11],h1*e^(-2*b*h1),1); % LMI #1: h1*e^(-2*b*h1)*y11
lmiterm([1 2 1 w11],h1*e^(-2*b*h1),1); % LMI #1: h1*e^(-2*b*h1)*w11
lmiterm([1 2 1 S2],1,-A); % LMI #1: -S2*A
lmiterm([1 2 1 -M21],h1*e^(-2*b*h1)*h1,-1); % LMI #1: -h1*e^(-2*b*h1)*h1*M21'
lmiterm([1 2 1 -N21],h1*e^(-2*b*h1)*(h1/3),-1); % LMI #1: -h1*e^(-2*b*h1)*(h1/3)*N21'
lmiterm([1 2 1 -y21],h1*e^(-2*b*h1),-1); % LMI #1: -h1*e^(-2*b*h1)*y21'
lmiterm([1 2 1 -w21],h1*e^(-2*b*h1),1); % LMI #1: h1*e^(-2*b*h1)*w21'
lmiterm([1 2 2 Q],.5*e^(2*b*h1),1,'s'); % LMI #1: e^(2*b*h1)*Q (NON SYMMETRIC?)
lmiterm([1 2 2 M22],.5*h1*e^(-2*b*h1)*h1,-1,'s'); % LMI #1: -h1*e^(-2*b*h1)*h1*M22 (NON SYMMETRIC?)
lmiterm([1 2 2 N22],.5*h1*e^(-2*b*h1)*(h1/3),-1,'s'); % LMI #1: -h1*e^(-2*b*h1)*(h1/3)*N22 (NON SYMMETRIC?)
lmiterm([1 2 2 y21],.5*0.5*h1*e^(-2*b*h1),-1,'s');      % LMI #1: -0.5*h1*e^(-2*b*h1)*y21 (NON SYMMETRIC?)
lmiterm([1 2 2 -y21],.5*0.5*h1*e^(-2*b*h1),-1,'s');      % LMI #1: -0.5*h1*e^(-2*b*h1)*y21' (NON SYMMETRIC?)
lmiterm([1 2 2 w21],.5*0.5*h1*e^(-2*b*h1),1,'s');      % LMI #1: 0.5*h1*e^(-2*b*h1)*w21 (NON SYMMETRIC?)
lmiterm([1 2 2 -w21],.5*0.5*h1*e^(-2*b*h1),1,'s');      % LMI #1: 0.5*h1*e^(-2*b*h1)*w21' (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h2*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -h2*e^(-2*b*h2)*h2*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h2*e^(-2*b*h2)*h1,1,'s');      % LMI #1: h2*e^(-2*b*h2)*h1*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -h2*e^(-2*b*h2)*(h2/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h2*e^(-2*b*h2)*(h1/3),1,'s');      % LMI #1:
```

```
h2*e^(-2*b*h2)*(h1/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 y11],.5*h2*e^(-2*b*h2),-1,'s');      % LMI #1: -h2*e^(-
2*b*h2)*y11 (NON SYMMETRIC?)
lmiterm([1 2 2 w11],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*w11 (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h1*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*h2*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h1*e^(-2*b*h2)*h1,1,'s');      % LMI #1: h1*e^(-
2*b*h2)*h1*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h1*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*(h2/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h1*e^(-2*b*h2)*(h1/3),1,'s');      % LMI #1:
h1*e^(-2*b*h2)*(h1/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 y11],.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -h1*e^(-
2*b*h2)*y11 (NON SYMMETRIC?)
lmiterm([1 2 2 w11],.5*h1*e^(-2*b*h2),1,'s');      % LMI #1: h1*e^(-
2*b*h2)*w11 (NON SYMMETRIC?)
lmiterm([1 3 1 M12],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M12
lmiterm([1 3 1 N12],h2*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h1/3)*N12
lmiterm([1 3 1 y11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*y11
lmiterm([1 3 1 w11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*w11
lmiterm([1 3 1 S3],1,-A);                        % LMI #1: -S3*A
lmiterm([1 3 1 -M21],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M21'
lmiterm([1 3 1 -N21],h2*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h1/3)*N21'
lmiterm([1 3 1 -y21],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*y21'
lmiterm([1 3 1 -w21],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w21'
lmiterm([1 3 2 M12],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M12
lmiterm([1 3 2 M12],h2*e^(-2*b*h2)*h1,1);        % LMI #1: h2*e^(-
2*b*h2)*h1*M12
lmiterm([1 3 2 N12],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N12
lmiterm([1 3 2 N12],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N12
lmiterm([1 3 2 y11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*y11
lmiterm([1 3 2 w11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*w11
lmiterm([1 3 2 M11],h1*e^(-2*b*h2)*h2,-1);       % LMI #1: -h1*e^(-
2*b*h2)*h2*M11
lmiterm([1 3 2 M11],h1*e^(-2*b*h2)*h1,-1);       % LMI #1: -h1*e^(-
2*b*h2)*h1*M11
lmiterm([1 3 2 N12],h1*e^(-2*b*h2)*(h2/3),1);      % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N12
lmiterm([1 3 2 N12],h1*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N12
lmiterm([1 3 2 y11],h1*e^(-2*b*h2),-1);          % LMI #1: -h1*e^(-
2*b*h2)*y11
lmiterm([1 3 2 w11],h1*e^(-2*b*h2),-1);          % LMI #1: -h1*e^(-
2*b*h2)*w11
```

```
lmiterm([1 3 2 -M12],h2*e^(-2*b*h2)*h1,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h1*M12'
lmiterm([1 3 2 -M21],h2*e^(-2*b*h2)*h1,1);       % LMI #1: h2*e^(-
2*b*h2)*h1*M21'
lmiterm([1 3 2 -N21],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N21'
lmiterm([1 3 2 -N21],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N21'
lmiterm([1 3 2 -y21],h2*e^(-2*b*h2),-1);         % LMI #1: -h2*e^(-
2*b*h2)*y21'
lmiterm([1 3 2 -w21],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w21'
lmiterm([1 3 2 -M21],h1*e^(-2*b*h2)*h2,-1);      % LMI #1: -h1*e^(-
2*b*h2)*h2*M21'
lmiterm([1 3 2 -M21],h1*e^(-2*b*h2)*h1,1);       % LMI #1: h1*e^(-
2*b*h2)*h1*M21'
lmiterm([1 3 2 -N21],h1*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h2/3)*N21'
lmiterm([1 3 2 -N21],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N21'
lmiterm([1 3 2 -y21],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*y21'
lmiterm([1 3 2 -w21],h1*e^(-2*b*h2),1);          % LMI #1: h1*e^(-
2*b*h2)*w21'
lmiterm([1 3 2 Q],0.5*e^(2*b*h1),-1);            % LMI #1: -
0.5*e^(2*b*h1)*Q
lmiterm([1 3 2 Q],0.5*e^(2*b*h2),-1);            % LMI #1: -
0.5*e^(2*b*h2)*Q
lmiterm([1 3 3 Q],.5*e^(-2*b*h2),-1,'s');        % LMI #1: -e^(-
2*b*h2)*Q (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h2*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*h2*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 y21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*y21 (NON SYMMETRIC?)
lmiterm([1 3 3 w21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*w21 (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h2*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*h2*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h2*e^(-2*b*h2)*h1,1,'s');      % LMI #1: h2*e^(-
2*b*h2)*h1*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h2*e^(-2*b*h2)*(h1/3),1,'s');      % LMI #1:
h2*e^(-2*b*h2)*(h1/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 -y21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*y21' (NON SYMMETRIC?)
lmiterm([1 3 3 -w21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*w21' (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h1*e^(-2*b*h2)*h2,1,'s');      % LMI #1: h1*e^(-
2*b*h2)*h2*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h1*e^(-2*b*h2)*h1,-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*h1*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h1*e^(-2*b*h2)*(h2/3),1,'s');      % LMI #1:
h1*e^(-2*b*h2)*(h2/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h1*e^(-2*b*h2)*(h1/3),-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*(h1/3)*N22 (NON SYMMETRIC?)
```

```
lmiterm([1 3 3 y21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*y21 (NON SYMMETRIC?)
lmiterm([1 3 3 -y21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*y21' (NON SYMMETRIC?)
lmiterm([1 3 3 w21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*w21 (NON SYMMETRIC?)
lmiterm([1 3 3 -w21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*w21' (NON SYMMETRIC?)
lmiterm([1 4 1 P],0.5*D',1);                            % LMI #1: 0.5*D'*P
lmiterm([1 4 1 P],0.5,D);                               % LMI #1: 0.5*P*D
lmiterm([1 4 1 -S1],D',-1);                             % LMI #1: -D'*S1'
lmiterm([1 4 1 S4],1,-A);                               % LMI #1: -S4*A
lmiterm([1 4 2 -S2],D',-1);                             % LMI #1: -D'*S2'
lmiterm([1 4 3 -S3],D',-1);                             % LMI #1: -D'*S3'
lmiterm([1 4 4 S4],1,-D,'s');                           % LMI #1: -S4*D-D'*S4'
lmiterm([1 5 1 M13],h1*e^(-2*b*h1)*h1,-1);              % LMI #1: -h1*e^(-
2*b*h1)*h1*M13
lmiterm([1 5 1 N13],h1*e^(-2*b*h1)*(h1/3),-1);          % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N13
lmiterm([1 5 1 w11],h1*e^(-2*b*h1)*2,-1);               % LMI #1: -h1*e^(-
2*b*h1)*2*w11
lmiterm([1 5 1 S5],1,-A);                               % LMI #1: -S5*A
lmiterm([1 5 1 -M31],h1*e^(-2*b*h1)*h1,-1);             % LMI #1: -h1*e^(-
2*b*h1)*h1*M31'
lmiterm([1 5 1 -N31],h1*e^(-2*b*h1)*(h1/3),-1);          % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N31'
lmiterm([1 5 1 -y31],h1*e^(-2*b*h1),-1);                % LMI #1: -h1*e^(-
2*b*h1)*y31'
lmiterm([1 5 1 -w31],h1*e^(-2*b*h1),1);                 % LMI #1: h1*e^(-
2*b*h1)*w31'
lmiterm([1 5 2 M23],h1*e^(-2*b*h1)*h1,-1);              % LMI #1: -h1*e^(-
2*b*h1)*h1*M23
lmiterm([1 5 2 N23],h1*e^(-2*b*h1)*(h1/3),-1);          % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N23
lmiterm([1 5 2 w21],h1*e^(-2*b*h1)*2,-1);               % LMI #1: -h1*e^(-
2*b*h1)*2*w21
lmiterm([1 5 2 -M32],h1*e^(-2*b*h1)*h1,-1);             % LMI #1: -h1*e^(-
2*b*h1)*h1*M32'
lmiterm([1 5 2 -N32],h1*e^(-2*b*h1)*(h1/3),-1);          % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N32'
lmiterm([1 5 2 -y31],h1*e^(-2*b*h1),1);                 % LMI #1: h1*e^(-
2*b*h1)*y31'
lmiterm([1 5 2 -w31],h1*e^(-2*b*h1),1);                 % LMI #1: h1*e^(-
2*b*h1)*w31'
lmiterm([1 5 3 0],O);                                   % LMI #1: O
lmiterm([1 5 4 S5],1,-D);                               % LMI #1: -S5*D
lmiterm([1 5 5 M33],.5*h1*e^(-2*b*h1)*h1,-1,'s');       % LMI #1: -
h1*e^(-2*b*h1)*h1*M33 (NON SYMMETRIC?)
lmiterm([1 5 5 N33],.5*h1*e^(-2*b*h1)*(h1/3),-1,'s');      % LMI #1: -
h1*e^(-2*b*h1)*(h1/3)*N33 (NON SYMMETRIC?)
lmiterm([1 5 5 w31],.5*h1*e^(-2*b*h1),-1,'s');          % LMI #1: -h1*e^(-
2*b*h1)*w31 (NON SYMMETRIC?)
lmiterm([1 5 5 -w31],.5*h1*e^(-2*b*h1),-1,'s');          % LMI #1: -h1*e^(-
2*b*h1)*w31' (NON SYMMETRIC?)
lmiterm([1 6 1 M13],h2*e^(-2*b*h2)*h2,-1);              % LMI #1: -h2*e^(-
2*b*h2)*h2*M13
lmiterm([1 6 1 N13],h2*e^(-2*b*h2)*(h2/3),-1);          % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N13
```

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lmiterm([1 6 1 w11],h2*e^(-2*b*h2)*2,-1);        % LMI #1: -h2*e^(-
2*b*h2)*2*w11
lmiterm([1 6 1 S6],1,-A);                        % LMI #1: -S6*A
lmiterm([1 6 1 -M31],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M31'
lmiterm([1 6 1 -N31],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N31'
lmiterm([1 6 1 -y31],h2*e^(-2*b*h2),-1);         % LMI #1: -h2*e^(-
2*b*h2)*y31'
lmiterm([1 6 1 -w31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w31'
lmiterm([1 6 2 0],0);                            % LMI #1: 0
lmiterm([1 6 3 M23],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M23
lmiterm([1 6 3 N23],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N23
lmiterm([1 6 3 w11],h2*e^(-2*b*h2)*2,-1);        % LMI #1: -h2*e^(-
2*b*h2)*2*w11
lmiterm([1 6 3 -M32],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M32'
lmiterm([1 6 3 -N32],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N32'
lmiterm([1 6 3 -y31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*y31'
lmiterm([1 6 3 w31],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*w31
lmiterm([1 6 4 S6],1,-D);                        % LMI #1: -S6*D
lmiterm([1 6 5 0],0);                            % LMI #1: 0
lmiterm([1 6 6 M33],.5*h2*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*h2*M33 (NON SYMMETRIC?)
lmiterm([1 6 6 N33],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N33 (NON SYMMETRIC?)
lmiterm([1 6 6 w31],.5*h2*e^(-2*b*h2),-1,'s');      % LMI #1: -h2*e^(-
2*b*h2)*w31 (NON SYMMETRIC?)
lmiterm([1 6 6 -w31],.5*h2*e^(-2*b*h2),-1,'s');      % LMI #1: -h2*e^(-
2*b*h2)*w31' (NON SYMMETRIC?)
lmiterm([1 7 1 S7],1,-A);                        % LMI #1: -S7*A
lmiterm([1 7 2 M13],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M13
lmiterm([1 7 2 M13],h2*e^(-2*b*h2)*h1,1);        % LMI #1: h2*e^(-
2*b*h2)*h1*M13
lmiterm([1 7 2 N13],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N13
lmiterm([1 7 2 N13],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N13
lmiterm([1 7 2 w11],h2*e^(-2*b*h2)*2,-1);        % LMI #1: -h2*e^(-
2*b*h2)*2*w11
lmiterm([1 7 2 M13],h1*e^(-2*b*h2)*h2,1);        % LMI #1: h1*e^(-
2*b*h2)*h2*M13
lmiterm([1 7 2 M13],h1*e^(-2*b*h2),-1);          % LMI #1: -h1*e^(-
2*b*h2)*M13
lmiterm([1 7 2 N13],h1*e^(-2*b*h2)*(h2/3),1);      % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N13
lmiterm([1 7 2 N13],h1*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N13
lmiterm([1 7 2 w11],h1*e^(-2*b*h2)*2,1);         % LMI #1: h1*e^(-
2*b*h2)*2*w11
lmiterm([1 7 2 -M31],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
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2*b*h2)*h2*M31'
lmiterm([1 7 2 -M31],h2*e^(-2*b*h2)*h1,1);       % LMI #1: h2*e^(-
2*b*h2)*h1*M31'
lmiterm([1 7 2 -N31],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N31'
lmiterm([1 7 2 -N31],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N31'
lmiterm([1 7 2 -y31],h2*e^(-2*b*h2),-1);         % LMI #1: -h2*e^(-
2*b*h2)*y31'
lmiterm([1 7 2 -w31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w31'
lmiterm([1 7 2 -M31],h1*e^(-2*b*h2)*h2,1);       % LMI #1: h1*e^(-
2*b*h2)*h2*M31'
lmiterm([1 7 2 -M31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*M31'
lmiterm([1 7 2 -N31],h1*e^(-2*b*h2)*(h2/3),1);      % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N31'
lmiterm([1 7 2 -N31],h1*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N31'
lmiterm([1 7 2 -y31],h1*e^(-2*b*h2),1);          % LMI #1: h1*e^(-
2*b*h2)*y31'
lmiterm([1 7 2 -w31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*w31'
lmiterm([1 7 3 M23],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M23
lmiterm([1 7 3 M23],h2*e^(-2*b*h2)*h1,1);        % LMI #1: h2*e^(-
2*b*h2)*h1*M23
lmiterm([1 7 3 N23],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N23
lmiterm([1 7 3 N23],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N23
lmiterm([1 7 3 w21],h2*e^(-2*b*h2)*2,-1);        % LMI #1: -h2*e^(-
2*b*h2)*2*w21
lmiterm([1 7 3 M23],h1*e^(-2*b*h2)*h2,1);        % LMI #1: h1*e^(-
2*b*h2)*h2*M23
lmiterm([1 7 3 M23],h1*e^(-2*b*h2)*h1,-1);       % LMI #1: -h1*e^(-
2*b*h2)*h1*M23
lmiterm([1 7 3 N23],h1*e^(-2*b*h2)*(h2/3),1);      % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N23
lmiterm([1 7 3 N23],h1*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N23
lmiterm([1 7 3 w21],h1*e^(-2*b*h2)*2,1);         % LMI #1: h1*e^(-
2*b*h2)*2*w21
lmiterm([1 7 3 -M32],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M32'
lmiterm([1 7 3 -M32],h2*e^(-2*b*h2)*h1,1);       % LMI #1: h2*e^(-
2*b*h2)*h1*M32'
lmiterm([1 7 3 -N32],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N32'
lmiterm([1 7 3 -N32],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N32'
lmiterm([1 7 3 -y31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*y31'
lmiterm([1 7 3 -w31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w31'
lmiterm([1 7 3 -M32],h1*e^(-2*b*h2)*h2,1);       % LMI #1: h1*e^(-
2*b*h2)*h2*M32'
lmiterm([1 7 3 -M32],h1*e^(-2*b*h2)*h1,-1);      % LMI #1: -h1*e^(-
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```
2*b*h2)*h1*M32'
lmiterm([1 7 3 -N32],h1*e^(-2*b*h2)*(h2/3),1);     % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N32'
lmiterm([1 7 3 -N32],h1*e^(-2*b*h2)*(h1/3),-1);     % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N32'
lmiterm([1 7 3 -y31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*y31'
lmiterm([1 7 3 -w31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*w31'
lmiterm([1 7 4 S7],1,-D);                        % LMI #1: -S7*D
lmiterm([1 7 5 0],O);                            % LMI #1: O
lmiterm([1 7 6 0],O);                            % LMI #1: O
lmiterm([1 7 7 M33],.5*h2*e^(-2*b*h2)*h2,-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*h2*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 M33],.5*h2*e^(-2*b*h2)*h1,1,'s');     % LMI #1: h2*e^(-
2*b*h2)*h1*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h2*e^(-2*b*h2)*(h1/3),1,'s');     % LMI #1:
h2*e^(-2*b*h2)*(h1/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 w31],.5*h2*e^(-2*b*h2),-1,'s');     % LMI #1: -h2*e^(-
2*b*h2)*w31 (NON SYMMETRIC?)
lmiterm([1 7 7 -w31],.5*h2*e^(-2*b*h2),-1,'s');     % LMI #1: -h2*e^(-
2*b*h2)*w31' (NON SYMMETRIC?)
lmiterm([1 7 7 M33],.5*h1*e^(-2*b*h2)*h2,1,'s');     % LMI #1: h1*e^(-
2*b*h2)*h2*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 M33],.5*h1*e^(-2*b*h2)*h1,-1,'s');     % LMI #1: -
h1*e^(-2*b*h2)*h1*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h1*e^(-2*b*h2)*(h2/3),1,'s');     % LMI #1:
h1*e^(-2*b*h2)*(h2/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h1*e^(-2*b*h2)*(h1/3),-1,'s');     % LMI #1: -
h1*e^(-2*b*h2)*(h1/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 w31],.5*h1*e^(-2*b*h2),1,'s');     % LMI #1: h1*e^(-
2*b*h2)*w31 (NON SYMMETRIC?)
lmiterm([1 7 7 -w31],.5*h1*e^(-2*b*h2),1,'s');     % LMI #1: h1*e^(-
2*b*h2)*w31' (NON SYMMETRIC?)
lmiterm([1 8 1 -S1],1,1);                        % LMI #1: S1'
lmiterm([1 8 1 S8],1,-A);                        % LMI #1: -S8*A
lmiterm([1 8 2 -S2],1,1);                        % LMI #1: S2'
lmiterm([1 8 3 -S3],1,1);                        % LMI #1: S3'
lmiterm([1 8 4 -S4],1,1);                        % LMI #1: S4'
lmiterm([1 8 4 S8],1,-D);                        % LMI #1: -S8*D
lmiterm([1 8 5 -S5],1,1);                        % LMI #1: S5'
lmiterm([1 8 6 S6],1,1);                         % LMI #1: S6
lmiterm([1 8 7 S7],1,1);                         % LMI #1: S7
lmiterm([1 8 8 R],.5*h1^2,1,'s');                % LMI #1: h1^2*R (NON
SYMMETRIC?)
lmiterm([1 8 8 R],.5*h2^2,-1,'s');               % LMI #1: -h2^2*R (NON
SYMMETRIC?)
lmiterm([1 8 8 U],.5*h2^2,1,'s');                % LMI #1: h2^2*U (NON
SYMMETRIC?)
lmiterm([1 8 8 U],.5*2*h2*h1,1,'s');             % LMI #1: 2*h2*h1*U
(NON SYMMETRIC?)
lmiterm([1 8 8 U],.5*h1^2,-1,'s');               % LMI #1: -h1^2*U (NON
SYMMETRIC?)
lmiterm([1 8 8 S8],1,1,'s');                     % LMI #1: S8+S8'
```

```
lmiterm([-2 1 1 P],1,1);                       % LMI #2: P

lmiterm([-3 1 1 Q],1,1);                       % LMI #3: Q

lmiterm([-4 1 1 R],1,1);                       % LMI #4: R

lmiterm([-5 1 1 U],1,1);                       % LMI #5: U

lmiterm([-6 1 1 S6],1,1);                      % LMI #6: S6

lmiterm([-7 1 1 S7],1,1);                      % LMI #7: S7

lmiterm([-8 1 1 S8],1,1);                      % LMI #8: S8

lmiterm([-9 1 1 M11],1,1);                     % LMI #9: M11

lmiterm([-10 1 1 M22],1,1);                    % LMI #10: M22

lmiterm([-11 1 1 M33],1,1);                    % LMI #11: M33

lmiterm([-12 1 1 N11],1,1);                    % LMI #12: N11

lmiterm([-13 1 1 N22],1,1);                    % LMI #13: N22

lmiterm([-14 1 1 N33],1,1);                    % LMI #14: N33

t=getlmis;

[tmin,xfeas]=feasp(t)
P=dec2mat(t,xfeas,P);
Q =dec2mat(t,xfeas,Q);
R =dec2mat(t,xfeas,R);
U =dec2mat(t,xfeas,U);
S1=dec2mat(t,xfeas,S1);
S2=dec2mat(t,xfeas,S2);
S3=dec2mat(t,xfeas,S3);
S4=dec2mat(t,xfeas,S5);
S6=dec2mat(t,xfeas,S6);
S7=dec2mat(t,xfeas,S7);
S8=dec2mat(t,xfeas,S8);
M11=dec2mat(t,xfeas,M11);
M21=dec2mat(t,xfeas,M21);
M31=dec2mat(t,xfeas,M31);
M22=dec2mat(t,xfeas,M22);
M32=dec2mat(t,xfeas,M32);
M12=dec2mat(t,xfeas,M12);
M13=dec2mat(t,xfeas,M13);
M23=dec2mat(t,xfeas,M23);
M33=dec2mat(t,xfeas,M33);
N11=dec2mat(t,xfeas,N11);
N12=dec2mat(t,xfeas,N12);
N13=dec2mat(t,xfeas,N13);
N21=dec2mat(t,xfeas,N21);
N22=dec2mat(t,xfeas,N22);
N23=dec2mat(t,xfeas,N23);
```

```
N31=dec2mat(t,xfeas,N31);
N32=dec2mat(t,xfeas,N32);
N33=dec2mat(t,xfeas,N33);
y11=dec2mat(t,xfeas,y11);
y12=dec2mat(t,xfeas,y12);
y13=dec2mat(t,xfeas,y13);
y21=dec2mat(t,xfeas,y21);
y22=dec2mat(t,xfeas,y22);
y23=dec2mat(t,xfeas,y23);
y31=dec2mat(t,xfeas,y31);
y32=dec2mat(t,xfeas,y32);
y33=dec2mat(t,xfeas,y33);
w11=dec2mat(t,xfeas,w11);
w12=dec2mat(t,xfeas,w12);
w13=dec2mat(t,xfeas,w13);
w21=dec2mat(t,xfeas,w21);
w22=dec2mat(t,xfeas,w22);
w23=dec2mat(t,xfeas,w23);
w31=dec2mat(t,xfeas,w31);
w32=dec2mat(t,xfeas,w32);
w33=dec2mat(t,xfeas,w33);
tmin
```

2. MATLAB CODE for finding solution of examples 2

```
A=[0.5,1;-1,-1]
 D=[-1,0.3;0.2,-0.5]
 O=[0,0;0,0]
 h1=0.1
 h2=3.1495
b=5
e=2.7182
Ha=[0.04,1;-1,-1]
Hd=[0.03,-0.002;0.001,0.06]
Ea=[-0.07,0.004;0.005,0.075]
Ed=[-0.045,0.002;0.001,0.04]

setlmis([]);
   P=lmivar(1,[2,1]);
   Q=lmivar(1,[2,1]);
   R=lmivar(1,[2,1]);
   U=lmivar(1,[2,1]);
   S1=lmivar(2,[2,2]);
   S2=lmivar(2,[2,2]);
   S3=lmivar(2,[2,2]);
   S4=lmivar(2,[2,2]);
   S5=lmivar(2,[2,2]);
   S6=lmivar(1,[2,1]);
   S7=lmivar(1,[2,1]);
   S8=lmivar(1,[2,1]);
   M11=lmivar(1,[2,1]);
   M21=lmivar(2,[2,2]);
   M31=lmivar(2,[2,2]);
   M22=lmivar(1,[2,1]);
   M32=lmivar(2,[2,2]);
   M12=lmivar(2,[2,2]);
   M13=lmivar(2,[2,2]);
   M23=lmivar(2,[2,2]);
   M33=lmivar(1,[2,1]);
   N11=lmivar(1,[2,1]);
   N12=lmivar(2,[2,2]);
   N13=lmivar(2,[2,2]);
   N21=lmivar(2,[2,2]);
   N22=lmivar(1,[2,1]);
   N23=lmivar(2,[2,2]);
   N31=lmivar(2,[2,2]);
   N32=lmivar(2,[2,2]);
   N33=lmivar(1,[2,1]);
   y11=lmivar(2,[2,2]);
   y12=lmivar(2,[2,2]);
   y13=lmivar(2,[2,2]);
   y21=lmivar(2,[2,2]);
   y22=lmivar(2,[2,2]);
   y23=lmivar(2,[2,2]);
   y31=lmivar(2,[2,2]);
   y32=lmivar(2,[2,2]);
   y33=lmivar(2,[2,2]);
   w11=lmivar(2,[2,2]);
   w12=lmivar(2,[2,2]);
   w13=lmivar(2,[2,2]);
```

```
w21=lmivar(2,[2,2]);
w22=lmivar(2,[2,2]);
w23=lmivar(2,[2,2]);
w31=lmivar(2,[2,2]);
w32=lmivar(2,[2,2]);
w33=lmivar(2,[2,2]);

lmiterm([1 1 1 P],A',1,'s');                          % LMI #1: A'*P+P*A
lmiterm([1 1 1 Q],.5*2,1,'s');                        % LMI #1: 2*Q (NON
SYMMETRIC?)
lmiterm([1 1 1 M11],.5*h1*e^(-2*b*h1)*h1,-1,'s');     % LMI #1: -
h1*e^(-2*b*h1)*h1*M11 (NON SYMMETRIC?)
lmiterm([1 1 1 N11],.5*h1*e^(-2*b*h1)*(h1/3),-1,'s');     % LMI #1: -
h1*e^(-2*b*h1)*(h1/3)*N11 (NON SYMMETRIC?)
lmiterm([1 1 1 y11],.5*h1*e^(-2*b*h1),-1,'s');     % LMI #1: -h1*e^(-
2*b*h1)*y11 (NON SYMMETRIC?)
lmiterm([1 1 1 w11],.5*h1*e^(-2*b*h1),1,'s');     % LMI #1: h1*e^(-
2*b*h1)*w11 (NON SYMMETRIC?)
lmiterm([1 1 1 M11],.5*h2*e^(-2*b*h2)*h2,-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*h2*M11 (NON SYMMETRIC?)
lmiterm([1 1 1 N11],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N11 (NON SYMMETRIC?)
lmiterm([1 1 1 y11],.5*h2*e^(-2*b*h2),-1,'s');     % LMI #1: -h2*e^(-
2*b*h2)*y11 (NON SYMMETRIC?)
lmiterm([1 1 1 w11],.5*h2*e^(-2*b*h2),1,'s');     % LMI #1: h2*e^(-
2*b*h2)*w11 (NON SYMMETRIC?)
lmiterm([1 1 1 S1],1,-A,'s');                         % LMI #1: -S1*A-A'*S1'
lmiterm([1 2 1 M12],h1*e^(-2*b*h1)*h1,-1);           % LMI #1: -h1*e^(-
2*b*h1)*h1*M12
lmiterm([1 2 1 N12],h1*e^(-2*b*h1)*(h1/3),-1);     % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N12
lmiterm([1 2 1 y11],h1*e^(-2*b*h1),1);               % LMI #1: h1*e^(-
2*b*h1)*y11
lmiterm([1 2 1 w11],h1*e^(-2*b*h1),1);               % LMI #1: h1*e^(-
2*b*h1)*w11
lmiterm([1 2 1 S2],1,-A);                             % LMI #1: -S2*A
lmiterm([1 2 1 -M21],h1*e^(-2*b*h1)*h1,-1);          % LMI #1: -h1*e^(-
2*b*h1)*h1*M21'
lmiterm([1 2 1 -N21],h1*e^(-2*b*h1)*(h1/3),-1);     % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N21'
lmiterm([1 2 1 -y21],h1*e^(-2*b*h1),-1);             % LMI #1: -h1*e^(-
2*b*h1)*y21'
lmiterm([1 2 1 -w21],h1*e^(-2*b*h1),1);              % LMI #1: h1*e^(-
2*b*h1)*w21'
lmiterm([1 2 2 Q],.5*e^(2*b*h1),1,'s');              % LMI #1: e^(2*b*h1)*Q
(NON SYMMETRIC?)
lmiterm([1 2 2 M22],.5*h1*e^(-2*b*h1)*h1,-1,'s');     % LMI #1: -
h1*e^(-2*b*h1)*h1*M22 (NON SYMMETRIC?)
lmiterm([1 2 2 N22],.5*h1*e^(-2*b*h1)*(h1/3),-1,'s');     % LMI #1: -
h1*e^(-2*b*h1)*(h1/3)*N22 (NON SYMMETRIC?)
lmiterm([1 2 2 y21],.5*0.5*h1*e^(-2*b*h1),-1,'s');     % LMI #1: -
0.5*h1*e^(-2*b*h1)*y21 (NON SYMMETRIC?)
lmiterm([1 2 2 -y21],.5*0.5*h1*e^(-2*b*h1),-1,'s');     % LMI #1: -
0.5*h1*e^(-2*b*h1)*y21' (NON SYMMETRIC?)
lmiterm([1 2 2 w21],.5*0.5*h1*e^(-2*b*h1),1,'s');     % LMI #1:
0.5*h1*e^(-2*b*h1)*w21 (NON SYMMETRIC?)
lmiterm([1 2 2 -w21],.5*0.5*h1*e^(-2*b*h1),1,'s');     % LMI #1:
```

```
0.5*h1*e^(-2*b*h1)*w21' (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h2*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*h2*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h2*e^(-2*b*h2)*h1,1,'s');      % LMI #1: h2*e^(-
2*b*h2)*h1*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h2*e^(-2*b*h2)*(h1/3),1,'s');      % LMI #1:
h2*e^(-2*b*h2)*(h1/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 y11],.5*h2*e^(-2*b*h2),-1,'s');      % LMI #1: -h2*e^(-
2*b*h2)*y11 (NON SYMMETRIC?)
lmiterm([1 2 2 w11],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*w11 (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h1*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*h2*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 M11],.5*h1*e^(-2*b*h2)*h1,1,'s');      % LMI #1: h1*e^(-
2*b*h2)*h1*M11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h1*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*(h2/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 N11],.5*h1*e^(-2*b*h2)*(h1/3),1,'s');      % LMI #1:
h1*e^(-2*b*h2)*(h1/3)*N11 (NON SYMMETRIC?)
lmiterm([1 2 2 y11],.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -h1*e^(-
2*b*h2)*y11 (NON SYMMETRIC?)
lmiterm([1 2 2 w11],.5*h1*e^(-2*b*h2),1,'s');      % LMI #1: h1*e^(-
2*b*h2)*w11 (NON SYMMETRIC?)
lmiterm([1 3 1 M12],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M12
lmiterm([1 3 1 N12],h2*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h1/3)*N12
lmiterm([1 3 1 y11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*y11
lmiterm([1 3 1 w11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*w11
lmiterm([1 3 1 S3],1,-A);                        % LMI #1: -S3*A
lmiterm([1 3 1 -M21],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M21'
lmiterm([1 3 1 -N21],h2*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h1/3)*N21'
lmiterm([1 3 1 -y21],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*y21'
lmiterm([1 3 1 -w21],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w21'
lmiterm([1 3 2 M12],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M12
lmiterm([1 3 2 M12],h2*e^(-2*b*h2)*h1,1);        % LMI #1: h2*e^(-
2*b*h2)*h1*M12
lmiterm([1 3 2 N12],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N12
lmiterm([1 3 2 N12],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N12
lmiterm([1 3 2 y11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*y11
lmiterm([1 3 2 w11],h2*e^(-2*b*h2),1);           % LMI #1: h2*e^(-
2*b*h2)*w11
lmiterm([1 3 2 M11],h1*e^(-2*b*h2)*h2,-1);       % LMI #1: -h1*e^(-
2*b*h2)*h2*M11
lmiterm([1 3 2 M11],h1*e^(-2*b*h2)*h1,-1);       % LMI #1: -h1*e^(-
2*b*h2)*h1*M11
```

```
lmiterm([1 3 2 N12],h1*e^(-2*b*h2)*(h2/3),1);      % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N12
lmiterm([1 3 2 N12],h1*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N12
lmiterm([1 3 2 y11],h1*e^(-2*b*h2),-1);          % LMI #1: -h1*e^(-
2*b*h2)*y11
lmiterm([1 3 2 w11],h1*e^(-2*b*h2),-1);          % LMI #1: -h1*e^(-
2*b*h2)*w11
lmiterm([1 3 2 -M12],h2*e^(-2*b*h2)*h1,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h1*M12'
lmiterm([1 3 2 -M21],h2*e^(-2*b*h2)*h1,1);       % LMI #1: h2*e^(-
2*b*h2)*h1*M21'
lmiterm([1 3 2 -N21],h2*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N21'
lmiterm([1 3 2 -N21],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N21'
lmiterm([1 3 2 -y21],h2*e^(-2*b*h2),-1);         % LMI #1: -h2*e^(-
2*b*h2)*y21'
lmiterm([1 3 2 -w21],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w21'
lmiterm([1 3 2 -M21],h1*e^(-2*b*h2)*h2,-1);      % LMI #1: -h1*e^(-
2*b*h2)*h2*M21'
lmiterm([1 3 2 -M21],h1*e^(-2*b*h2)*h1,1);       % LMI #1: h1*e^(-
2*b*h2)*h1*M21'
lmiterm([1 3 2 -N21],h1*e^(-2*b*h2)*(h2/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h2/3)*N21'
lmiterm([1 3 2 -N21],h2*e^(-2*b*h2)*(h1/3),1);      % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N21'
lmiterm([1 3 2 -y21],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*y21'
lmiterm([1 3 2 -w21],h1*e^(-2*b*h2),1);          % LMI #1: h1*e^(-
2*b*h2)*w21'
lmiterm([1 3 2 Q],0.5*e^(2*b*h1),-1);            % LMI #1: -
0.5*e^(2*b*h1)*Q
lmiterm([1 3 2 Q],0.5*e^(2*b*h2),-1);            % LMI #1: -
0.5*e^(2*b*h2)*Q
lmiterm([1 3 3 Q],.5*e^(-2*b*h2),-1,'s');        % LMI #1: -e^(-
2*b*h2)*Q (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h2*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*h2*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 y21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*y21 (NON SYMMETRIC?)
lmiterm([1 3 3 w21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*w21 (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h2*e^(-2*b*h2)*h2,-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*h2*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h2*e^(-2*b*h2)*h1,1,'s');      % LMI #1: h2*e^(-
2*b*h2)*h1*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');      % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h2*e^(-2*b*h2)*(h1/3),1,'s');      % LMI #1:
h2*e^(-2*b*h2)*(h1/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 -y21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*y21' (NON SYMMETRIC?)
lmiterm([1 3 3 -w21],.5*h2*e^(-2*b*h2),1,'s');      % LMI #1: h2*e^(-
2*b*h2)*w21' (NON SYMMETRIC?)
```

```
lmiterm([1 3 3 M22],.5*h1*e^(-2*b*h2)*h2,1,'s');      % LMI #1: h1*e^(-
2*b*h2)*h2*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 M22],.5*h1*e^(-2*b*h2)*h1,-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*h1*M22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h1*e^(-2*b*h2)*(h2/3),1,'s');      % LMI #1:
h1*e^(-2*b*h2)*(h2/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 N22],.5*h1*e^(-2*b*h2)*(h1/3),-1,'s');      % LMI #1: -
h1*e^(-2*b*h2)*(h1/3)*N22 (NON SYMMETRIC?)
lmiterm([1 3 3 y21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*y21 (NON SYMMETRIC?)
lmiterm([1 3 3 -y21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*y21' (NON SYMMETRIC?)
lmiterm([1 3 3 w21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*w21 (NON SYMMETRIC?)
lmiterm([1 3 3 -w21],.5*0.5*h1*e^(-2*b*h2),-1,'s');      % LMI #1: -
0.5*h1*e^(-2*b*h2)*w21' (NON SYMMETRIC?)
lmiterm([1 4 1 P],0.5*D',1);                         % LMI #1: 0.5*D'*P
lmiterm([1 4 1 P],0.5,D);                            % LMI #1: 0.5*P*D
lmiterm([1 4 1 -S1],D',-1);                          % LMI #1: -D'*S1'
lmiterm([1 4 1 S4],1,-A);                            % LMI #1: -S4*A
lmiterm([1 4 2 -S2],D',-1);                          % LMI #1: -D'*S2'
lmiterm([1 4 3 -S3],D',-1);                          % LMI #1: -D'*S3'
lmiterm([1 4 4 S4],1,-D,'s');                        % LMI #1: -S4*D-D'*S4'
lmiterm([1 5 1 M13],h1*e^(-2*b*h1)*h1,-1);           % LMI #1: -h1*e^(-
2*b*h1)*h1*M13
lmiterm([1 5 1 N13],h1*e^(-2*b*h1)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N13
lmiterm([1 5 1 w11],h1*e^(-2*b*h1)*2,-1);            % LMI #1: -h1*e^(-
2*b*h1)*2*w11
lmiterm([1 5 1 S5],1,-A);                            % LMI #1: -S5*A
lmiterm([1 5 1 -M31],h1*e^(-2*b*h1)*h1,-1);          % LMI #1: -h1*e^(-
2*b*h1)*h1*M31'
lmiterm([1 5 1 -N31],h1*e^(-2*b*h1)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N31'
lmiterm([1 5 1 -y31],h1*e^(-2*b*h1),-1);             % LMI #1: -h1*e^(-
2*b*h1)*y31'
lmiterm([1 5 1 -w31],h1*e^(-2*b*h1),1);              % LMI #1: h1*e^(-
2*b*h1)*w31'
lmiterm([1 5 2 M23],h1*e^(-2*b*h1)*h1,-1);           % LMI #1: -h1*e^(-
2*b*h1)*h1*M23
lmiterm([1 5 2 N23],h1*e^(-2*b*h1)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N23
lmiterm([1 5 2 w21],h1*e^(-2*b*h1)*2,-1);            % LMI #1: -h1*e^(-
2*b*h1)*2*w21
lmiterm([1 5 2 -M32],h1*e^(-2*b*h1)*h1,-1);          % LMI #1: -h1*e^(-
2*b*h1)*h1*M32'
lmiterm([1 5 2 -N32],h1*e^(-2*b*h1)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h1)*(h1/3)*N32'
lmiterm([1 5 2 -y31],h1*e^(-2*b*h1),1);              % LMI #1: h1*e^(-
2*b*h1)*y31'
lmiterm([1 5 2 -w31],h1*e^(-2*b*h1),1);              % LMI #1: h1*e^(-
2*b*h1)*w31'
lmiterm([1 5 3 0],O);                                % LMI #1: O
lmiterm([1 5 4 S5],1,-D);                            % LMI #1: -S5*D
lmiterm([1 5 5 M33],.5*h1*e^(-2*b*h1)*h1,-1,'s');      % LMI #1: -
h1*e^(-2*b*h1)*h1*M33 (NON SYMMETRIC?)
lmiterm([1 5 5 N33],.5*h1*e^(-2*b*h1)*(h1/3),-1,'s');      % LMI #1: -
h1*e^(-2*b*h1)*(h1/3)*N33 (NON SYMMETRIC?)
```

```
lmiterm([1 5 5 w31],.5*h1*e^(-2*b*h1),-1,'s');     % LMI #1: -h1*e^(-
2*b*h1)*w31 (NON SYMMETRIC?)
lmiterm([1 5 5 -w31],.5*h1*e^(-2*b*h1),-1,'s');     % LMI #1: -h1*e^(-
2*b*h1)*w31' (NON SYMMETRIC?)
lmiterm([1 6 1 M13],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M13
lmiterm([1 6 1 N13],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N13
lmiterm([1 6 1 w11],h2*e^(-2*b*h2)*2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*2*w11
lmiterm([1 6 1 S6],1,-A);                       % LMI #1: -S6*A
lmiterm([1 6 1 -M31],h2*e^(-2*b*h2)*h2,-1);     % LMI #1: -h2*e^(-
2*b*h2)*h2*M31'
lmiterm([1 6 1 -N31],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N31'
lmiterm([1 6 1 -y31],h2*e^(-2*b*h2),-1);        % LMI #1: -h2*e^(-
2*b*h2)*y31'
lmiterm([1 6 1 -w31],h2*e^(-2*b*h2),1);         % LMI #1: h2*e^(-
2*b*h2)*w31'
lmiterm([1 6 2 0],0);                           % LMI #1: 0
lmiterm([1 6 3 M23],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M23
lmiterm([1 6 3 N23],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N23
lmiterm([1 6 3 w11],h2*e^(-2*b*h2)*2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*2*w11
lmiterm([1 6 3 -M32],h2*e^(-2*b*h2)*h2,-1);     % LMI #1: -h2*e^(-
2*b*h2)*h2*M32'
lmiterm([1 6 3 -N32],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N32'
lmiterm([1 6 3 -y31],h2*e^(-2*b*h2),1);         % LMI #1: h2*e^(-
2*b*h2)*y31'
lmiterm([1 6 3 w31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w31
lmiterm([1 6 4 S6],1,-D);                       % LMI #1: -S6*D
lmiterm([1 6 5 0],0);                           % LMI #1: 0
lmiterm([1 6 6 M33],.5*h2*e^(-2*b*h2)*h2,-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*h2*M33 (NON SYMMETRIC?)
lmiterm([1 6 6 N33],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N33 (NON SYMMETRIC?)
lmiterm([1 6 6 w31],.5*h2*e^(-2*b*h2),-1,'s');     % LMI #1: -h2*e^(-
2*b*h2)*w31 (NON SYMMETRIC?)
lmiterm([1 6 6 -w31],.5*h2*e^(-2*b*h2),-1,'s');     % LMI #1: -h2*e^(-
2*b*h2)*w31' (NON SYMMETRIC?)
lmiterm([1 7 1 S7],1,-A);                       % LMI #1: -S7*A
lmiterm([1 7 2 M13],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M13
lmiterm([1 7 2 M13],h2*e^(-2*b*h2)*h1,1);       % LMI #1: h2*e^(-
2*b*h2)*h1*M13
lmiterm([1 7 2 N13],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N13
lmiterm([1 7 2 N13],h2*e^(-2*b*h2)*(h1/3),1);     % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N13
lmiterm([1 7 2 w11],h2*e^(-2*b*h2)*2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*2*w11
lmiterm([1 7 2 M13],h1*e^(-2*b*h2)*h2,1);       % LMI #1: h1*e^(-
2*b*h2)*h2*M13
lmiterm([1 7 2 M13],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
```

```
2*b*h2)*M13
lmiterm([1 7 2 N13],h1*e^(-2*b*h2)*(h2/3),1);     % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N13
lmiterm([1 7 2 N13],h1*e^(-2*b*h2)*(h1/3),-1);     % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N13
lmiterm([1 7 2 w11],h1*e^(-2*b*h2)*2,1);         % LMI #1: h1*e^(-
2*b*h2)*2*w11
lmiterm([1 7 2 -M31],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M31'
lmiterm([1 7 2 -M31],h2*e^(-2*b*h2)*h1,1);       % LMI #1: h2*e^(-
2*b*h2)*h1*M31'
lmiterm([1 7 2 -N31],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N31'
lmiterm([1 7 2 -N31],h2*e^(-2*b*h2)*(h1/3),1);     % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N31'
lmiterm([1 7 2 -y31],h2*e^(-2*b*h2),-1);         % LMI #1: -h2*e^(-
2*b*h2)*y31'
lmiterm([1 7 2 -w31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w31'
lmiterm([1 7 2 -M31],h1*e^(-2*b*h2)*h2,1);       % LMI #1: h1*e^(-
2*b*h2)*h2*M31'
lmiterm([1 7 2 -M31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*M31'
lmiterm([1 7 2 -N31],h1*e^(-2*b*h2)*(h2/3),1);     % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N31'
lmiterm([1 7 2 -N31],h1*e^(-2*b*h2)*(h1/3),-1);     % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N31'
lmiterm([1 7 2 -y31],h1*e^(-2*b*h2),1);          % LMI #1: h1*e^(-
2*b*h2)*y31'
lmiterm([1 7 2 -w31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*w31'
lmiterm([1 7 3 M23],h2*e^(-2*b*h2)*h2,-1);       % LMI #1: -h2*e^(-
2*b*h2)*h2*M23
lmiterm([1 7 3 M23],h2*e^(-2*b*h2)*h1,1);        % LMI #1: h2*e^(-
2*b*h2)*h1*M23
lmiterm([1 7 3 N23],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N23
lmiterm([1 7 3 N23],h2*e^(-2*b*h2)*(h1/3),1);     % LMI #1: h2*e^(-
2*b*h2)*(h1/3)*N23
lmiterm([1 7 3 w21],h2*e^(-2*b*h2)*2,-1);        % LMI #1: -h2*e^(-
2*b*h2)*2*w21
lmiterm([1 7 3 M23],h1*e^(-2*b*h2)*h2,1);        % LMI #1: h1*e^(-
2*b*h2)*h2*M23
lmiterm([1 7 3 M23],h1*e^(-2*b*h2)*h1,-1);       % LMI #1: -h1*e^(-
2*b*h2)*h1*M23
lmiterm([1 7 3 N23],h1*e^(-2*b*h2)*(h2/3),1);     % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N23
lmiterm([1 7 3 N23],h1*e^(-2*b*h2)*(h1/3),-1);     % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N23
lmiterm([1 7 3 w21],h1*e^(-2*b*h2)*2,1);         % LMI #1: h1*e^(-
2*b*h2)*2*w21
lmiterm([1 7 3 -M32],h2*e^(-2*b*h2)*h2,-1);      % LMI #1: -h2*e^(-
2*b*h2)*h2*M32'
lmiterm([1 7 3 -M32],h2*e^(-2*b*h2)*h1,1);       % LMI #1: h2*e^(-
2*b*h2)*h1*M32'
lmiterm([1 7 3 -N32],h2*e^(-2*b*h2)*(h2/3),-1);     % LMI #1: -h2*e^(-
2*b*h2)*(h2/3)*N32'
lmiterm([1 7 3 -N32],h2*e^(-2*b*h2)*(h1/3),1);     % LMI #1: h2*e^(-
```

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2*b*h2)*(h1/3)*N32'
lmiterm([1 7 3 -y31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*y31'
lmiterm([1 7 3 -w31],h2*e^(-2*b*h2),1);          % LMI #1: h2*e^(-
2*b*h2)*w31'
lmiterm([1 7 3 -M32],h1*e^(-2*b*h2)*h2,1);       % LMI #1: h1*e^(-
2*b*h2)*h2*M32'
lmiterm([1 7 3 -M32],h1*e^(-2*b*h2)*h1,-1);      % LMI #1: -h1*e^(-
2*b*h2)*h1*M32'
lmiterm([1 7 3 -N32],h1*e^(-2*b*h2)*(h2/3),1);      % LMI #1: h1*e^(-
2*b*h2)*(h2/3)*N32'
lmiterm([1 7 3 -N32],h1*e^(-2*b*h2)*(h1/3),-1);      % LMI #1: -h1*e^(-
2*b*h2)*(h1/3)*N32'
lmiterm([1 7 3 -y31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*y31'
lmiterm([1 7 3 -w31],h1*e^(-2*b*h2),-1);         % LMI #1: -h1*e^(-
2*b*h2)*w31'
lmiterm([1 7 4 S7],1,-D);                        % LMI #1: -S7*D
lmiterm([1 7 5 0],0);                            % LMI #1: 0
lmiterm([1 7 6 0],0);                            % LMI #1: 0
lmiterm([1 7 7 M33],.5*h2*e^(-2*b*h2)*h2,-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*h2*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 M33],.5*h2*e^(-2*b*h2)*h1,1,'s');     % LMI #1: h2*e^(-
2*b*h2)*h1*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h2*e^(-2*b*h2)*(h2/3),-1,'s');     % LMI #1: -
h2*e^(-2*b*h2)*(h2/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h2*e^(-2*b*h2)*(h1/3),1,'s');     % LMI #1:
h2*e^(-2*b*h2)*(h1/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 w31],.5*h2*e^(-2*b*h2),-1,'s');     % LMI #1: -h2*e^(-
2*b*h2)*w31 (NON SYMMETRIC?)
lmiterm([1 7 7 -w31],.5*h2*e^(-2*b*h2),-1,'s');     % LMI #1: -h2*e^(-
2*b*h2)*w31' (NON SYMMETRIC?)
lmiterm([1 7 7 M33],.5*h1*e^(-2*b*h2)*h2,1,'s');     % LMI #1: h1*e^(-
2*b*h2)*h2*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 M33],.5*h1*e^(-2*b*h2)*h1,-1,'s');     % LMI #1: -
h1*e^(-2*b*h2)*h1*M33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h1*e^(-2*b*h2)*(h2/3),1,'s');     % LMI #1:
h1*e^(-2*b*h2)*(h2/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 N33],.5*h1*e^(-2*b*h2)*(h1/3),-1,'s');     % LMI #1: -
h1*e^(-2*b*h2)*(h1/3)*N33 (NON SYMMETRIC?)
lmiterm([1 7 7 w31],.5*h1*e^(-2*b*h2),1,'s');     % LMI #1: h1*e^(-
2*b*h2)*w31 (NON SYMMETRIC?)
lmiterm([1 7 7 -w31],.5*h1*e^(-2*b*h2),1,'s');     % LMI #1: h1*e^(-
2*b*h2)*w31' (NON SYMMETRIC?)
lmiterm([1 8 1 -S1],1,1);                        % LMI #1: S1'
lmiterm([1 8 1 S8],1,-A);                        % LMI #1: -S8*A
lmiterm([1 8 2 -S2],1,1);                        % LMI #1: S2'
lmiterm([1 8 3 -S3],1,1);                        % LMI #1: S3'
lmiterm([1 8 4 -S4],1,1);                        % LMI #1: S4'
lmiterm([1 8 4 S8],1,-D);                        % LMI #1: -S8*D
lmiterm([1 8 5 -S5],1,1);                        % LMI #1: S5'
lmiterm([1 8 6 S6],1,1);                         % LMI #1: S6
lmiterm([1 8 7 S7],1,1);                         % LMI #1: S7
lmiterm([1 8 8 R],.5*h1^2,1,'s');                % LMI #1: h1^2*R (NON
SYMMETRIC?)
lmiterm([1 8 8 R],.5*h2^2,-1,'s');               % LMI #1: -h2^2*R (NON
SYMMETRIC?)
lmiterm([1 8 8 U],.5*h2^2,1,'s');                % LMI #1: h2^2*U (NON
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  SYMMETRIC?)
  lmiterm([1 8 8 U],.5*2*h2*h1,1,'s');           % LMI #1: 2*h2*h1*U
  (NON SYMMETRIC?)
  lmiterm([1 8 8 U],.5*h1^2,-1,'s');             % LMI #1: -h1^2*U (NON
  SYMMETRIC?)
  lmiterm([1 8 8 S8],1,1,'s');                   % LMI #1: S8+S8'

  lmiterm([-2 1 1 P],1,1);                       % LMI #2: P

  lmiterm([-3 1 1 Q],1,1);                       % LMI #3: Q

  lmiterm([-4 1 1 R],1,1);                       % LMI #4: R

  lmiterm([-5 1 1 U],1,1);                       % LMI #5: U

  lmiterm([-6 1 1 S6],1,1);                      % LMI #6: S6

  lmiterm([-7 1 1 S7],1,1);                      % LMI #7: S7

  lmiterm([-8 1 1 S8],1,1);                      % LMI #8: S8

  lmiterm([-9 1 1 M11],1,1);                     % LMI #9: M11

  lmiterm([-10 1 1 M22],1,1);                    % LMI #10: M22

  lmiterm([-11 1 1 M33],1,1);                    % LMI #11: M33

  lmiterm([-12 1 1 N11],1,1);                    % LMI #12: N11

  lmiterm([-13 1 1 N22],1,1);                    % LMI #13: N22

  lmiterm([-14 1 1 N33],1,1);                    % LMI #14: N33

  t=getlmis;

  [tmin,xfeas]=feasp(t)
  P=dec2mat(t,xfeas,P);
Q =dec2mat(t,xfeas,Q);
R =dec2mat(t,xfeas,R);
U =dec2mat(t,xfeas,U);
S1=dec2mat(t,xfeas,S1);
S2=dec2mat(t,xfeas,S2);
S3=dec2mat(t,xfeas,S3);
S4=dec2mat(t,xfeas,S5);
S6=dec2mat(t,xfeas,S6);
S7=dec2mat(t,xfeas,S7);
S8=dec2mat(t,xfeas,S8);
M11=dec2mat(t,xfeas,M11);
M21=dec2mat(t,xfeas,M21);
M31=dec2mat(t,xfeas,M31);
M22=dec2mat(t,xfeas,M22);
M32=dec2mat(t,xfeas,M32);
M12=dec2mat(t,xfeas,M12);
M13=dec2mat(t,xfeas,M13);
M23=dec2mat(t,xfeas,M23);
```

```
M33=dec2mat(t,xfeas,M33);
N11=dec2mat(t,xfeas,N11);
N12=dec2mat(t,xfeas,N12);
N13=dec2mat(t,xfeas,N13);
N21=dec2mat(t,xfeas,N21);
N22=dec2mat(t,xfeas,N22);
N23=dec2mat(t,xfeas,N23);
N31=dec2mat(t,xfeas,N31);
N32=dec2mat(t,xfeas,N32);
N33=dec2mat(t,xfeas,N33);
y11=dec2mat(t,xfeas,y11);
y12=dec2mat(t,xfeas,y12);
y13=dec2mat(t,xfeas,y13);
y21=dec2mat(t,xfeas,y21);
y22=dec2mat(t,xfeas,y22);
y23=dec2mat(t,xfeas,y23);
y31=dec2mat(t,xfeas,y31);
y32=dec2mat(t,xfeas,y32);
y33=dec2mat(t,xfeas,y33);
w11=dec2mat(t,xfeas,w11);
w12=dec2mat(t,xfeas,w12);
w13=dec2mat(t,xfeas,w13);
w21=dec2mat(t,xfeas,w21);
w22=dec2mat(t,xfeas,w22);
w23=dec2mat(t,xfeas,w23);
w31=dec2mat(t,xfeas,w31);
w32=dec2mat(t,xfeas,w32);
w33=dec2mat(t,xfeas,w33);

tmin
```

BIOGRAPHY

BIOGRAPHY

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Exponential stability of linear systems with interval time-varying delays using a new bounding technique 2018